Δ-Jonsson theories and their classes of models

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References

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Outline

Jonsson theories

Definition [1]

A theory T is Jonsson if :

1. theory T has infinite models ;
2. theory T is inductive ;
3. theory T has the joint embedding property (JEP) ;
4. theory T has the property of amalgam (AP).

Homogeneous-universal model

Definition [2]

Let $\kappa \geq \omega$. Model M of theory T is called

- κ-universal for T, if each model T with the power strictly less κ isomorphically imbedded in M;
- κ-homogeneous for T, if for any two models A and A_1 of theory T, which are submodels of M with the power strictly less then κ and for isomorphism $f : A \to A_1$ for each extension B of model A, wich is a submodel of M and is model of T with the power strictly less then κ there is exist the extension B_1 of model A_1, which is a submodel of M and an isomorphism $g : B \to B_1$ which extends f.

Semantic model

Definition [2]

Model C of Jonsson theory T is called semantic model, if it is ω^+-homogeneous-universal.

Definition [2]

The center of Jonsson theory T is called an elementary theory of the its semantic model. And denoted through T^*, i.e. $T^* = Th(C)$.

Existence of semantic model

Fact 1. [2]

Each Jonsson theory T has k^+-homogeneous-universal model of power 2^k. Conversely, if a theory T is inductive and has infinite model and ω^+-homogeneous-universal model then the theory T is a Jonsson theory.

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Fact 2. [2]

Let T is a Jonsson theory. Two k-homogeneous-universal models M and M_1 of T are elementary equivalent.

Definition [2]

Jonsson theory T is called a perfect theory, if each a semantic model of theory T is saturated model of T^*.

Theorem [2]

Let T is a Jonsson theory. Then the following conditions are equivalent :

1. theory T is perfect ;
2. theory T^* is a model companion of theory T.

Let E_T be a class of all existentially closed models of theory T.

Theorem [2]

If T is a perfect Jonsson theory then $E_T = ModT^*$.

Agreements about positiveness and Δ

Let L is the language of the first order.

At is a set of atomic formulas of this language.

$B^+(At)$ is containing all the atomic formulas, and closed under positive Boolean combination and for sub-formulas and substitution of variables.

$L^+ = Q(B^+(At)$ is the set of formulas in prenex normal type obtained by application of quantifiers ($\forall$ and $\exists$) to $B^+(At)$.

$B(L^+)$ is any Boolean combination of formulas from L^+

$\Delta \subseteq B(L^+)$

Δ-Homomorphism

Let M and N are the structure of language, $\Delta \subseteq B(L^+)$.
The map $h : M \to N$ Δ-homomorphism (symbolically $h : M \leftrightarrow_\Delta N$, if for any $\varphi(\bar{x}) \in \Delta, \forall \bar{a} \in M$ such that $M \models \varphi(\bar{a})$, we have that $N \models \varphi(h(\bar{a})))$.

The model M is said to begin in N and we say that M continues to N, with $h(M)$ is a continuation of M. If the map h is injective, we say that h immersion M into N (symbolically $h : M \longleftrightarrow_\Delta N$). In the following we will use the terms Δ-continuation and Δ-immersion.

Δ-Joint Embedding Property and Δ-Amalgamation Property

Definition

The theory T admits $\Delta - JEP$, if for any $A, B \in ModT$ there are exist $C \in ModT$ and Δ-homomorphism's $h_1 : A \to_\Delta C$, $h_2 : B \to_\Delta C$.

Definition

The theory T admits $\Delta - AP$, if for any $A, B, C \in ModT$ with $h_1 : A \to_\Delta C$, $g_1 : A \to_\Delta B$, where h_1, g_1 are Δ-homomorphism's, there are exist $D \in Modt$ and $h_2 : C \to_\Delta D$, $g_2 : B \to_\Delta D$ where h_2, g_2 are Δ-homomorphism's such that $h_2 \circ h_1 = g_2 \circ g_1$.

Definition

Theory T called Δ-positive mustafinien (Δ-M)-theory, if as the morphisms considered only immersions and following conditions are true :

(1) theory T has infinite models,

(2) theory T is Π^{+}_{n+2}-axiomatizable,

(3) theory T admits Δ-JEP,

(4) theory T admits Δ-AP.

