

# Convex relaxation for a class of free boundary problems

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This lecture is based on a duality scheme for non convex variational problems which has been developed in collaboration with I Fragala (2016 Arxiv) and Minh Pahn (Toulon). Typically this theory concerns minimum problems of the kind

$$\inf \left\{ \int_{\Omega} (|\nabla u|^p + g(u)) dx \quad : \quad u \in W^{1,p}(\Omega) , u = u_0 \text{ on } \partial\Omega \right\}$$

where  $g : \mathbb{R} \rightarrow \mathbb{R} \cup \{+\infty\}$  is a non convex l.s.c function with possibly many jumps. In this talk, I will report on two recent related results specific to the case  $p = 1$ :

- the first one concerns an exclusion principle which states that minimizers take values outside the set  $\{g^{**} < g\}$ . This principle allows convex relaxation and then we focus on a multiphase problem that we treat numerically by means of a primal-dual algorithm.
- the second one concerns a variant of the Cheeger problem in a convex subset  $D \subset \mathbb{R}^2$  for which we construct explicit calibrating fields. This is done by using a locally Lipschitz potential whose trace on  $\partial D$  coincides with the normal distance to the cut-locus.