Asymptotic Behavior of systems of PDE arising in physics and biology: theoretical and numerical points of view

3rd edition

August 28th-31st 2018 – Lille

Asymptotic Behavior of systems of PDE arising in physics and biology: theoretical and numerical points of view
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LILLIAD Learning Center

Tuesday

14:00 **M.-T. Wolfram**
Segregation phenomena in population dynamics

14:45 **A. Gerstenmayer**
A finite-volume scheme for a degenerate cross-diffusion model motivated from ion transport

15:20 **J. Berendsen**
TBA

15:55 Coffee break

16:15 **M. Burger**
TBA

17:00 **S. Bulteau**
An asymptotic-preserving and well-balanced scheme for the shallow-water equations with Manning friction

Wednesday

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11:45	**H. Hivert** *An asymptotic-preserving scheme for a kinetic equation describing propagation phenomena*
12:20	**F. Blachère** *Benchmark of asymptotic preserving schemes for the hyperbolic to diffusive degeneracy*
12:55	Lunch
14:30	**J.-A. Carrillo de la Plata** *Swarming models with local alignment effects: phase transitions & hydrodynamics*
15:15	**L. M. K. Sabbagh** *On the Motion of Several Disks in an Unbounded Viscous Incompressible Fluid*
15:50	Coffee break
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Thursday

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9:30	**A.-L. Dalibard** *TBA*
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11:10	**R. Müller** *Dynamics of electrochemical interfaces*
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12:20	**H. Yoldaş** *Asymptotic behaviour of some biological models stemming from structured population dynamics*
12:55	Lunch
14:30	**C. Chalons** *TBA*
15:15	**O. Tse** *TBA*
15:50	Coffee break
16:10	**C. Choquet** *TBA*
16:55	**M. Heida** *On convergences of the square root approximation scheme to the Fokker-Planck operator*
17:30	**A.-M. Al Izeri** *Spectral theory of a transport equation with elastic and inelastic collision operators*

Friday

8:45 Coffee

9:30 **A. Crestetto**
Particle Micro-Macro schemes for collisional kinetic equations in the diffusive scaling

10:15 **E. Franck**
High order implicit relaxation schemes for nonlinear hyperbolic systems

10:50 Coffee break

11:10 **P. Biler**
Critical singularities in the higher dimensional minimal Keller-Segel model

11:55 **S. Salem**
Propagation of chaos for some 2 dimensional fractional Keller-Segel equation in Dominated diffusion and fair competition cases

Poster session

F. Boutaous
Fractional Powers Approach for the Resolution of some Elliptic PDEs

C. Cancès
TBA

N. Chatterjee
Convergence analysis of a numerical scheme for a general class of Mean field Equation

Y. Kosad
Time asymptotic behavior for singular neutron transport equation with bounce-back boundary conditions in L^1 *spaces*

Y. Løvbak
A Multilevel Monte Carlo Method For Kinetic Transport Equations Using Asymptotic-Preserving Particle Schemes

A. Mecherbet
Sedimentation of particles in Stokes flow

P. Mennuni
Existence of traveling waves for the nonlocal Gross-Pitaevskii-equation in dimension one

D. Mukherjee
Nonlocal elliptic equations: existence and multiplicity results

A. Nicolopoulos
Maxwell's equations with sign changing permittivity tensor

M. Veruete
Evolutionary branching via replicator-mutator equations

L. Zielinski
Long time asymptotics for solutions of the Short Pulse Equation

Thu. August 30th
17:30 – 18:05

Spectral theory of a transport equation with elastic and inelastic collision operators

A.-M. Al Izeri

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We are concerned with the time asymptotic behavior of the solution to the following Cauchy problem for partly elastic collisions operators, that is,

$$\begin{aligned}\frac{\partial\varphi}{\partial t}(x,v,t) = -v.\frac{\partial\varphi}{\partial x}(x,v,t) - \sigma(v)\varphi(x,v,t) + \int_V k_c(x,v,v')\varphi(x,v',t)dv' \\ + \sum_{j=1}^{l}\int_{\mathbb{S}^{N-1}} k_d^j(x,\rho_j,\omega,\omega')\varphi(x,\rho_j\omega',t)d\omega' \\ + \int_{\mathbb{S}^{N-1}} k_e(x,\rho,\omega,\omega')\phi(x,\rho\omega',t)d\omega'\end{aligned}$$

with the initial distribution

$$\varphi(x,v,0) = \varphi_0(x,v).$$

and we restrict ourselves to periodic boundary conditions, that is,

$$\varphi(x,v,t)_{|x_i=-a_i} = \varphi(x,v,t)_{|x_i=a_i},$$

where $\Omega = \prod_{i=1}^{N}(-a_i,a_i)$, $a_i > 0, i = 1,\dots,N$, is an open paved of $\mathbb{R}^N$. The set $V \subset \mathbb{R}^N$, is called the space of admissible velocities, $v = \rho\omega \in V =: I \times \mathbb{S}^{N-1}$ with $\omega \in \mathbb{S}^{N-1}$, $\rho \in I := (\rho_{\min},\rho_{\max})$ and $\mu(\cdot)$ is a positive Radon measure on $\mathbb{R}^N$, we denote by V the support of μ. The function $\varphi(x,v,t)$ represents the number density of gas particles having the position x and the velocity v at time t. The function $\sigma(\cdot)$ is called the collision frequency and the functions $k_c(\cdot,\cdot,\cdot)$, $k_e(\cdot,\cdot,\cdot,\cdot)$ and $k_d^j(\cdot,\cdot,\cdot,\cdot)$, $j = 1,\dots,l$, denote the scattering kernels of the operators K_c, K_e and $K_d = \sum_{j=1}^{l} K_d^j$ (called classical, elastic and inelastic collision operators respectively).

The purpose of this work is to extend the results obtained in [1,2,3] to periodic boundary conditions.