If there is no symbol Δ, it is means that we have deal with Jonsson theory.
Easy to see that Δ-Jonsson (Δ-positive generalization of Jonsson theories) theory is a special case Δ-Mustafinien theory.

Positively existentially closeness

Definition

The model A of the theory T is Δ-positively existentially closed, if for any Δ-homomorphism's $h : A \to_{\Delta} B$ and any $\bar{a} \in A$ and $\varphi(\bar{x}, \bar{y}) \in \Delta$, $B \models \exists \bar{y} \varphi(h(\bar{a}), \bar{y}) \Rightarrow A \models \exists \bar{y} \varphi(\bar{a}, \bar{y})$.

Let L is a countable language of first order. Let T be Δ-Jonsson perfect theory complete for existential sentences in the language L and its semantic model is a C.

Definition [2]

We say that a set X - Δ-definable if it is definable by some formula from Δ.

Definition [2]

The set X is called Δ-Jonsson in theory T, if it satisfies the following properties :

(1) X is Δ-definable subset of C ;

(2) $Dcl(X)$ is the support of some existentially closed submodel of C.

In what follows, for the sake of simplicity, we shall deal only with purely Jonsson theories. But all these results are also true for an corresponding Δ-M-theory.

Definition [2]

Let Jonsson theory T is complete for the existential sentences in the language L and C is its semantic model.
Let X is Jonsson subset of T, and M is existentially closed submodel of semantic model C of considered Jonsson theory T, where $dcl(X) = M$.
$Th_{\forall\exists}(M) = T_M$, T_M is the Jonsson fragment of Jonsson set X.

Let L is a countable first-order language. Let T is a arbitrary Jonsson theory in the language of first-order signature σ, C is a semantic model theory T. Let $A \subseteq C$ is Jonsson set in the theory T. Let $\sigma_\Gamma(A) = \sigma \cup \{c_a | a \in A\} \cup \Gamma$, where $\Gamma = \{P\} \cup \{c\}$.

Let $T_A^C = T \cup Th_{\forall\exists}(C, a)_{a \in A} \cup \{P(c_a) | a \in A\} \cup \{P(c)\} \cup \{"P \subseteq"\}$, where $\{"P \subseteq"\}$ is an infinite set of sentences, which says that a interpretation of symbol P is existentially closed submodel in the language of the signature $\sigma_\Gamma(A)$ and this model is a definable closure of the set A.

Let T^* is the center of Jonsson theory T_A^C and $T^* = Th(C')$, where C' is a semantic model of the theory T_A^C. By restriction of theory T_A^C to signature $\sigma_\Gamma(A) \setminus \{c\}$ the theory T_A^C becomes complete type. This type we call a central type of theory T relative to the Jonsson set A and let it denote through P_A^C.

The lattices of formulas

Let X is a Jonsson set in the theory T_A^C and M is existentially closed submodel of the semantic model C considered Jonsson theory T_A^C, where $dcl(X) = M$. Then let $Th_{\forall\exists}(M) = Fr(X)$, $Fr(X)$ is the Jonsson fragment of the Jonsson set X.

Consider an arbitrary $Fr(A)$-theory, then $E(Fr(A)) = \bigcup\limits_{n<\omega} E_n(Fr(A))$, where $E_n(Fr(A))$ is a lattice of positive existential formulas with n free variables.

Definition

Let A_1, A_2 is a Jonsson subsets of submodel of semantic model C of the theory T_A^C. We say that $Fr(A_1)$ and $Fr(A_2)$ is syntactically similar, if there is a bijection $f : E(Fr(A_1)) \to E(Fr(A_2))$ such that :

(1) the restriction of f to $E_n(Fr(A_1))$ is an isomorphism of lattices $E_n(Fr(A_1))$ and $E_n(Fr(A_2))$, $n < \omega$;

(2) $f(\exists v_{n+1}\varphi) = \exists v_{n+1} f(\varphi)$, $\varphi \in E_n(T)$, $n < \omega$;

(3) $f(v_1 = v_2) = (v_1 = v_2)$.

