Long time asymptotics for solutions of the Short Pulse Equation

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The non-linear Schrödinger (NLS) equation is one of the universal integrable models describing the slow modulation of the amplitude of a weakly nonlinear wave packet in a moving medium. However in the case of ultra-short pulses in high-speed fiber-optic communication the NLS model should be replaced by a short pulse (SP) model which can be reduced to Cauchy problem of studying $u : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\begin{cases} u_{xt} = u + \frac{1}{6}(u^3)_{xx} \\ u(x,0) = u_0(x) \end{cases}$$

We assume that $u_0(x)$ is rapidly decaying as $|x| \to \infty$, and we are looking for the solution $u(x,t)$ which is also rapidly decaying as $|x| \to \infty$, for any fixed t. Our purpose is to investigate the asymptotic behavior of $u(x,t)$ for large time t using an adaptation of the inverse scattering transform method, in the form of a Riemann–Hilbert factorization problem. We explain how to obtain different types of asymptotics: rapidly or slowly decaying solutions, soliton type solutions or wave breaking. The talk is based on a joint work with A. Boutet de Monvel (Institut de Mathématiques de Jussieu) et D. Shepelsky (V. N. Karazin Kharkiv National University, Ukraine).