ABSTRACTS

Simon BRENDLE *(Columbia University)*

Robert BRYANT *(Duke University)*
Algebraically Constrained Special Holonomy Metrics and Second-order Associative 3-folds

There are various methods known now for constructing more-or-less explicit metrics with special holonomy; most of these rely on assumptions of symmetry and/or reduction. Another promising method for constructing special solutions to look for metrics that satisfy algebraic curvature conditions. This often leads to a study of structure equations that satisfy an overdetermined system of PDE, sometimes involutive sometimes not, and the theory of exterior differential systems is particularly well-suited for analyzing these problems.

In this talk, I will describe the ideas and the underlying techniques needed from the theory of exterior differential systems, illustrate the application in the most basic cases, and describe the results so far.

A similar program has been implemented for finding explicit calibrated submanifolds of the associated geometries and, time permitting, I will describe some of this work and the current results.

Yaiza CANZANI GARCIA *(University of North Carolina)*
Statistics of randomized Laplace eigenfunctions.

There are several questions about the behavior of Laplace eigenfunctions that are extremely hard to tackle and hence remain unsolved. Among the features that we don't fully understand yet are: the number of critical points, the size of the zero set, the number of components of the zero set, and the topology of such components. A natural approach is then to randomize the problem and study these features for a randomized version of the eigenfunctions. In this talk I will present several results that tackle the problems described above for random linear combinations of eigenfunctions (with Gaussian coefficients) on a compact Riemannian manifold. This talk is based on joint works with Boris Hanin and Peter Sarnak.

Gilles CARRON *(Nantes)*
Compactness of conformal metric with a critical integrability assumption.

This is a joint work with Clara Aldana (Luxembourg) and Samuel Tapie (Nantes).
In 1993, M. Gursky has shown a compactness result in $C^{1,\alpha}$ for conformal metrics $g = e^{2f}\ g_0$ of unit volume and uniform L^p-bound on the full curvature tensor (where $p > dim/2$). We will describe what kind of compactness theorem can be obtained for conformal metric with unit volume and uniform $L^{dim/2}$ bound on the scalar curvature.

Jeff CHEEGER *(Courant Institute of Mathematical Sciences)*
Noncollapsed Gromov-Hausdorff limit spaces with Ricci curvature bounded below.

We will discuss joint work with Aaron Naber and Wenshuai Jiang on the structure of noncollapsed pointed Gromov-Hausdorff limit spaces (X^n, d, p), with $\mathrm{Ric}_{M_i^n} \geq -(n-1)$ and $\mathrm{Vol}(B_1(p_i)) \geq v$. The main result states that for all $\epsilon > 0$, there is a decomposition, $X^n = \mathcal{R}_\epsilon \cup S_\epsilon$ with the following properties. The ϵ-regular set, $\mathcal{R}_\epsilon$ is $\eta(\epsilon)$-bi-Hölder equivalent to a smooth riemannian manifold, with $\eta(\epsilon) \to 1$ as $\epsilon \to 0$. Moreover, S_ϵ is rectfiable and the r-tubular neighborhood $T_r(S_\epsilon)$ satisfies $\mathrm{Vol}(B_1(p) \cap T_r(S_\epsilon)) \leq c(n, v, \epsilon) \cdot r^2$. In particular, the $(n-2)$-dimensional Hausdorff measure of $B_1(p) \cap S_\epsilon$ is $\leq c(n, v)$.

Tobias COLDING *(MIT)*
Optimal regularity for geometric flows

Many physical phenomema lead to tracking moving fronts whose speed depends on the curvature. The level set method has been tremendously succesful for this, but the solutions are typically only continuous. We will discuss results that show that the level set flow has twice differentiable solutions. This is optimal.

These analytical questions crucially rely on understanding the underlying geometry. The proofs draws inspiration from real algebraic geometry and the theory of analytical functions. Further developing these geometric techniques gives solutions to other analytical questions like Rene Thom's gradient conjecture for degenerate equations. We believe these results are the first instances of a general principle: Solutions of many degenerate equations should behave as if they are analytic, even when they are not. If so, this would explain various conjectured phenomena.

Finally, the techniques should have applications to other geometric flows. If time permits, then we will discuss results about this

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This is joint work with Bill Minicozzi.

Karsten GROVE *(Notre Dame)*
Positive curvature and beyond: A status report and future peek.

We will provide an overview of some of the main results around manifolds with positive and non-negative sectional curvature and discuss established and evolving approaches to the area.

Colin GUILLARMOU (*CNRS and Université Paris-Sud*)
Boundary and lens rigidity for non-convex manifolds

we show that the boundary distance function determine a simply connected surface with no conjugate points and that the lens data determine a non-trapping surface with no conjugate points. The novelty is that the manifold is not assumed to be « simple », i.e the boundary is not assumed convex. We also show new result on the geodesic ray transform when there is trapped set. This is joint work with Mazzucchelli and Tzou.