Jonsson similarity

Theorem

Let $Fr(A_1)$ and $Fr(A_2)$ – Σ-complete, perfect Jonsson theory. Then the following conditions are equivalent :

1. $Fr^*(A_1)$ and $Fr^*(A_2)$ is syntactically similar in the sense of [2] ;
2. $Fr(_1)$ and $Fr(_2)$ is syntactically similar in previous definition.

The question of A.D.Taymanov

A well-known question of academician A.D.Taymanov :

(*) What properties must have a Boolean algebras B_n, $n \in \omega$ that exists a complete theory T, such that B_n to be isomorphic to $F_n(T)$, $n \in \omega$?
The above mentioned question of A.D.Taymanov (*) in this case can be summarized as follows :

(**) What properties must have a lattice E_n, $n \in \omega$, that exists the theory $Fr^*(A)$, such that E_n to be isomorphic to $E_n(Fr^*(A))$, $n \in \omega$? Where $Fr^*(A)$ is the center of $Fr(A)$.

We say that the question (**) has a positive solution for the theory of $Fr^*(A)$, if there is a sequence of lattices E_n, $n \in \omega$, that E_n is isomorphic to $E_n(Fr^*(A))$, $n \in \omega$.

Definition

Let T_A^C is a theory. Its #-companion called theory $T^{\#}$ such that

(1) $(T^{\#})_{\forall} = T_{\forall}$;

(2) if the $T_{\forall} = T'_{\forall}$, then $T^{\#} = (T')^{\#}$;

(3) $T \subseteq T^{\#}$.

We have the following natural examples : if $\# \in \{o, *, e, f\}$, then we have accordingly Kaiser's hull of theory T, center of theory T, $Th(E_T)$, forcing companion theory T.

Theorem

Let T_A^C a perfect, complete for existential sentences Jonsson theory. Then the following conditions are equivalent :

1. a positive solution to the question (**) regarding of the theory $Fr^*(A)$;
2. a positive solution to the question (*) regarding $\#$-companion of theory $Fr(A)$, $\# \in \{*\,,\ 0\,,\ m\,,\ f\,,\ e\}$, where 0-companion is Kaiser's hull, $*$ - companion is a center, m-companion is a model companion, f-companion is the forcing companion, e-companion is an elementary theory of the class of existentially closed models of the theory T.

The existentially primness

Definition

The model of the theory is called algebraically prime if it is to be embedded in any model of this theory.

Definition

The inductive theory T is called the existentially prime if :

1. it has a algebraically prime model, the class of its AP (algebraically prime models) denote by AP_T ;
2. class E_T non trivial intersects with class AP_T, i.e. $AP_T \bigcap E_T \neq 0$.

Theorem

If T is a Jonsson theory and X is a Jonsson set in the theory T. And $X \subseteq C$, where C is semantic model of T, then T existentially prime.

Theorem

Let T is existentially prime Jonsson theory, X is a Jonsson set in the theory T. Then the following conditions are equivalent :

1. $Fr(X)$ is perfect ;
2. $E_n(Fr(X))$ is weakly complemented [2] ;
3. $E_n(Fr(X))$ is Stone algebra [2].

Theorem

Let T is existentially prime Jonsson theory, X is a Jonsson set in the theory T. Then the following conditions are equivalent :

(1) $Fr^*(X)$ is a Jonsson theory ;

(2) every $\varphi^T \in E_n(Fr(X))$ is weak quantifier-free addition [2].

Theorem

Let T_A^C is Jonsson theory. Then the following conditions are equivalent :

(1) $Fr(A)$ is perfect ;

(2) T_A^C has a model companion.

Theorem

Let T_A^C is a perfect Jonsson theory. Then the following conditions are equivalent :

1. $Fr(A)$ is complete ;
2. $Fr(A)$ model complete.

