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Title: Facets of tensor cone of symmetrizable Kac-Moody Lie algebras

Abstract: This is a joint work with Nicolas Ressayre. In this work, we are interested in the decomposition of the saturated tensor product of two representations of a symmetrizable Kac-Moody Lie algebra $\mathfrak{g}$. Let P_+ be the set of dominant integral weights. For $\lambda \in P_+$, $L(\lambda)$ denotes the (irreducible) integrable, highest weight representation of $\mathfrak{g}$ with highest weight λ. Let $P_{+,\mathbb{Q}}$ be the rational convex cone generated by P_+. Consider the *saturated tensor cone*

$$\Gamma(\mathfrak{g}) := \{(\lambda_1, \lambda_2, \mu) \in P^3_{+,\mathbb{Q}} \mid \exists N \geq 1 \text{ with } L(N\mu) \subset L(N\lambda_1) \otimes L(N\lambda_2)\}.$$

If $\mathfrak{g}$ is finite dimensional, $\Gamma(\mathfrak{g})$ is a convex polyhedral cone described by Belkale-Kumar by an explicit finite list of inequalities, which was proved to be an irredundant set of inequalities by Ressayre. In general, $\Gamma(\mathfrak{g})$ is neither polyhedral, nor closed. Brown-Kumar obtained a list of inequalities that describe $\Gamma(\mathfrak{g})$ conjecturally. In this work, we prove that each of the Brown-Kumar's inequalities corresponds to a codimension one face of $\Gamma(\mathfrak{g})$. Ressayre has proved that this set of inequalities is sufficient in the case of affine Kac-Moody Lie algebras. The sufficiency question is still open for non-affine $\mathfrak{g}$.