On the black hole interior and thermalization

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IHES May 2017

Motivations

I will present a new natural class of non-equilibrium states, which exist in any chaotic quantum system. These may be interesting for

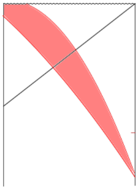
- Their relevance for black hole horizon and interior
- Their role in statistical mechanics

Black holes and thermodynamics

Laws of BH dynamics $\Leftrightarrow$ Laws of thermodynamics

Bekenstein-Hawking entropy

$$S = \frac{A}{4G}$$

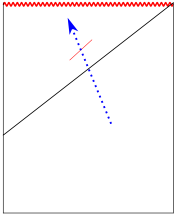


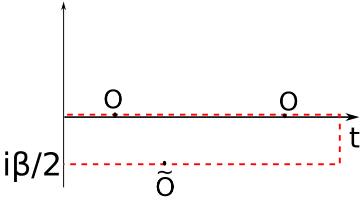
Dynamical properties, transport coefficients

Universality of black hole exterior geometry

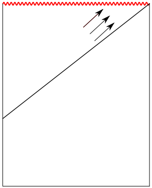
Role of interior?

Universality of interior geometry $\Rightarrow$ What does it mean in CFT?





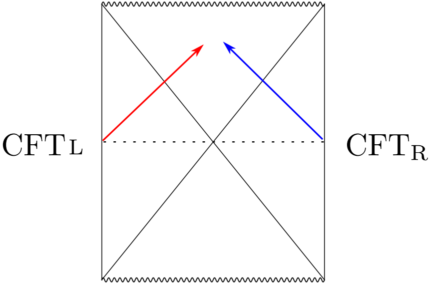
Stages of thermalization



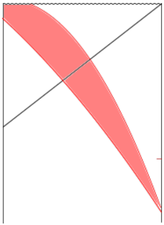
Main Claim

- I will describe a new class of non-equilibrium states which exist in any chaotic statistical system. In holographic CFTs these states correspond to excitations behind the black hole horizon.
- The number of such states is in 1-1 correspondence with possible ways to excite the horizon in General Relativity
- The existence of these states is motivated by, but logically independent from state-dependent operators.
- Their existence is robust (no subtleties about "generalization of quantum mechanics")
- Strong evidence the CFT contains the states in its Hilbert space which describe excitations of the interior and hence that the black hole interior is as predicted by GR

Behind the horizon in AdS/CFT?



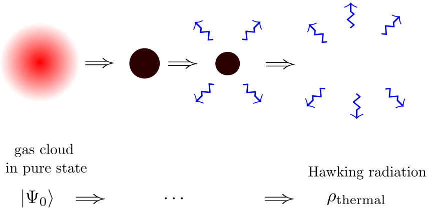
$$|TFD\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\frac{\beta E}{2}} |E\rangle \otimes |E\rangle$$

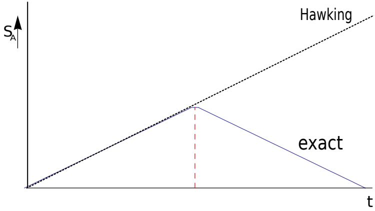


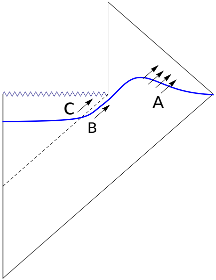
Interior for 1-sided black hole?

Firewall paradox in AdS/CFT

The information/firewall paradox







Violation of strong subadditivity of entanglement entropy

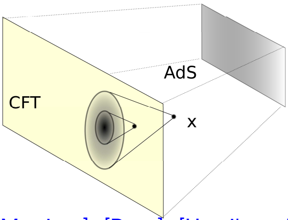
$$S_{AB} + S_{BC} \geq S_A + S_C$$

[Mathur], [Almheiri, Marolf, Polchinski, Sully]

Firewall paradox in AdS/CFT

- Large black holes in AdS are holographically dual to QGP states of $\mathcal{N} = 4$ in deconfined phase
- These black holes are in equilibrium with their Hawking radiation and do not evaporate
- Nevertheless the firewall paradox has been formulated even for these stable black holes [Almheiri, Marolf, Polchinski, Stanford, Sully], [Marolf, Polchinski]
- It suggests that big AdS black holes have a singular horizon.
- Most precise formulation of the paradox.