Theorem

Let T_A^C is perfect Jonsson theory. Then the following conditions are equivalent :

1. $Fr(A)$ is perfect ;
2. $Fr^*(A)$ model complete ;

Definition

The theory is called convex if for any its model $\mathfrak{A}$ and for any family $\{\mathfrak{B}_i \mid i \in \mathfrak{I}\}$ of substructures of $\mathfrak{A}$, which are models of the theory T, the intersection $\bigcap_{i \in I} \mathfrak{B}_i$ is a model of T.
If it is assumed that this intersection is not empty.
If this intersection is never empty, then the theory is called strongly convex.

Definition

The model of the signature of this theory (hereinafter structure) is called core if it is isomorphic to a single substructure of each model of this theory.

Definition

Model A of the theory T is called (Σ, Σ)-atomic if for each n each n-tuple of elements from A satisfies in A a certain formula from Σ which is complete for Σ-formulas.

Theorem

Let $Fr(A)$ is a theory in the language $\sigma_\Gamma(A)$ and M_1, M_2 is countable (Σ, Σ)-atomic models of the theory $Fr^*(A)$. Then these models M_1 and M_2 are isomorphic.

Theorem

Let T is perfect strongly convex existential prime $\exists$-complete Jonsson theory. M is countable model of the theory T. Then the following conditions are equivalent :

1. M is (Σ, Σ) atomic model ;
2. M is algebraically prime model.

Definition

Jonsson theory T is called Robinson's (R), if it is universally axiomatizable.

Theorem

Let T is strongly convex existentially prime, $\exists$-complete perfect R-theory. Then the following conditions are equivalent :

(1) theory T^* has core structure ;

(2) theory T_A^C has core model ;

(3) whenever $\varphi(x)$ is an existential formula and is provable in T, then there is some existential formula of $\psi(x)$ and integer number n such that in T $\exists^{=n} x \varphi \wedge \exists x (\varphi \wedge \psi)$, and if $T \models (\delta_1 \vee \delta_2)$, where δ_1, δ_2 is some existential sentences, then $T \models \delta_1$ or $T \models \delta_2$.

Theorem

Let theory T is perfect existentially prime strongly convex R-theory. Then $\mathfrak{M}$ is the core structure of T_A^C if and only if $\mathfrak{M}$ is the core model of the center T^* in the above mentioned enrichment.

Theorem

If the theory T_A^C is ω-categorical, then $Fr(A)$ is perfect.

Theorem

If the theory T_A^C is κ-categorical, then $\#$-companion for $Fr^*(A)$ is κ-categorical, $\kappa \geq \omega$.

Theorem

If the theory T_A^C is totally categorical, then T^* is not finite axiomatizable.

Through S^J_A denoted the set of all $\exists$-completion of the theory T^C_A. Let λ is arbitrary cardinal.

Definition

Jonsson theory T called Jonsson A-λ-stable (J-A-λ-stable) if $|S^J_A(X)| \leq \lambda$ for any set A power $\leq \lambda$.

Definition

Jonsson theory T is called a J-A-stable if T is a J-A-λ-stable for some λ.

Theorem

Let λ an arbitrary infinite cardinal, the T is convex, existentially prime, perfect, complete for $\exists$-sentences Jonsson theory. Then the following conditions are equivalent :

1. $(T^*)^F$ is λ-stable in the classical sense, where $(T^*)^F$ - forcing companion of the theory T^* in the rich signature ;
2. T^* is λ-stable in the classical sense.

Theorem

Let λ an arbitrary infinite cardinal, the T is convex, existentially prime, perfect, complete for $\exists$-sentences Jonsson theory. Then the following conditions are equivalent :

1. $(T_A^C)^F$ is J-A-λ-stable in the classical sense, where $(T_A^C)^F$ - forcing companion of the theory T_A^C in the rich signature ;
2. $(T_A^C)^*$ is λ-stable in the classical sense.

$J-\lambda-$stability

Let T - Jonsson theory, $S^J(X)$ - the set of all existential complete n-types over X, consistent with T, for every finite n, where $X \subset C$.

Definition

We say that a Jonsson theory T is $J-\lambda-$stable if, for any T-existentially closed model A, for any subset X of A, $|X| \leq \lambda \Rightarrow |S^J(X)| \leq \lambda$.