We shall prove that the second order remainder term, $R_2(t)$ of the Dyson-Philips expansion is compact on L $L^p(\Omega \times V, dxd\mu(v)), (1 \le p < \infty)$. As an immediate consequence of the result of the compactness, we have

$$\omega_{ess}\big(e^{t(T_p+K_c+K_e+K_d)}\big) = \omega_{ess}\big(e^{t(T_p+K_e+K_d)}\big). \tag{1}$$

Equality (1) is of fundamental importance to describe the analysis of the asymptotic behaviour ($t \to \infty$) of the solution.

References

[1] E. W. Larsen, P. F. Zweifel, *On the spectrum of the linear transport operator*, J. Mathematical Phys. **15** (1974), pp. 1987-1997.

[2] M. Sbihi, *Spectral theory of neutron transport semigroups with partly elastic collision operators*, J. Math. Phys. **47**-12 (2006), 123502.

[3] M. Sbihi, *Analyse Spectrale De Modèles Neutroniques*, Thèse de Doctorat de l'Université de Franche-Comté, Besançon, 2005.

TBA

Tue. August 28th
15:20 – 15:55

J. Berendsen

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Abstract

Critical singularities in the higher dimensional minimal Keller-Segel model

Fri. August 31st
11:10 – 11:55

P. Biler

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Existence of global in time radially symmetric solutions is studied for "large" initial data. Criteria for blowup of solutions in terms of Morrey norms are derived.

Wed. August 29th
12:20 – 12:55

Benchmark of asymptotic preserving schemes for the hyperbolic to diffusive degeneracy

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In the spirit of the benchmark from FVCA5 [6] we compare several schemes for system of conservations laws which degenerates to diffusive equation when the source term becomes stiff or with late time. For instance, the Telegraph equations (2):

$$\begin{cases} \partial_t z + a\,\partial_x w = 0\,, \\ \partial_t w + a\,\partial_x z = -2\sigma\, w\,, \end{cases} \tag{2}$$

degenerate to the following diffusive equation when $\sigma t \to \infty$:

$$\partial_t(z) - \partial_x \frac{a^2}{2\sigma} \partial_x(z) = 0\,.$$

Several asymptotic-preserving schemes exist to preserve at the discrete level this degeneracy. A non-exhaustive list may contains the following schemes [7, 5, 1, 2, 3, 4]. The aim of this talk is to compare those schemes with various test cases in different configurations.

This current work will lead to an open-source code to allow an easy implementation of new schemes.

This is a joint work with S. Guisset (CEA-DAM DIF, Arpajon, France).

References

[1] D. Aregba-Driollet, M. Briani and R. Natalini, *Asymptotic high-order schemes for* 2×2 *dissipative hyperbolic systems*, SIAM J. Numer. Anal. **46**-2 (2008) pp. 869-894.

[2] C. Berthon and R. Turpault, *Asymptotic preserving HLL schemes*, Numer. Methods Partial Differential Equations **27**-6 (2011), pp. 1396-1422.

[3] C. Chalons and S. Guisset, *An Antidiffusive HLL Scheme for the Electronic* M_1 *Model in the Diffusion Limit*, Multiscale Model. Simul. **16**-2 (2018), pp. 991-1016.

[4] C. Chalons and R. Turpault, *High-order asymptotic-preserving schemes for linear systems. Application to the Goldstein-Taylor equations*, in preparation.

[5] L. Gosse and G. Toscani, *An asymptotic-preserving well-balanced scheme for the hyperbolic heat equations*, C. R. Math. Acad. Sci. Paris **334**-4 (2002), pp. 337-342.

[6] R. Herbin and F. Hubert, *Benchmark on discretization schemes for anisotropic diffusion problems on general grids*, Finite volumes for complex applications V, ISTE, London (2008), pp. 659-692.

[7] S. Jin and C. D. Levermore, *Numerical schemes for hyperbolic conservation laws with stiff relaxation terms*, J. Comput. Phys. **126**-2 (2002), pp. 449-467.

Flux vector approximation schemes for systems of conservation laws

Wed. August 29th
10:15 – 10:50

J. ten Thije Boonkkamp

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Conservation laws in continuum physics are often coupled, for example the continuity equations for a reacting gas mixture or a plasma are coupled through multi-species diffusion and a complicated reaction mechanism. For space discretisation of these equations we employ the finite volume method. The purpose of this talk is to present novel flux vector approximation schemes that incorporate this coupling in the discretisation. More specifically, we consider as model problems linear advection-diffusion systems with a nonlinear source and linear diffusion-reaction systems, also with a nonlinear source.

The new flux approximation schemes are inspired by the complete flux scheme for scalar equations, see [1]. An extension to systems of equations is presented in [2]. The basic idea is to compute the numerical flux vector at a cell interface from a local inhomogeneous ODE-system, thus including the nonlinear source. As a consequence, the numerical flux vector is the superposition of a homogeneous flux, corresponding to the homogeneous ODE-system, and an inhomogeneous flux, taking into account the effect of the nonlinear source. The homogeneous ODE-system is either an advection-diffusion system or a diffusion-reaction system. In the first case, the homogeneous flux contains only real-valued exponentials, on the other hand, in the second case, also complex-valued components are possible, generating oscillatory solutions. The inclusion of the inhomogeneous flux makes that all schemes display second order convergence, uniformly in all parameters (Peclet and Damköhler numbers).

The performance of the novel schemes is demonstrated for several test cases, moreover, we investigate several limiting cases.

This is a joint work with J. van Dijk and R.A.M. van Gestel (Department of Applied Physics, Eindhoven University of Technology).

References

[1] J.H.M. ten Thije Boonkkamp and M.J.H. Anthonissen, *The finite volume-complete flux scheme for advection-diffusion-reaction equations*, J. Sci. Comput. **46** (2011) pp. 47-70.

[2] J.H.M. ten Thije Boonkkamp, J. van Dijk, L. Liu and K.S.C. Peerenboom, *Extension of the complete flux scheme to systems of comservation laws*, J. Sci. Comput. **53** (2012), pp. 552-568.

Wed. August 29th
16:10 – 18:30

Fractional Powers Approach for the Resolution of some Elliptic PDEs

F. Boutaous

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In this work, we will study an operational second order differential equation with variable operators as coefficients, with Dirichlet boundary conditions. In the framework of Hölderian spaces and using some differentiability assumptions on the resolvent operators of the square roots characterizing the ellipticity of the previous problem, we give necessary and sufficient conditions on the data for the existence and uniqueness of the strict solution of the Dirichlet problem.