Local observables in AdS

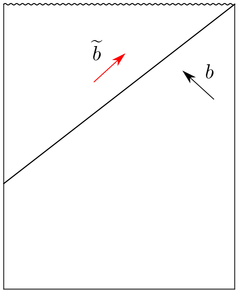


[Banks, Douglas, Horowitz, Martinec], [Bena], [Hamilton, Kabat, Lifschytz, Lowe]

$$\phi(x) = \int dY\, K(x, Y)\, \mathcal{O}(Y)$$

$\mathcal{O}$= local single trace operator dual to ϕ

K =known kernel



For smooth horizon effective field theory requires:

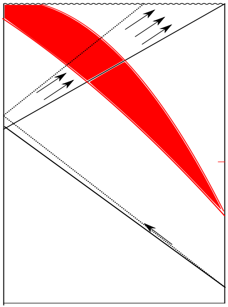
I) $\widetilde{b}$ commute with b

II) $\widetilde{b}$ entangled with b: $\quad (\widetilde{b} - e^{-\frac{\beta\omega}{2}} b^\dagger)|\Psi\rangle = 0$

$$\begin{array}{ccc} b & \Leftrightarrow & \mathcal{O} \\ \widetilde{b} & \Leftrightarrow & ? \end{array}$$

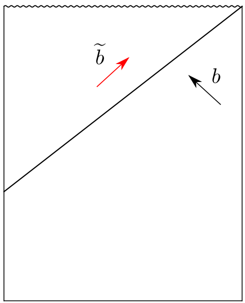
Which CFT operators $\widetilde{\mathcal{O}}$ correspond to $\widetilde{b}$?

Direct reconstruction?



- Transplanckian problem
- States formed by collapse form a small subset of typical BH microstates.

Firewall paradox for large AdS black holes



$$[b, b^\dagger] = 1$$
$$[H, b^\dagger] = \omega b^\dagger$$

$$[\widetilde{b}, \widetilde{b}^\dagger] = 1$$
$$[H, \widetilde{b}^\dagger] = -\omega \widetilde{b}^\dagger$$

- [AMPSS, MP] paradox: if typical CFT states have smooth horizon, using $[H, \widetilde{\mathcal{O}}^\dagger_\omega] = -\omega \widetilde{\mathcal{O}}^\dagger_\omega$ we find

$$Tr[e^{-\beta H} \widetilde{\mathcal{O}}^\dagger_\omega \widetilde{\mathcal{O}}_\omega] < 0$$

which is inconsistent

$$[H, \widetilde{\mathcal{O}}^\dagger_\omega] = -\omega \widetilde{\mathcal{O}}^\dagger_\omega \qquad [\widetilde{\mathcal{O}}_\omega, \widetilde{\mathcal{O}}^\dagger_\omega] = 1$$

$$\widetilde{\mathcal{O}}^\dagger_\omega = \text{“creation operator”}$$

$\Rightarrow$ $\widetilde{\mathcal{O}}^\dagger_\omega$ should not annihilate (typical) states of the CFT.

On the other hand

$$[H, \widetilde{\mathcal{O}}^\dagger_\omega] = -\omega \widetilde{\mathcal{O}}^\dagger_\omega$$

implies that $\widetilde{\mathcal{O}}^\dagger_\omega$ lowers the energy so it maps CFT states of energy E to $E - \omega$.

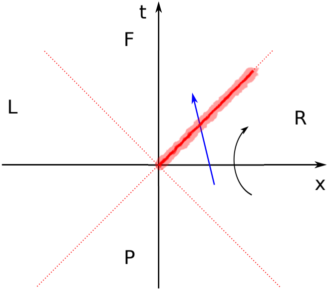
But in the CFT, we have $S(E) > S(E - \omega)$.

$\Rightarrow$ if $\widetilde{\mathcal{O}}^\dagger_\omega$ is an ordinary linear operator, it must have a nontrivial kernel.

Inconsistent with statement that $\widetilde{\mathcal{O}}^\dagger_\omega$ does not annihilate states.

