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About the equilibria of a cross-diffusion system in population dynamics

Cross-diffusion is a mechanism used in population dynamics to model a repulsive effect between individuals. Mathematically, this corresponds to adding a nonlinear diffusion term to classical reaction-diffusion systems. Cross-diffusion allows to obtain a richer variety of solutions, whose qualitative behavior seems to better fit observations (*spatial segregation* phenomenom), but it also complicates the mathematical study of these solutions.

In this talk, I will explain how this problem can be tackled by combining numerical simulations with a posteriori estimates, to obtain *computer-assisted proofs*. First, I will briefly present the general strategy behind this kind of computer-assisted techniques, namely to apply a fixed point theorem in a neighborhood of a numerical solution, which then yields the existence of a true solution. Then, I will illustrate how this techniques can be applied to study inhomogeneous steady states of the SKT triangular system:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta((d_1 + d_{12}v)u) + (r_1 - a_1 u - b_1 v)u \\ \dfrac{\partial v}{\partial t} = \Delta(d_2 v) + (r_2 - b_2 u - a_2 v)v, \end{cases}$$

This is the result of a joint work with R. Castelli (VU Amsterdam).