By using the approach based on the semigroups theory, the fractional powers of linear operators, the Dunfords functional calculus and the interpolation theory, we will prove the main results on the existence, the uniqueness and the optimal regularity of the strict solution. All the abstract results obtained in this study are applied to an example concret of partial differential equations.

References

[1] F. Boutaous, R. Labbas and B.-K. Sadallah, *Fractional-power approach for solving complete elliptic abstract differential equations with variable operator coefficients*, Electronic Journal of Differential Equations **2012**-5, pp. 1-33 (2012).

[2] S.-G. Krein, *Linear Differential Equations in Banach Spaces*, Moscow, 1967, English transl. AMS, Providence, 1971.

About the equilibria of a cross-diffusion system in population dynamics

Wed. August 29th
11:10 – 11:45

M. Breden

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Cross-diffusion is a mechanism used in population dynamics to model a repulsive effect between individuals. Mathematically, this corresponds to adding a nonlinear diffusion term to classical reaction-diffusion systems. Cross-diffusion allows to obtain a richer variety of solutions, whose qualitative behavior seems to better fit observations (spatial segregation phenomenom), but it also complicates the mathematical study of these solutions.

In this talk, I will explain how this problem can be tackled by combining numerical simulations with a posteriori estimates, to obtain computerassisted proofs. First, I will briefly present the general strategy behind this kind of computer-assisted techniques, namely to apply a fixed point theorem in a neighborhood of a numerical solution, which then yields the existence of a true solution. Then, I will illustrate how this techniques can be applied to study inhomogeneous steady states of the SKT triangular system:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta((d_1 + d_{12}v)\, u) + (r_1 - a_1 u - b_1 v)\, u\,, \\ \dfrac{\partial v}{\partial t} = \Delta(d_2\, v) + (r_2 - b_2 u - a_2 v)\, v\,. \end{cases}$$

This is the result of a joint work with R. Castelli (VU Amsterdam).

An asymptotic-preserving and well-balanced scheme for the shallow-water equations with Manning friction

Tue. August 28th
17:00 – 17:35

S. Bulteau

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The purpose of this work concerns the derivation of a non-negative, well-balanced and asymptotic preserving scheme for the shallow-water equations with Manning friction. This problem, used to model the flow of water in a one-dimensional channel with a flat bottom that applies a friction force, reads:

$$\begin{cases} \partial_t h + \partial_x q = 0\,, \\ \partial_t q + \partial_x \left(\dfrac{q^2}{h} + \dfrac{g h^2}{2} \right) = -kq|q|h^{-\eta}\,. \end{cases} \tag{3}$$

The unknowns involved are the positive water height $h(t, x)$ and the depth-averaged discharge $q(t, x)$. The parameters of the model are the gravity constant g, the Manning coefficient k and the parameter η, usually equal to $\frac{7}{3}$.

The objectives of the proposed work are twofold. First, we are insterested by the steady states of (3). They are governed by

$$\begin{cases} \partial_x q = 0\,, \\ \partial_x \left(\dfrac{q^2}{h} + \dfrac{gh^2}{2}\right) = -kq|q|h^{-\eta}\,. \end{cases} \tag{4}$$

In addition, we are also interested in asymptotic regime satisfied by the solutions of (3). More precisely we study the long time and dominant friction. In order to model it, we proceed to a rescaling of (3) using a small parameter ε as follows:

$$\begin{cases} \varepsilon\partial_t h + \partial_x q = 0\,, \\ \varepsilon\partial_t q + \partial_x \left(\dfrac{q^2}{h} + \dfrac{gh^2}{2}\right) = -\dfrac{k}{\varepsilon^2} q|q|h^{-\eta}\,. \end{cases} \tag{5}$$

Arguing a formal Chapman-Enskog expansion of the solutions of (5) in the limit of ε goes to zero, h satisfies the following non-linear parabolic equation:

$$\partial_t h + \partial_x \left(-\mathrm{sign}(\partial_x h) \sqrt{\frac{h^\eta}{k} \left| \partial_x \frac{gh^2}{2} \right|} \right) = 0\,.$$

The objective of this work is to derive a numerical method to capture both all the steady states and the correct diffusive limit of (5). In a first part, we present a technique to extend the asymptotic preserving scheme of Berthon and Turpault [1] in order to take into account asymptotic regimes governed by quadratic terms, as in this case. Next we present a method to get, in addition, the fully well-balanced property. To access such an issue, we propose an extension of a scheme recently introduced by Michel-Dansac in [2].

References

[1] C. Berthon and R. Turpault, *Asymptotic preserving HLL schemes*, Numer. Methods Partial Differential Equations **27** (2011) pp. 1396-1422.

[2] V. Michel-Dansac, C. Berthon, S. Clain , AND F. Foucher, *A well-balanced scheme for the shallow-water equations with topography or Manning friction*, J. Comput. Phys. **335** (2017), pp. 115-154.

Tue. August 28th
16:15 – 17:00

TBA

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Abstract

TBA

Wed. August 29th
16:10 – 18:30

C. Cancès
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Abstract

Swarming models with local alignment effects: phase transitions & hydrodynamics

Wed. August 29th
14:30 – 15:15

J.-A. Carrillo de la Plata
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We will discuss a collective behavior model in which individuals try to imitate each other's velocity and have a preferred asymptotic speed. It is a variant of the well-known Cucker-Smale model in which the alignment term is localized. We showed that a phase change phenomenon takes place as diffusion decreases, bringing the system from a disordered "to anordered" state. This effect is related to recently noticed phenomena for the diffusive Vicsek model. We analysed the expansion of the large friction limit around the limiting Vicsek model on the sphere leading to the so-called Self-Organized Hydrodynamics (SOH). This talk is based on papers in collaboration with Bostan, and with Barbaro, Cañizo and Degond.

