AN ASYMPTOTIC PRESERVING AND WELL-BALANCED SCHEME FOR THE SHALLOW-WATER EQUATIONS WITH MANNING FRICTION

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The purpose of this work concerns the derivation of a non-negative, well-balanced and asymptotic preserving scheme for the shallow-water equations with Manning friction. This problem, used to model the flow of water in a one-dimensional channel with a flat bottom that applies a friction force, reads:

$$\begin{cases} \partial_t h + \partial_x q = 0, \\ \partial_t q + \partial_x \left(\dfrac{q^2}{h} + \dfrac{gh^2}{2} \right) = -kq|q|h^{-\eta}. \end{cases} \tag{1}$$

The unknowns involved are the positive water height $h(t,x)$ and the depth-averaged discharge $q(t,x)$. The parameters of the model are the gravity constant g, the Manning coefficient k and the parameter η, usually equal to $7/3$.

The objectives of the proposed work are twofold. First, we are insterested by the steady states of (1). They are governed by

$$\begin{cases} \partial_x q = 0, \\ \partial_x \left(\dfrac{q^2}{h} + \dfrac{gh^2}{2} \right) = -kq|q|h^{-\eta}. \end{cases} \tag{2}$$

In addition, we are also interested in asymptotic regime satisfied by the solutions of (1). More precisely we study the long time and dominant friction. In order to model it, we proceed to a rescaling of (1) using a small parameter ε as follows:

$$\begin{cases} \varepsilon \partial_t h + \partial_x q = 0, \\ \varepsilon \partial_t q + \partial_x \left(\dfrac{q^2}{h} + \dfrac{gh^2}{2} \right) = -\dfrac{k}{\varepsilon^2} q|q|h^{-\eta}. \end{cases} \tag{3}$$

Arguing a formal Chapman-Enskog expansion of the solutions of (3) in the limit of ε goes to zero, h satisfies the following non-linear parabolic equation:

$$\partial_t h + \partial_x \left(-\text{sign}\,(\partial_x h) \sqrt{\frac{h^\eta}{k} \left| \partial_x \frac{gh^2}{2} \right|} \right) = 0. \tag{4}$$

The objective of this work is to derive a numerical method to capture both all the steady states and the correct diffusive limit of (3). In a first part, we present a technique to extend the asymptotic preserving scheme of Berthon and Turpault [1] in order to take into account asymptotic regimes governed by quadratic terms, as in this case. Next we present a method to get, in addition, the fully well-balanced property. To access such an issue, we propose an extension of a scheme recently introduced by Michel-Dansac in [2].

REFERENCES

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