Theorem

Let A_1, A_2 is a Jonsson sets in J-ω-stable Jonsson theory T, a_1 is the realization of the central type $P^C_{A_1}$ and a_2 is the realization of the central type $P^C_{A_2}$. Then the following conditions are equivalent :

1. $T^C_{A_1}$ and $T^C_{A_2}$ is syntactically similar as Jonsson theories ;
2. $RM(a_1) = RM(a_2)$, where RM is Morley rank ;
3. $\exists \varphi \in Aut(C) : \varphi(a_1) = a_2$.

SB and JSB properties

Definition

The SB property for T is the same as any two elementarily bi-embeddable models being isomorphic.

Jonsson theory T has the Schroder-Bernstein (JSB) property if for any two models $A, B \in E_T$ from the fact that they are mutually isomorphically embeddable each other follows that they are isomorphic.

Lemma

Theory of Abelian groups is a Jonsson theory.

Lemma

Theory of Abelian groups is perfect.

Theorem

Let T be a Jonsson theory of Abelian groups, then the following conditions are equivalent :

1. T is $J-\omega-$stable ;
2. T^* is $\omega-$stable ;
3. T has JSB property.

Cosemanticness

Let $A \in Mod\ \sigma_{AG}$, where $\sigma_{AG} = \langle +, -, 0 \rangle$, i.e. our considered theories are universal. Denote through $JSp(A)$ Jonsson spectrum of Abelian group A, where $JSp(A) = \{T | T$ is a Jonsson theory in language σ_{AG} and $A \in ModT\}$.

We say that T_1 is cosemantic to T_2 ($T_1 \bowtie T_2$) if $C_{T_1} = C_{T_2}$, where C_{T_i} is semantic model of T_i, $i = 1, 2$. Then it is easy to notice that $JSp(A)/_{\bowtie}$ is a factor set by relation $\bowtie$.

Jonsson invariant of Abelian Group

Theorem

Let T is Jonsson theory of Abelian groups then $C_T \in E_T$ and C_T is divisible group and its a Shmelev's standart group is $\oplus_p \mathbb{Z}_{p^\infty}^{(\alpha_p)} \oplus \mathbb{Q}^{(\beta)}$, where $\alpha_p, \beta \in \omega^+$, $2^\omega = |C_T|$.

Let's call a pair $(\alpha_p, \beta)_{C_{[T]}}^A$ as Jonsson invariant of Abelian group A if a Shmelev's standart group of a group A is a group of the following form

$$\oplus_p \mathbb{Z}_{p^\infty}^{(\alpha_p)} \oplus \mathbb{Q}^{(\beta)},$$

where $C_{[T]}$ is semantic model of $[T] \in JSp(A)/_{\bowtie}$.

Definition

We will say that the model A is Jonnson elementarily equivalent to the model B ($A \underset{J}{\equiv} B$) if $JSp(A) = JSp(B)$.

Lemma

$\forall A, B \in \mathrm{Mod}\sigma \;\; JSp(A) \cap JSp(B) \neq \emptyset$.

We say that A JSp-cosemantic to B ($A \underset{JSp}{\bowtie} B$) if

$JSp(A)/_{\bowtie} = JSp(B)/_{\bowtie}$.

Lemma

We have the following implications : $A \equiv B \Rightarrow A \underset{J}{\equiv} B \Rightarrow A \underset{JSp}{\bowtie} B$,

where A, B - arbitrary Jonsson Abelian groups.

Main result

The following result is a Jonsson analogue of the well-known Shmelev's theorem about the elementary classification of Abelian groups.

We define the following set $\{(\alpha_p, \beta)^A_{C_{[T]}} : [T] \in JSp(A)/_{\bowtie}$, for all prime $p\}$ as the Jonsson invariant of the factor set $JSp(A)/_{\bowtie}$ and we denote it through $JInv(JSp(A)/_{\bowtie})$.

Theorem

Let $A, B \in Mod\ \sigma_{AG}$, then the following conditions are equivalent :

1. $A \underset{JSp}{\bowtie} B$;
2. $JInv(JSp(A)/_{\bowtie}) = JInv(JSp(B)/_{\bowtie})$.