Numerical simulations of slurry pipeline for water-slurry-water

Thu. August 30th
11:45 – 12:20

T. Chakkour
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Numerous slurry transportation pipeline systems have been built in the past 10 years. At the same time, T. Chakkour & F. Benkhaldoun study in [2, 3] the hydraulic transport of particles in tubes. We investigated in [1] the hydraulic transport of slurry system in horizontal tubes (The Khouribga mines). This mineral pipeline has often been referred to as one of the most challenging projects in terms of operating complexity and system configuration in Morocco. This physical model features a mass and momentum balance for three-fluid model in 1D. It allows to predict the pressure drop and flow patterns. The originality of this work is to present in simplified form a homogeneous single-phase model. The most important advantage of this model is the considerably smaller number of variables to be solved compared to the multiphase model. In this presentation, we give the asymptotic behavior of friction-disharge term fQ^2 that is involved in the last term of motions equation, taking into account the Reynolds number. This allows to understand how the elevation varies, when the flow is very laminar.

References

[1] T. Chakkour, F. Benkhaldoun and M. Boubekeur, *Slurry Pipeline for fluid transients in pressurized conduits*, submitted

[2] T. Chakkour, *Simulations numériques des tubes avec contraction brusque sur openfoam*, Thermodynamique des interfaces et mécanique des fluides **17** (2017).

[3] F. Benkhaldoun, I. Elmahi and M. Sea, *Well-balanced finite volume schemes for pollutant transport by shallow water equations on unstructured meshes*, J. Comput. Phys. **226**-1 (2007), pp. 180-203.

Thu. August 30th
14:30 – 15:15

TBA

C. Chalons

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Abstract

Wed. August 29th
16:10 – 18:30

Convergence analysis of a numerical scheme for a general class of Mean field Equation

N. Chatterjee

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A widely used prototype phase model to describe the synchronous behavior of weakly coupled limit-cycle oscillators is the Kuramoto model whose dynamics for sufficiently large ensemble of oscillators can be effectively approximated by the corresponding mean-field equation 'the Kuramoto Sakaguchi Equation'. In the recent past, it has been extensively studied to analyze the phase transition of between different kind of ordered states.In the talk, we are going to derive and analyze a numerical method for a general class of mean-field equations, including the Kuramoto Sakaguchi equation. Along the way, we will prove the strong convergence of the scheme to the unique weak solution whenever the initial datum has bounded variation. We also show convergence in the sense of measures, thereby relaxing the assumption of bounded variation. The theoretical results will be verified with several numerical experiments.

This is a joint work with U. S. Fjordholm.

TBA

Thu. August 30th
16:10 – 16:55

C. Choquet

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Abstract

Particle Micro-Macro schemes for collisional kinetic equations in the diffusive scaling

Fri. August 31st
9:30 – 10:15

A. Crestetto

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In this talk, I will present a new asymptotic preserving scheme for kinetic equations of Boltzmann-BGK type in the diffusive scaling. The scheme is a suitable combination of micro-macro decomposition, the micro part being discretized by a particle method, and Monte Carlo techniques. Thanks to the Monte Carlo particle approximation, the computational cost of the method automatically reduces when the system approaches the diffusive limit. However, this approximation requires a splitting between the transport part and the collisional one, so that both stiff terms can not offset each other a priori, which prevents from uniform stability. That is why we propose a suitable reformulation of the micro-macro system, without stiff terms. The scheme will be presented in detail and illustrated by several numerical results (including in the 3D in space - 3D in velocity framework).

This work is a collaboration with Nicolas Crouseilles, Giacomo Dimarco and Mohammed Lemou.

TBA

Thu. August 29th
9:30 – 10:15

A.-L. Dalibard

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Abstract

Thu. August 29th
9:30 – 10:15

TBA

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Abstract

Fri. August 31st
10:15 – 10:50

High order implicit relaxation schemes for nonlinear hyperbolic systems

E. Franck
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In this work we consider the time discretization of compressible fluid models which appear in gas dynamics, biology, astrophysics or plasma physics for Tokamaks. In general, for the hyperbolic system we use an explicit scheme in time. However, for some applications, the characteristic velocity of the fluid is very small compared to the fastest velocity speed. In this case, to filter the fast scales it is common to use an implicit scheme. The implicit schemes allows to filter the fast scale that we do not want to consider and choose a time step independent of the mesh step and adapted to the characteristic velocity of the fluid. The matrices induced by the discretization of the hyperbolic system are ill-conditioned in the regime considered and very hard to invert. In this work we propose an alternative method to the classical preconditioning, based on the BGK relaxation methods. The idea is, to propose a larger and simpler model (here a BGK model [1]-[2]) depending of the small parameter which approximate the original system. Designing an AP scheme based on splitting method [1] for the BGK model, stable without CFL condition, we obtain at the end a very simple method avoiding matrix inversion and unconditionally stable for the initial model. This method can approximate any hyperbolic models and can be generalized to treat models including additional small diffusion terms. After the presentation of the method we will show how obtained high-order schemes in time and space. To finish we will focus on two non trivial applications for the BGK relaxation methods: the low-Mach regime for Euler equations and the parabolic models.

References

[1] D. Coulette, E. Franck, P. Helluy, M. Mehrenberger and L. Navoret, *High-order implicit palindromic discontinuous Galerkin method for kinetic-relaxation approximation*, submitted, arXiv:1802.04590

[2] D. Aregba-Driollet and R. Natalini, *Discrete Kinetic Schemes for Multidimensional Conservation Laws*, SIAM J. Num. Anal. **37** (2000), pp. 1973-2004.

A finite-volume scheme for a degenerate cross-diffusion model motivated from ion transport

Tue. August 28$^{\text{th}}$
14:45 – 15:20

A. Gerstenmayer

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An implicit Euler finite-volume scheme for a degenerate cross-diffusion system describing the ion transport through biological membranes is proposed. We consider the model developed in [1] for describing size exclusion effects in narrow channels. The strongly coupled equations for the ion concentrations include drift terms involving the electric potential, which is coupled to the concentrations through a Poisson equation. The cross-diffusion system possesses a formal gradient-flow structure revealing nonstandard degeneracies, which lead to considerable mathematical difficulties.