Ursula HAMENSTÄDT *(Rheinische Friedrich-Wilhelms-Universität Bonn)*
The geometry of 3-manifolds before and after Perelman

The rank of a hyperbolic manifold is the smallest number of generators of its fundamental group. McMullen conjectured that for all $k\geq 2$, the pointwise injectivity radius of a closed hyperbolic 3-manifold of rank at most k is uniformly bounded from above. We explain some methods which were introduced before and after the foundational work of Perelman to study these manifolds, and we show how these methods can be used to prove McMullen's conjecture in many cases including random 3-manifolds.

Dominique HULIN *(Université Paris-Sud)*
Harmonic coarse embeddings

The Schoen conjecture, recently proved by V. Markovic, states that any quasi-isometric map from the hyperbolic plane to itself is within bounded distance from a unique harmonic map. We generalize this result to coarse embeddings between two Hadamard manifolds with pinched curvature. This is a joint work with Yves Benoist.

Bruce KLEINER *(Courant Institute of Mathematical Sciences)*
Ricci flow, diffeomorphism groups, and the Generalized Smale Conjecture.

The Smale Conjecture (1961) may be stated in any of the following equivalent forms:

- The space of embedded 2-spheres in R^3 is contractible.
- The inclusion of the orthogonal group O(4) into the group of diffeomorphisms of the 3-sphere is a homotopy equivalence.
- The space of all Riemannian metrics on S^3 with constant sectional curvature is contractible.

While the analogous statement one dimension lower can be proven in many ways - for instance using the Riemann mapping theorem - Smale's conjecture turned out to be surprisingly difficult, and remained open until 1983, when it was proven by Hatcher using a deep combinatorial argument. Smale's Conjecture has a natural generalization to other spherical space forms: if *M* is a spherical space form with a Riemannian metric of constant sectional curvature, then the inclusion of the isometry group into the diffeomorphism group is a homotopy equivalence. The lecture will explain how Ricci flow through singularites, as developed in the last few years by John Lott, Richard Bamler, and myself, can be used to address this conjecture.

This is joint work with Richard Bamler.

Blaine LAWSON *(Stony Brook University)*
A Monge-Ampère Operator in Symplectic Geometry

The point of this talk is to introduce a polynomial differential operator which is an analogue of the classical real and complex Monge-Ampère equations. This operator makes sense on any symplectic manifold with a Gromov metric, and its solutions are exactly the functions obtained as upper envelopes of Lagrangian plurisubharmonic functions. Both the homogeneous and inhomogeneous Dirichlet problems for this operator are solved on Lagrangian convex domains, and the homogeneous result also holds for all other branches of the equation. In C^n a fundamental solution is established, where the inhomogeneous term is a delta function. There are many interesting open questions.

(Talk presented by P. PANSU)

André NEVES *(University of Chicago)*
Gromov's Weyl Law and Denseness of minimal hypersurfaces

Minimal surfaces are ubiquitous in Geometry but they are quite hard to find. For instance, Yau in 1982 conjectured that any 3-manifold admits infinitely many closed minimal surfaces but the best one knows is the existence of at least two.
In a different direction, Gromov conjectured a Weyl Law for the volume spectrum that was proven last year by Liokumovich, Marques, and myself.
I will cover a bit the history of the problem and then talk about recent work with Irie, Marques, and myself: we combined Gromov's Weyl Law with the Min-max theory Marques and I have been developing over the last years to prove that, for generic metrics, not only there are infinitely many minimal hypersurfaces but they are also dense.

Dorothee SCHÜTH *(Berlin)*
Spectrum and Curvature.

Spectral geometry is one of the many areas in which Marcel Berger did seminal work. He was the first to compute, in 1966, an explicit formula for the heat invariant a_2. The heat invariants are the coefficients which appear in the asymptotic expansion of the heat trace of a compact Riemannian manifold, and thus are determined by the eigenvalue spectrum of the Laplace operator. They are expressible in terms of the curvature of the manifold. In this context, we review old and new results and questions surrounding the intriguing interplay between spectrum and curvature, with an emphasis on polygons in surfaces.

Robert YOUNG *(Courant Institute of Mathematical Sciences)*

Quantifying nonorientability and filling multiples of embedded curves

Filling a curve with an oriented surface can sometimes be "cheaper by the dozen". For example, L. C. Young constructed a smooth curve drawn on a projective plane in R^n which is only about 1.5 times as hard to fill twice as it is to fill once and asked whether this ratio can be bounded below. This phenomenon is based on the nonorientability of the projective plane; we will define a new invariant that quantifies the nonorientability of a manifold in R^n and use it to answer L. C. Young's question.