Optimal shape and location of sensors or actuators in PDE models

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We consider the problem of optimizing the shape and the location of sensors or actuators for systems whose evolution is driven by a linear PDE model. This problem is frequently encountered in applications where one wants for instance to maximize the quality of the reconstruction of solutions by using only partial observations. For example, we model and solve the following informal question: What is the optimal shape and location of a thermometer? We stress that we want to optimize not only the placement but also the shape of the observation domain, over the class of all possible measurable subsets of the domain having a prescribed measure. We model this optimal design problem as the one of maximizing a functional that we call the randomized observability constant, which reflects what happens for random initial data, and which is of a spectral nature. Solving this problem is then strongly dependent on the PDE model under consideration. For parabolic equations, we prove the existence and uniqueness of a best domain, regular enough, and whose algorithmic construction depends in general on a finite number of modes. In contrast, for wave or Schrodinger equations, relaxation may occur, and our analysis reveals intimate relations with quantum chaos, more precisely with quantum ergodicity properties of the eigenfunctions. These works are in collaboration with Y. Privat and E. Zuazua.