The proposed finite-volume scheme is based on two-point flux approximations with "double" upwind mobilities. The existence of solutions to the fully discrete scheme is proved. When the particles are not distinguishable and the dynamics are driven by cross-diffusion only, it is shown that the scheme preserves the structure of the equations like nonnegativity, upper bounds, and entropy dissipation. The degeneracy is overcome by proving a new discrete Aubin-Lions lemma of "degenerate" type. Numerical simulations of a calcium-selective ion channel in two space dimensions show that the scheme is efficient even in the general case of ion transport.

This is a joint work with C. Cancès (Inria Lille), C. Chainais-Hillairet (Univ. Lille) and A. Jüngel (TU Wien).

References

[1] M. Burger, B. Schlake, and M.-T. Wolfram, *Nonlinear Poisson-Nernst-Planck equations for ion flux through confined geometries*, Nonlinearity **25** (2012) pp. 961-990.

[2] C. Cancès, C. Chainais-Hillairet, A. Gerstenmayer and A. Jüngel, *Convergence of a Finite-Volume Scheme for a Degenerate Cross-Diffusion Model for Ion Transport*, submitted, arXiv:1801.09408.

[3] A. Gerstenmayer and A. Jüngel, *Analysis of a degenerate parabolic cross-diffusion system for ion transport*, submitted, arXiv:1706.07261.

Thu. August 30$^{\text{th}}$
16:55 – 17:30

On convergences of the square root approximation scheme to the Fokker-Planck operator

M. Heida

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We study the qualitative convergence behavior of a novel FV-discretization scheme of the Fokker-Planck equation, the squareroot approximation scheme (SQRA), that recently was proposed by [Lie, Fackeldey and Weber 2013] in the context of conformation dynamics. We show that SQRA has a natural gradient structure related to the Wasserstein gradient flow structure of the Fokker-Planck equation and that solutions to the SQRA converge to solutions of the Fokker-Planck equation. This is done using a discrete notion of G-convergence for the underlying discrete elliptic operator. The gradient structure of the FV-scheme guaranties positivity of solutions and preserves asymptotic behavior of the Fokker-Planck equation for large times. Furthermore, the SQRA does not need to account for the volumes of cells and interfaces and is taylored for high dimensional spaces. However, based on FV-discretizations of the Laplacian it can also be used in lower dimensions taking into account the volumes of the cells. As an example, in the special case of stationary Voronoi tessellations we use stochastic two-scale convergence to prove that this setting satisfies the G-convergence property.

Wed. August 29$^{\text{th}}$
11:45 – 12:20

An asymptotic-preserving scheme for a kinetic equation describing propagation phenomena

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The run-and-tumble motion of bacteria such as E. coli can be represented by a nonlinear kinetic equation. It will be considered under an hyperbolic scaling, and rewritten using the Hopf-Cole transform of the distribution function. It has been shown that the asymptotic model is either a Hamilton-Jacobi equation in which the Hamiltonian is implicitely defined, or a non-local HamiltonJacobi-like equation.

Since the kinetic equation becomes a stiff problem when reaching the asymptotic, its numerical computation must be performed with care to avoid instabilities when reaching it. Asymptotic Preserving (AP) schemes have been introduced to avoid these difficulties, since they enjoy the property of being stable along the transition towards the asymptotic regime.

I will present an AP scheme for this nonlinear kinetic equation, which is based on a formal asymptotic analysis of the problem. The discretization of the limit Hamilon-Jacobi equation will also be discussed.

Time asymptotic behavior for singular neutron transport equation with bounce-back boundary conditions in L^1 spaces

Wed. August 29th
16:10 – 18:30

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This communication is concerned with the well-posedness of a Cauchy problem governed by a singular neutron transport equation in L^1 spaces with bounce-back boundary conditions. To be more precise, we are concerned with the following equation

$$\begin{cases} \dfrac{\partial \psi}{\partial t}(x,v,t) & = & A_H\psi(x,v,t) := T_H\psi(x,v,t) + K\,\psi(x,v,t) \\ & = & -v\,\nabla_x\psi(x,v,t) - \sigma(v)\,\psi(x,v,t) + \displaystyle\int_{\mathbb{R}^n} \kappa(x,v,v')\,\psi(x,v',t)\,dv'\,, \\ \psi(x,v,0) = \psi_0(x,v)\,, \end{cases}$$

where $(x,v) \in D \times \mathbb{R}^n$ and κ is the partial integral part of A_H, it is called the collision operator. Here D is a bounded open subset of $\mathbb{R}^n$. As usually in transport theory, the function $\psi(x,v,t)$ represents the number (or probability) density of gas particles having the position x and the velocity v at the time t. The functions $\sigma(\cdot)$ and $\kappa(\cdot,\cdot,\cdot)$ are called, respectively, the collision frequency and the scattering kernel. We prove the weak compactness of the second-order remainder term of the Dyson-Phillips expansion which implies that the essential types of the streaming semigroup T_H and to that of the transport semigroup A_H coincide and we derive, via classical arguments, the time asymptotic behavior of the solution.

A Multilevel Monte Carlo Method For Kinetic Transport Equations Using Asymptotic-Preserving Particle Schemes

Wed. August 29th
16:10 – 18:30

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In many applications of particle simulation, we encounter issues with time-scale separation, where we are required to perform simulations with a small time step, in order to track the system's fast dynamics, but also require simulation over a long time-horizon, in order to capture the system's slow dynamics. When considering hyperbolic transport equations, a scaling parameter ε (related to the mean free particle path) is commonly used to characterize the time-scale separation. The value of ε can vary by many orders of magnitude in different regions of the space-time domain, causing stiffness in the problem formulation. As this scaling parameter tends to zero, the hyperbolic transport equation converges, in the limit, to a parabolic diffusion equation. Simulations of the transport equation in this small ε region however, suffer from extreme time step reduction constraints to maintain stability.

