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RĒGA, 13 jūn 2012

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## Deligne's Riemann-Roch Theorem

Some standing assumptions:

everything is over an alg. closed field  $k = \bar{k}$

My varieties will (almost always) be smooth

Riemann-Roch for curves:

$C$  curve

$$D = \sum_{P \in C} n_P P \text{ divisor on } C$$

let  $L = \mathcal{O}(D)$  be the associated line bundle.

Then, if  $\chi(L) = h^0(L) - h^1(L)$

$$\text{R-R: } \chi(L) = \deg L + 1 - g, \quad g = g(C)$$

$$\text{Here: } \deg L = \sum n_P$$

Sketch of proof

$\chi(F) = \sum (-1)^i h^i(F)$  has the property

that for an exact sequence of sheaves:

$$0 \rightarrow F' \rightarrow F \rightarrow F'' \rightarrow 0 / C$$

$$\chi(F) = \chi(F') + \chi(F'')$$

This means that an expansion of the form

$$\chi(L) = \chi(L - \mathcal{O}_c) + \chi(\mathcal{O}_c)$$

where  $\chi(L - \mathcal{O}_c) := \chi(L) - \chi(\mathcal{O}_c)$

Two statements hold:

$$\chi(L - \mathcal{O}_c) = \deg L$$

$$\chi(\mathcal{O}_c) = 1 - g_c$$

$$\begin{array}{ccc} h^0(\mathcal{O}_c) & - & h^1(\mathcal{O}_c) \\ \text{"} & & \text{"} \\ 1 & & g_c \end{array} \quad \text{by Serre duality}$$

The first point can be proven by playing with exact sequences of the form

$$0 \rightarrow L(-p) \rightarrow L \rightarrow L|_p \rightarrow 0 \quad \square$$

General interest is understanding of  $h^0(L)$ , but somehow we can mostly say something about  $\chi(L)$ .

Next, what does the Riemann-Roch theorem say for surfaces?