Let L be a countable first-order language and T is some inductive theory in this language, E_T and AP_T are denoting correspondingly the following classes of this theory : class of all existentially closed models and class of all algebraically prime models.

Definition

The inductive theory T called existential-prime (EP), if it has a algebraically prime model and $AP_T \cap E_T \neq \emptyset$.

Definition

The theory T is called convex (C) if for any model $\mathfrak{A}$ and any family $\{\mathfrak{B}_i | i \in I\}$ of its substructures, which are models of the theory T, the intersection $\bigcap_{i \in I} \mathfrak{B}_i$ is a model theory T. It is assumed that this intersection is not empty. If this intersection is never empty, then the theory is called the strongly convex (SC).

Definition

An inductive theory is called an existentially prime strongly convex theory if it satisfies the above definitions simultaneously and is denoted by $EPSC$.

Let X be the Jonsson set in the theory T and M is existentially closed submodel semantic model $\mathfrak{C}$, where $dcl(X) = M$. Then let $Th_{\forall\exists}(M) = Fr(X)$, $Fr(X)$ is Jonsson fragment of Jonsson set X. Let A_1 and A_2 are Jonsson subset of the semantic model the some of Jonsson $EPSC$-theory. Where $Fr(A_1)$ and $Fr(A_2)$ are fragments of Jonsson sets A_1 and A_2.

We have the following result.

Theorem

Let $Fr(A_1)$ and $Fr(A_2)$ are $\exists$-complete perfect Jonsson theories. Then following conditions are equivalent :

1. $Fr(A_1)$ and $Fr(A_2)$ are J-syntactically similar as Jonsson theories [2] ;
2. $Fr(A_1)^*$ and $Fr(A_2)^*$ are syntactically similar as the complete theories [2], where $Fr(A_1)^*$ and $Fr(A_2)^*$ respectively be the centers of fragments of considered sets A_1, A_2.

We have deal with some J-ω-stable theory [2] and its semantic model. This thesis introduced and discussed the concepts of minimal Jonsson subsets and respectively strongly minimal Jonsson subsets of this semantic model.
We want to each Jonsson subset X of the semantic model assign ordinal number (or, perhaps, -1 or ∞) and it is the rank Morley of this set, denoted by $MR(X)$.

Let T is a fragment of some Jonsson set and it is a perfect Jonsson theory, C will be its semantic model. X is a definable subset of C.

Definition

$MR(X) \geq 0$ if and only if X is nonempty;
$MR(X) \geq \lambda$ if and only if $MR(X) \geq \alpha$ for all $\alpha < \lambda$ (λ is the limit ordinal);
$MR(X) \geq (\alpha + 1)$ if and only if in X exists an infinite family (X_i) disjoint $\exists$-definable subsets, such that $MR[(X_i)] \geq \alpha$ for all i.

Then Morley rank of set X is $MR(X) = \sup\{\alpha \mid MR(X) \geq \alpha\}$. Moreover, we assume that $MR(\varnothing) = -1$ and $MR(X) = \infty$ if $MR(X) \geq \alpha$ for all α (in the latter case we say that X has not rank).

Definition

An algebraically Jonsson set A is said to be independent if $\mathrm{acl}(A) \neq \mathrm{acl}(A')$ for any proper subset $A' \subset A$.
A maximal independent subset of a algebraically Jonsson set A is said to be a basis of A.

Lemma

Any two bases B and C of a algebraically Jonsson set A are of the same cardinality

Definition

Let M be an existentially closed minimal model of theory T and let $D \subseteq M^n$ be an infinite algebraically Jonsson set in theory T. We say that D is minimal in M if for any definable $Y \subseteq D$ either Y is finite or $D \setminus Y$ is finite. If $\phi(\bar{v}, \bar{a})$ is the formula that defines D, then we also say that $\phi(\bar{v}, \bar{a})$ is minimal.
We say that D and ϕ are strongly minimal if ϕ is minimal in any existentially closed extension N of M.
We say that a Jonsson theory T is strongly minimal if any its existential formula $\phi(\bar{v}, \bar{a})$ is finite or cofinite(i.e., if $M \models E_T$, then M is strongly minimal).