In [1], a new Monte Carlo scheme was developed that converges in the diffusive limit, while avoiding these stability constraints, this is achieved at the cost of a linear model error in the time step size. In this new work, we apply the multilevel Monte Carlo method to this scheme, which reduces simulation costs by first computing a rough estimate with a large time step size, and correspondingly large bias. This estimate is then improved upon with more accurate Monte Carlo simulations using a finer time step. The bias corrections and variances of differences of coupled simulations in this scheme have a different structure from that which is typically observed in multilevel Monte Carlo applications.

In this talk we will present an analysis of this multilevel scheme and show numerical simulations confirming the efficiency of the proposed approach.

This is a joint work with Stephan Vandewalle and Giovanni Samaey.

References

[1] G. Dimarco, L. Pareschi and G. Samaey. *Asymptotic-Preserving Monte Carlo methods for transport equations in the diffusive limit*, SIAM J. Sci. Comput. **40** (2018) pp. 504-528.

Wed. August 29$^{\text{th}}$
16:10 – 18:30

Sedimentation of particles in Stokes flow

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We consider the sedimentation of N identical spherical particles in a uniform gravitational field. Particle rotation is included in the model while inertia is neglected.

In the dilute case, the result in [5] shows that the particles do not get closer in finite time. The rigorous convergence of the dynamics to the solution of a Vlasov-Stokes equation is proven in [4] in a certain averaged sense. The result holds true in the case of particles that are not so dilute as in [5] and for which the interactions between particles are still important.

In this paper, using the method of reflections, we extend the investigation of [4] by discussing the optimal particle distance which is conserved in finite time. The set of particle configurations considered herein is the one introduced in [3] for the analysis of the homogenization of the Stokes equation. We also prove that the particles interact with a singular interaction force given by the Oseen tensor and justify the mean field approximation of Vlasov-Stokes equations in the spirit of [1] and [2].

Key-words: Suspension flows, Interacting particle systems, Stokes equations, Vlasov-like equations

References

[1] M. Hauray, *Wasserstein distances for vortices approximation of Euler-type equations*, Math. Models Methods Appl. Sci. **19** (2009) pp. 1357-1384.

[2] M. Hauray and P.-E. Jabin. *Particle approximation of Vlasov equations with singular forces: propagation of chaos*, Ann. Sci. Ec. Norm. Super. **48**-4 (2015), pp. 891-940.

[3] M. Hillairet, *On the homogenization of the Stokes problem in a perforated domain.* Arch. Rational Mech. Anal. (2018), https://doi.org/10.1007/s00205-018-1268-7

[4]R.-M. Höfer, *Sedimentation of Inertialess Particles in Stokes Flows*, Commun. Math. Phys. **360**-1 (2018), pp. 55-101.

[5]P.-E. Jabin and F. Otto, *Identification of the dilute regime in particle sedimentation*, Commun. Math. Phys. **250** (2004), pp. 415-432.

Existence of traveling waves for the nonlocal Gross-Pitaevskii-equation in dimension one

Wed. August 29th
16:10 – 18:30

P. Mennuni

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We consider the nonlocal GrossPitaevskii equation in dimension one

$$i\partial_t u = \Delta u + u\left(W * (1 - |u|^2)\right), \text{on } \mathbb{R} \times \mathbb{R}, \qquad \text{(NGP)}$$

where u is a complex-valued function and W is a tempered distribution. In the Bose-Einstein condensate, u represents a wave function whereas W describes the interaction between bosons.

If W is a real-valued even distribution, (NGP) is a Hamiltonian equation whose energy given by

$$E(u(t)) = \frac{1}{2}\int_{\mathbb{R}} |u'(t)|^2\, dx + \frac{1}{4}\int_{\mathbb{R}} (W * (1 - |u(t)|^2))(1 - |u(t)|^2)\, dx\,,$$

is formally conserved. If one considers finite energy solution, then u should not vanish at infinity and should in some sense tend to 1 when $|x| \to +\infty$. Thus, we will consider the Cauchy problem for (NGP) with an initial date $u(0) = u_0$ verifying $|u_0(x)| \longrightarrow_{|x|\to+\infty} 1$. We recall the concept of physical momentum

$$P(u) = \int_{\mathbb{R}} \langle iu', u\rangle\, dx\,,$$

which is also formally conserved but not always well-defined.

A traveling wave of speed $c \in \mathbb{R}$ is a solution of (NGP) of the form

$$u_c(t,x) = v(x-ct)\,.$$

Hence, the profile v satifies

$$icv' + \Delta v + v\,(W * (1|v|^2) = 0\,, \quad \text{in } \mathbb{R}, \tag{NTWc}$$

and by using complex conjugation, we can restrict ourselves to the case $c \geq 0$. Note that any constant complex-valued function v of modulus one verifies (NTWc), so that we refer to them as the trivial solutions. In the case $W = \delta$, the explicit formula of finite energy travelling waves is known (see [1]).

We will present a constraint minimization approaches to prove the existence of (non trivial) traveling waves for a wide class of tempered distributions. An important part of the proof is based on the study of the long-wave transonic limit of (NGP) which leads to the Korteweg-de-Vries equation. We will also present a numerical method based on projected gradient descent which will give us an approximation of traveling waves and energy-momentum diagrams.

References

[1] F. Béthuel, Ph. Gravejat and J.-C. Saut, *Existence and properties of travelling waves for the Gross-Pitaevskii equation, Stationary and time dependent GrossPitaevskii equations*, AMS Contemporary Mathematics **473** (2008), pp. 55-104, 978-0-8218-43.

[1]] A. De Laire, *Global well-posedness for a nonlocal Gross-Pitaevskii equation with non-zero condition at infinity*, Comm. Part. Diff. Eq. **35**-11 (2010), pp. 2021-2058.

Wed. August 29th
16:10 – 18:30

Nonlocal elliptic equations: existence and multiplicity results

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In this paper we prove the existence of infinitely many nontrivial solutions of the following equations driven by a nonlocal integro-differential operator L_K with concave-convex nonlinearities and homogeneous Dirichlet boundary conditions

$$\begin{aligned} L_K u + \mu\,|u|^{q-1} + \lambda\,|u|^{p-1}\,u &= 0 \quad \text{in} \quad \Omega\,, \\ u &= 0 \quad \text{in} \quad \mathbb{R}^N \backslash \Omega\,, \end{aligned}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N > 2s$, $s \in (0,1)$, $0 < q < 1 < p \leq \frac{N+2s}{N-2s}$. Moreover, when L_K reduces to the fractional laplacian operator $-(-\Delta)^s$, $p = \frac{N+2s}{N-2s}$, $\frac{1}{2}\,\frac{N+2s}{N-2s} < q < 1$, $N > 6s$, $\lambda = 1$, we find $\mu^* > 0$ such that for any $\mu \in (0, \mu^*)$, there exists at least one sign changing solution.

