Time asymptotic behavior for singular neutron transport equation with bounce-back boundary conditions in $L^1$ spaces

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Abstract: This communication is concerned with the well-posedness of a Cauchy problem governed by a singular neutron transport equation in $L^1$ spaces with bounce-back boundary conditions. To be more precise, we are concerned with the following equation

\[
\begin{aligned}
\frac{\partial \psi}{\partial t}(x,v,t) &= A_H \psi(x,v,t) := T_H \psi(x,v,t) + K \psi(x,v,t) \\
&= -v \nabla_x \psi(x,v,t) - \sigma(v) \psi(x,v,t) + \int_{\mathbb{R}^n} \kappa(x,v,v') \psi(x,v',t) dv', \\
\psi(x,v,0) &= \psi_0(x,v),
\end{aligned}
\]

where $(x,v) \in D \times \mathbb{R}^n$ and $K$ is the partial integral part of $A_H$, it is called the collision operator. Here $D$ is a bounded open subset of $\mathbb{R}^n$. As usually in transport theory, the function $\psi(x,v,t)$ represents the number (or probability) density of gas particles having the position $x$ and the velocity $v$ at the time $t$. The functions $\sigma(\cdot)$ and $\kappa(\cdot, \cdot, \cdot)$ are called, respectively, the collision frequency and the scattering kernel. We prove the weak compactness of the second-order remainder term of the Dyson-Phillips expansion which implies that the essential types of the streaming semigroup $T_H$ and to that of the transport semigroup $A_H$ coincide and we derive, via classical arguments, the time asymptotic behavior of the solution.