Abstract. **Spectral theory of a transport equation with elastic and inelastic collision operators** .

We are concerned with the time asymptotic behavior of the solution to the following Cauchy problem for partly elastic collisions operators, that is,

$$\frac{\partial \varphi}{\partial t}(x,v,t) = -v.\frac{\partial \varphi}{\partial x}(x,v,t) - \sigma(v)\varphi(x,v,t) + \int_V k_c(x,v,v')\varphi(x,v',t)dv'$$
$$+ \sum_{j=1}^{l} \int_{\mathbb{S}^{N-1}} k_d^j(x,\rho_j,\omega,\omega')\varphi(x,\rho_j\omega',t)d\omega'$$
$$+ \int_{\mathbb{S}^{N-1}} k_e(x,\rho,\omega,\omega')\phi(x,\rho\omega',t)d\omega' \tag{1}$$

with the initial distribution

$$\varphi(x,v,0) = \varphi_0(x,v). \tag{2}$$

and we restrict our selves to periodic boundary conditions, that is,

$$\varphi(x,v,t) \mid_{x_i=-a_i} = \varphi(x,v,t) \mid_{x_i=a_i}, \tag{3}$$

where $\Omega = \prod_{i=1}^{N}(-a_i,a_i)$, $a_i > 0, i = 1,\cdots,N$, is an open paved of $\mathbb{R}^N$. The set $V \subset \mathbb{R}^N$, is called the space of admissible velocities, $v = \rho\omega \in V =: I \times \mathbb{S}^{N-1}$ with $\omega \in \mathbb{S}^{N-1}$, $\rho \in I := (\rho_{\min},\rho_{\max})$ and $\mu(\cdot)$ is a positive Radon measure on $\mathbb{R}^N$, we denote by V the support of μ. The function $\varphi(x,v,t)$ represents the number density of gas particles having the position x and the velocity v at time t. The function $\sigma(\cdot)$ is called the collision frequency and the functions $k_c(\cdot,\cdot,\cdot)$, $k_e(\cdot,\cdot,\cdot,\cdot)$ and $k_d^j(\cdot,\cdot,\cdot,\cdot)$, $j = 1,\cdots,l$, denote the scattering kernels of the operators K_c, K_e and $K_d = \sum_{j=1}^{l} K_d^j$ (called classical, elastic and inelastic collision operators respectively).

The purpose of this work is to extend the results obtained in [1, 2, 3] to periodic boundary conditions.

We shall prove that the second order remainder term, $R_2(t)$ of the Dyson-Philips expansion is compact on L $L^p(\Omega \times V, dxd\mu(v)), (1 \leq p < \infty)$. As an immediate consequence of the result of the compactness, we have

$$\omega_{ess}\big(e^{t(T_p+K_c+K_e+K_d)}\big) = \omega_{ess}\big(e^{t(T_p+K_e+K_d)}\big). \tag{4}$$

Equality (4) is of fundamental importance to describe the analysis of the asymptotic behaviour $(t \to \infty)$ of the solution.

References

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