Dynamics of electrochemical interfaces

Thu. August 30th
11:10 – 11:45

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Interface processes play an important role in many electrochemical applications like batteries, fuel-cells or water purification. In boundary regions typically sharp layers form where electrostatic potential develops steep gradients and the ionic species accumulate to an extend that saturation effects become relevant. In contrast, the classical Nernst-Planck model for electrolyte transport is build on the assumption of dilute solutions and thus it is unable to accurately describe electrochemical interfaces.

Various modifications of the standard Nernst-Planck systems have been proposed. Recently, we derived an extended continuum model from consistent coupling of electro- and thermodynamics in bulk domains intersected by singular surfaces. We apply the model to the interface between a liquid electrolyte and a metal electrode. The interface consists of the surface and the adjacent boundary layers. The surface is assumed to be blocking to all species such that no Faradayic surface reactions occur but adsorption-desorption between volume and surface is permitted. By means of matched asymptotic analysis, the dynamic behavior of such electrochemical interfaces is investigated in the thin double layer limit, *i.e.* for small Debye length. We find three different time scales characterizing the time scales of the bulk diffusion, the double layer charging and the bulk polarization.

For small amplitudes of the applied potential, a linearization of the asymptotic thin double layer model leads to an equivalent circuit model. Electrochemical impedance spectroscopy then allows the identification of parameters in the original full PDE model.

This is a joint work with W. Dreyer and C. Guhlke (WIAS Berlin).

Maxwell's equations with sign changing permittivity tensor

Wed. August 29th
16:10 – 18:30

A. Nicolopoulos
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To model hybrid resonances in fusion plasma, Maxwell's equations feature a sign changing permittivity tensor. The problem can be expressed as a degenerate elliptic PDE. There is no uniqueness of the solution, and the solutions admit a singularity inside the domain.

A small regularizing viscosity parameter can be introduced, but the problem is still numerically challenging because of the competition of this small parameter with the discretization step.

The work presented will consist of the characterization of the limit solution in a mixed variational setting and will be numerically illustrated.

Thu. August 30th
10:15 – 10:50

A kinetic approach to the bi-temperature Euler model

C. Prigent

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The aim of this work is the study of out-of-equilibirum plasma physics. It is a multiscale problem involving both very small lengths (Debye length) and high frequency oscillations (electronic plasma frequency). Transport of charged particles (electrons and ions) in context of Inertial Confinement Fusion (ICF) can be modelled by the bi-temperature Euler equations, which are a non-conservative hyperbolic system. It contains so-called non-conservatives terms, which cannot be put in divergential form. Such terms are not well-defined, and, in situations involving shocks, computing exact or approximated solutions is a challenging issue.

The bi-temperature Euler model can be recovered by using a ChapmanEnskog expansion on an underlying kinetic approach of this system, the Vlasov-BGK-Ampère system, which is conservative. We are interested in the numerical resolution of this kinetic model, in a macroscopic setting. Hence, a scaling is performed on this model in order to exhibit the behaviour of the system in large scale configurations. The major issue of such a system is that the Maxwell equations are describing small scale electromagnetics. At the macroscopic level, these equations degenerate into algebraic relations, preventing their use for computation purposes. Hence, we derive an Asymptotic-Preserving numerical method, which is able to solve the system even when these small scales (Debye length, electronic plasma frequency) are not resolved, i.e Δt, $\Delta x \gg \varepsilon$, with $\varepsilon \to 0$ [2].

Numerical test cases are studied. Several well-known Riemann problems are solved with our method and then compared with methods for the macroscopic bi-temperature Euler model, derived in [1].

References

[1] D. Aregba-Driollet, J. Breil, S. Brull, B. Dubroca and E. Estibals, *Modelling and numerical approximation for the nonconservative bi-temperature Euler model*, Math. Model. Numer. Anal. (2017), DOI 10.1051/m2an/2017007

[2] S. Jin, *Efficient asymptotic-preserving (AP) schemes for some multiscale kinetic equations*, SIAM J. Sci. Comput. **21**-2 (1999), pp. 441-454.

On the Motion of Several Disks in an Unbounded Viscous Incompressible Fluid

Wed. August 29th
15:15 – 15:50

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In this talk, we will present a recent result on fluid solid interaction problem. We consider the system formed by the incompressible Navier Stokes equations coupled with Newton's laws to describe the motion of a finite number of homogeneous rigid disks within a viscous homogeneous incompressible fluid in the whole space $\mathbb{R}^2$. The motion of the rigid bodies inside the fluid makes the fluid domain time dependent and unknown a priori. First, we generalize the existence and uniqueness of strong solutions result of the considered system in the case of a single rigid body moving in a bounded cavity in [3], and then by careful analysis of how elliptic estimates for the Stokes operator depend on the geometry of the fluid domain, we extend these solutions up to collision. Finally, we prove contact between rigid bodies cannot occur for almost arbitrary configurations by studying the distance between solids by a multiplier approach [1]. This talk is based on the results of the preprint [2].

References

[1] D. Gérard-Varet and M. Hillairet, *Regularity issues in the problem of fluid structure interaction*, Arch. For ration. Mech. Anal. (2010) pp. 375-407.

[2] L. M. K. Sabbagh, *On the motion of several disks in an unbounded viscous incompressible fluid*, in preparation.

[3] T. Takahashi, *Analysis of strong solutions for the equation modelling the motion of a rigid-fluid system in a bounded domain*, Adv. Differential Equations **8**-12 (2003), pp. 1499-1532.

