Benchmark of asymptotic preserving schemes for the hyperbolic to diffusive degeneracy

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In the spirit of the benchmark from FVCA5 [6] we compare several schemes for system of conservations laws which degenerates to diffusive equation when the source term becomes stiff or with late time. For instance, the Telegraph equations (1):

$$\begin{cases} \partial_t z + a\partial_x w &= 0 \\ \partial_t w + a\partial_x z &= -2\sigma w \end{cases} \tag{1}$$

degenerate to the following diffusive equation when $\sigma t \to \infty$:

$$\partial_t(z) - \partial_x \frac{a^2}{2\sigma}\partial_x(z) = 0. \tag{2}$$

Several asymptotic-preserving schemes exist to preserve at the discrete level this degeneracy. A non-exhaustive list may contains the following schemes [7, 5, 1, 2, 3, 4]. The aim of this talk is to compare those schemes with various test cases in different configurations.

This current work will lead to an open-source code to allow an easy implementation of new schemes.

References

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