

## Asymptotic behaviour of some biological models stemming from structured population dynamics

*jeudi 30 août 2018 12:20 (35 minutes)*

We consider two different partial differential equation models structured by elapsed time for dynamics of neuron population and give some improved results for long time asymptotics. The first model we study is a nonlinear version of the renewal equation, while the second model is a conservative drift-fragmentation equation which adds adaptation and fatigue effects to the neural network model. These problems were introduced in [1] and [2].

We prove that both the problems are well-posed in a measure setting. Both have steady states which may or may not be unique depending on further assumptions. In order to show the exponential convergence to steady states we use a technique from the theory of Markov processes called Doeblin's method. This method was used in [3] for demonstrating exponential convergence of solutions of the renewal equation to its equilibrium. It is based on the idea of finding a positive quantitative bound for solutions to the linear problem. This leads us to prove the spectral gap property in the linear setting. Then by exploiting this property we prove that both models converge exponentially to a steady state.

We consider an extension of the Doeblin's Theorem which is called Harris' Theorem, in order to obtain asymptotic convergence result for growth-fragmentation equation which is a more general model for cell growth and division and other phenomena involving fragmentation. This part is still on progress.

This is a joint work with José A. Cañizo.

### References

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**Orateur:** YOLDAŞ, Havva (BCAM - Universidad de Granada)