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Title:
Minimal mass blow-up solutions of the L^2 critical NLS with inverse-square potential

Abstract:
I will discuss minimal mass (L^2 norm) blow-up solutions of the focusing L^2 critical nonlinear Schrödinger equation with inverse-square potential,

$$i\partial_t u + \Delta u + \frac{c}{|x|^2}u + |u|^{\frac{4}{N}}u = 0,$$

with $N \geq 3$ and $0 < c < (N-2)^2/4$. First, solutions with a mass strictly below that of the ground state(s) are global. (Uniqueness of the ground state is not known when $c \neq 0$.) Secondly, all solutions with ground state mass which blow in finite time are classified using the pseudo-conformal invariance of the equation. The classification result relies on a subtle combination of Hardy's inequality with a suitable Gagliardo–Nirenberg inequality. This joint work with Elek Csobo extends the famous results of Weinstein [2] and Merle [1], from the case $c = 0$ to $0 < c < (N-2)^2/4$.

References

[1] F. Merle, Determination of blow-up solutions with minimal mass for nonlinear Schrödinger equations with critical power, *Duke Math. J.* **69** (1993), no. 2, 427–454.

[2] M.I. Weinstein, Nonlinear Schrödinger equations and sharp interpolation estimates, *Comm. Math. Phys.* **87** (1982), 567–576.