A Multilevel Monte Carlo Method for Kinetic Transport Equations using Asymptotic-Preserving Particle Schemes

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Many applications in particle simulation suffer from time-scale separation. This means that we have to perform simulations with a small time step, in order to track the system's fast dynamics, but also have to simulate over a long time-horizon, in order to capture the system's slow dynamics. In hyperbolic transport equations, a scaling parameter ε (related to the mean free particle path) is commonly used to characterize this time-scale separation. As this scaling parameter tends to zero, the hyperbolic transport equation converges, in the limit, to a parabolic diffusion equation. Simulations of the transport equation in this small ε region however, suffer from extreme time step reduction constraints to maintain stability.

In [1], a new Monte Carlo scheme was developed that converges in the diffusive limit, while avoiding these stability constraints, this is achieved at the cost of a linear model error in the time step size. In this new work, we apply the multilevel Monte Carlo method [2] to this scheme, replacing a precise, but expensive, Monte Carlo simulation with a cheap, but biased, simulation and a sequence of corrections, based on correlated simulations with different precisions, to remove the bias. The bias corrections and variances of differences of coupled simulations in this scheme have a different structure from that which is typically observed in multilevel Monte Carlo applications. This poster will showcase these new derived results, as well as experimental results.

References

[1] G. Dimarco, L. Pareschi and G. Samaey. Asymptotic-Preserving Monte Carlo methods for transport equations in the diffusive limit. *SIAM Journal on Scientific Computing*, 40: A504–A528, 2018.

[2] M.B. Giles Multilevel Monte Carlo methods. *Acta Numerica*, 24: 259–328, 2015.