Propagation of chaos for some 2 dimensional fractional Keller-Segel equation in Dominated diffusion and fair competition cases

Fri. August 31st
11:55 – 12:30

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In this work we deal with the local in time propagation of chaos without cut-off for some two dimensional fractional Keller-Segel models. More precisely the diffusion considered here is given by the fractional Laplacian operator $-(-\Delta)^{\frac{a}{2}}$ with $a \in (1, 2)$ and the singularity of the interaction is of order $|x|^{1-\alpha}$ with $\alpha \in]1, a]$. In the case $\alpha \in (1, a)$ we prove a complete propagation of chaos result, proving the Γ-l.s.c

property of the fractional Fisher information, already known for the classical Fisher information, using a result of [4]. In the fair competition case ([1]) $a = \alpha$, we only prove a convergence/consistency result in a sub-critical mass regime, similarly as the result obtained for the classical Keller-Segel equation in [2].

References

[1] J.-A. Carrillo, V. Calvez and F. Hoffmann, *Equilibria of homogeneous functionals in the fair-competition regime*, Nonlinear Analysis **159** (2017), pp. 85-128.

[2] N. Fournier and B. Jourdain, *Stochastic particle approximation of the Keller-Segel equation and two-dimensional generalization of Bessel processes*, accepted in Ann. Appl. Probab., arXiv:1507.01087

[3] N. Fournier, M. Hauray and S. Mischler, *Propagation of chaos for the 2D viscous vortex model*, J. Eur. Math. Soc. **16**-7 (2014), pp. 1423-1466.

[4] M. Hauray and S. Mischler, *On Kac's chaos and related problems*, J. Funct. Anal. **266** (2014), pp. 6055-6157.

Thu. August 30th
15:15 – 15:50

TBA

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Abstract

Wed. August 29th
16:10 – 18:30

Evolutionary branching via replicator-mutator equations

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We consider a class of non-local reaction-diffusion problems, referred to as replicator-mutator equations in evolutionary genetics. For a confining fitness function, we prove well-posedness and write the solution explicitly, via some underlying Schrödinger spectral elements (for which we provide new and non-standard estimates). As a consequence, the long time behaviour is determined by the principal eigenfunction or ground state. Based on this, we discuss (rigorously and via numerical explorations) the conditions on the fitness function and the mutation rate for evolutionary branching to occur.

Key-words: Evolutionary genetics, branching phenomena, long time behaviour, non-local PDE.

Segregation phenomena in population dynamics

Tue. August 28th
14:00 – 14:45

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In this talk we consider two large groups of interacting agents, whose dynamics are influenced by the overall perceived density. Such dynamics can be used to describe two pedestrian groups, walking in opposite directions. Or to model the relocation behavior of two distinct populations, which have a preference to stay within their own group.

We discuss the mathematical modeling in different applications as well as the corresponding PDE models. Furthermore we analyze the long term behavior of solutions and show that already minimal interactions can lead to segregation. Finally we confirm our analytical results and illustrate the rich dynamics with numerical experiments.

Asymptotic behaviour of some biological models stemming from structured population dynamics

Thu. August 30th
12:20 – 12:55

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We consider two different partial differential equation models structured by elapsed time for dynamics of neuron population and give some improved results for long time asymptotics. The first model we study is a nonlinear version of the renewal equation, while the second model is a conservative drift-fragmentation equation which adds adaptation and fatigue effects to the neural network model. These problems were introduced in [1] and [2].

We prove that both the problems are well-posed in a measure setting. Both have steady states which may or may not be unique depending on further assumptions. In order to show the exponential convergence to steady states we use a technique from the theory of Markov processes called Doeblins method. This method was used in [3] for demonstrating exponential convergence of solutions of the renewal equation to its equilibrium. It is based on the idea of finding a positive quantitative bound for solutions to the linear problem. This leads us to prove the spectral gap property in the linear setting. Then by exploiting this property we prove that both models converge exponentially to a steady state.

We consider an extension of the Doeblin's Theorem which is called Harris' Theorem, in order to obtain asymptotic convergence result for growth-fragmentation equation which is a more general model for cell growth and division and other phenomena involving fragmentation. This part is still on progress.

This is a joint work with José A. Cañizo.

References

[1] K. Pakdaman, B. Perthame and D. Salort, *Dynamics of a structured neuron population*, Nonlinearity **23**-1 (2010), pp. 55-75.

[2] K. Pakdaman, B. Perthame and D. Salort, *Adaptation and fatigue model for neuron networks and large time asymptotics in a nonlinear fragmentation equation*, J. Math. Neurosci. **4**-1 (2014), pp. 1-26.

[3] P. Gabriel, *Measure solutions to the conservative renewal equation*, submitted, arXiv:1704.00582

[4] J. A. Cañizo and H. Yoldaş, *Asymptotic behaviour of neuron population models structured by elapsed-time*, submitted, arXiv:1803.07062

Wed. August 29th
16:10 – 18:30

Long time asymptotics for solutions of the Short Pulse Equation

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The non-linear Schrödinger (NLS) equation is one of the universal integrable models describing the slow modulation of the amplitude of a weakly nonlinear wave packet in a moving medium. However in the case of ultra-short pulses in high-speed fiberoptic communication the NLS model should be replaced by a short pulse (SP) model which can be reduced to Cauchy problem of studying $u : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\begin{cases} u_{xt} = u + \dfrac{1}{6}(u^3)_{xx}\,, \\ u(x,0) = u_0(x)\,. \end{cases}$$

We assume that $u_0(x)$ is rapidly decaying as $|x| \to \infty$, and we are looking for the solution $u(x,t)$ which is also rapidly decaying as $|x| \to \infty$, for any fixed t. Our purpose is to investigate the asymptotic behavior of $u(x,t)$ for large time t using an adaptation of the inverse scattering transform method, in the form of a RiemannHilbert factorization problem. We explain how to obtain different types of asymptotics: rapidly or slowly decaying solutions, soliton type solutions or wave breaking.

The talk is based on a joint work with A. Boutet de Monvel (Institut de Mathématiques de Jussieu, France) and D. Shepelsky (V. N. Karazin Kharkiv National University, Ukraine).
