

A finite-volume scheme for a degenerate cross-diffusion model motivated from ion transport

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An implicit Euler finite-volume scheme for a degenerate cross-diffusion system describing the ion transport through biological membranes is proposed. We consider the model developed in [1] for describing size exclusion effects in narrow channels. The strongly coupled equations for the ion concentrations include drift terms involving the electric potential, which is coupled to the concentrations through a Poisson equation. The cross-diffusion system possesses a formal gradient-flow structure revealing nonstandard degeneracies, which lead to considerable mathematical difficulties.

The proposed finite-volume scheme is based on two-point flux approximations with “double” upwind mobilities. The existence of solutions to the fully discrete scheme is proved. When the particles are not distinguishable and the dynamics are driven by cross-diffusion only, it is shown that the scheme preserves the structure of the equations like nonnegativity, upper bounds, and entropy dissipation. The degeneracy is overcome by proving a new discrete Aubin-Lions lemma of “degenerate” type. Numerical simulations of a calcium-selective ion channel in two space dimensions show that the scheme is efficient even in the general case of ion transport.

This is a joint work with C. Cancès (Inria Lille), C. Chainais-Hillairet (Univ. Lille) and A. Jüngel (TU Wien).

References

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- [2] C. Cancès, C. Chainais-Hillairet, A. Gerstenmayer and A. Jüngel, Convergence of a Finite-Volume Scheme for a Degenerate Cross-Diffusion Model for Ion Transport, submitted, arXiv:1801.09408.
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Orateur: GERSTENMAYER, Anita (Vienna University of Technology)