Long time asymptotics for solutions of the Short Pulse Equation

Lech Zielinski

The non-linear Schrödinger (NLS) equation is one of the universal integrable models describing the slow modulation of the amplitude of a weakly nonlinear wave packet in a moving medium. However in the case of ultra-short pulses in high-speed fiber-optic communication the NLS model should be replaced by a short pulse (SP) model which can be reduced to Cauchy problem of studying \( u : \mathbb{R}^2 \to \mathbb{R} \) such that

\[
\begin{cases}
  u_{x_1} = u + \frac{1}{6} (u^3)_{x_2} \\
  u(x, 0) = u_0(x)
\end{cases}
\]

We assume that \( u_0(x) \) is rapidly decaying as \( |x| \to \infty \), and we are looking for the solution \( u(x, t) \) which is also rapidly decaying as \( |x| \to \infty \), for any fixed \( t \). Our purpose is to investigate the asymptotic behavior of \( u(x, t) \) for large time \( t \) using an adaptation of the inverse scattering transform method, in the form of a Riemann–Hilbert factorization problem. We explain how to obtain different types of asymptotics: rapidly or slowly decaying solutions, soliton type solutions or wave breaking. The talk is based on a joint work with A. Boutet de Monvel (Institut de Mathématiques de Jussieu) et D. Shepelsky (V. N. Karazin Kharkiv National University, Ukraine).