Definition

The degree of Morley $d_M(X)$ of Jonsson set X is the maximum length n of its decomposition $X = X_1 \cup ... \cup X_n$ into disjoint existentially definable subsets of rank Morley α.

Next, we standardly define pregeometry on the set of all subsets of the semantic model and the concept of strongly minimal Jonsson sets.

On this basis, it introduces the concept of the independence in the frame of special pregeometry under subsets of some existentially closed submodel. The notion of independence leads to the concept of basis and then we have an analogue of the theorem on uncountable categoricity for fragments of Jonsson set.

We consider countable language L and complete for existential sentences perfect Jonsson theory T in language L and its semantic models $\mathfrak{C}$. Let X be the Jonsson set in T and M is existentially closed submodel of the semantic model $\mathfrak{C}$, where $dcl(X) = M$. Then let $Th_{\forall\exists}(M) = Fr(X)$, $Fr(X)$ is the Jonsson fragment of Jonsson set X.

With the help of the nonforking notion we will give the notion of independence for Jonsson sets. Let M $\exists$-saturated existentially closed model power k (k enough big cardinal) of Jonsson theory T. Let A be the class of all Jonsson subsets of M and P is the class of all $\exists$-types (not necessarily complete), let JNF (Jonsson nonforking) $\subseteq P \times A$ be a binary relation. There is the list of the axioms 1-7 which defined Jonsson nonforking notion JNF and we have result for fragment $Fr(X)$ of the Jonsson set X.

Let M $\exists$-saturated existentially closed model power k (k enough big cardinal) of Jonsson theory T ($\exists$-saturation means the saturation relative to existential types). Let A be the class of all subsets M, P- the class of all $\exists$-types (not necessarily complete), let $JNF \subseteq P \times A$ - a binary relation. We put on JNF the following axiom :

Axiom 1.

If $(p, A) \in JNF, f : A \to B$ - automorphism M, then $(f(p), f(A)) \in JNF$.

Axiom 2.

If $(p, A) \in JNF, q \subseteq p$, then $(q, A) \in JNF$.

Axiom 3.

If $A \subseteq B \subseteq C, p \in S^J(C)$, then $(p, A) \in JNF \Leftrightarrow (p, B) \in JNF$ and $(p \restriction B, A) \in JNF$.

Axiom 4.

If $A \subseteq B, dom(p) \subseteq B, (p, A) \in JNF$, then $\exists q \in S^J(B)$, $p \subseteq q$ and $(q, a) \in JNF$.

Axiom 5.

There is a cardinal κ such that if $A \subseteq B \subseteq C, p \in S^J(C)$, $(p, A) \in JNF$ then $\mid \{q \in S^J(C) : p \subseteq q$ and $(q, a) \in JNF\} \mid < \kappa$.

Axiom 6.

There is a cardinal ρ such that if $\forall p \in P, \forall A \in A$, if $(p, A) \in JNF$, then $\exists A_1 \subseteq A, (\mid A_1 \mid < \rho)$ and $p, A_1 \in JNF$.

Axiom 7.

If $p \in S^J(A)$, then $(p, A) \in JNF$.

Theorem *

The following conditions are equivalent :

1. the relation JNF satisfies the axioms 1-7 relative to the fragment $Fr(X)$;
2. $Fr(X)^*$ stable and for all $p \in P$, $A \in A$ $((p, A) \in JNF \Leftrightarrow p$ not fork over $A)$ (in the classical meaning of S.Shelah), where $Fr(X)^*$ is the center of the fragment $Fr(X)$.

Independence

The nonforking extensions will be the "free" ones. Forking as in Theorem * can be used to give a notion of independence in J-ω-stable theories [2].

Definition

We say that $\overline{a}$ is independent from B over A if $\mathrm{tp}(\overline{a}/\mathrm{A})$ does not fork over $\mathrm{A} \cup \mathrm{B}$. We will denote this fact through $\overline{a} \perp_A B$.

This notion of independence for above mentioned Jonsson sets has many desirable properties : monotonicity, transitivity, finite basis, symmetry and etc.

Thanks a lot for your attention !