$\Rightarrow$ The CFT does not contain $\widetilde{\mathcal{O}}^\dagger_\omega$ operators and cannot describe the BH interior (?) [AMPSS], [Marolf, Polchinski]

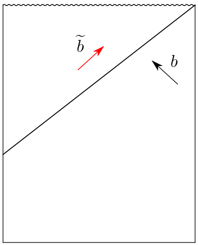
Entanglement vs typicality



$$\frac{1}{\sqrt{Z}} \prod_{\omega} \sum_{n=0}^{\infty} e^{-\pi\omega n} e^{i\theta_n} |n\rangle_L \otimes |n\rangle_R$$

$$\langle T_{\mu\nu} \rangle \neq 0$$

Rindler Horizon excited



BH $\Rightarrow$ large number of microstates

If $\mathcal{O}_\omega, \widetilde{\mathcal{O}}_\omega$ are fixed operators, how can typical CFT states have the precise entanglement needed for

$$(\widetilde{\mathcal{O}}_\omega - e^{-\frac{\beta\omega}{2}} \mathcal{O}_\omega^\dagger)|\Psi\rangle = 0$$

A construction of the BH interior

[KP and S.Raju]

- If we take a QFT state $|\Psi\rangle$ of high energy, we expect that at late times it will thermalize.

$$\langle\Psi|\mathcal{O}_1(x_1)...\mathcal{O}_n(x_n)|\Psi\rangle \approx Z^{-1}\mathrm{Tr}(e^{-\beta H}\mathcal{O}_1(x_1)...\mathcal{O}_n(x_n))$$

- This is true only for simple observables $n \ll N$

- Thermalization of pure state $\Rightarrow$ must have the notion of a small algebra of observables

- In a large N gauge theory, natural small "algebra" $\mathcal{A} =$ products of few, single trace operators

A construction of the BH interior

- Even though we are in an isolated CFT in a pure state, the small algebra of single trace operators probes the pure state $|\Psi\rangle$ as if it were an entangled state

$$\langle\Psi|..|\Psi\rangle \approx Tr[e^{-\beta H}...] \quad \leftrightarrow \quad |TFD\rangle = \sum_E \frac{e^{-\beta E/2}}{\sqrt{Z}} |E\rangle \otimes |E\rangle$$

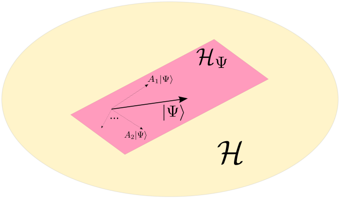
- What is the physical meaning of the second copy?

- The $O(N^2)$ d.o.f. of the CFT play the role of the "heat bath" for the small algebra of single trace operators

- Whatever operators the single trace operators $\mathcal{O}$ are entangled with, will play the role of $\widetilde{\mathcal{O}}$ behind the horizon.

- How do we identify these operators concretely?

The "small Hilbert space $\mathcal{H}_\Psi$"



We define

$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle = \{\text{span of} : \mathcal{O}(x_1)...\mathcal{O}(x_n))|\Psi\rangle\}$$

Simple EFT experiments in the bulk, around BH $|\Psi\rangle$ take place within $\mathcal{H}_\Psi$

$\mathcal{H}_\Psi$ is similar to "code subspace"

The interior operators $\widetilde{\mathcal{O}}$ will be defined to act only on this subspace.

Small algebra of observables

If $|\Psi\rangle$ is a BH microstate, we have the nontrivial property

$$A|\Psi\rangle \neq 0 \qquad \forall A \in \mathcal{A}\ ,\ A \neq 0$$

Physically this means that the state seems to be entangled when probed by the algebra $\mathcal{A}$.

Tomita-Takesaki modular theory

Algebra, cannot annihilate state.

$\Rightarrow$ the representation of the algebra is reducible, and the algebra has a nontrivial commutant acting on the same space.

Define antilinear map

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle$$

and

$$\Delta = S^\dagger S \qquad J = S\Delta^{-1/2}$$

Then the operators

$$\boxed{\widetilde{O} = JOJ}$$

i) commute with $\mathcal{O}$
ii) are correctly entangled with $\mathcal{O}$
These are the operators that we need for the Black Hole interior.

The modular Hamiltonian

The operator $\Delta = S^\dagger S$ is a positive, hermitian operator and can be written as

$$\Delta = e^{-K}$$

where

$$K = \text{modular Hamiltonian}$$

Using large N and the KMS condition for thermal correlators in equilibrium states

$$K = \beta(H_{CFT} - E_0)$$

The mirror operators

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta\omega}{2}}\mathcal{O}^\dagger_\omega|\Psi\rangle$$

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

$$[H,\widetilde{\mathcal{O}}_\omega]\mathcal{O}....\mathcal{O}|\Psi\rangle = \omega\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle$$

These equations define the operators $\widetilde{\mathcal{O}}$ on a subspace $\mathcal{H}_\Psi \subset \mathcal{H}_{\rm CFT}$, which is relevant for EFT around BH microstate $|\Psi\rangle$

$$\mathcal{H}_\Psi = {\rm span}\mathcal{A}|\Psi\rangle$$

Equations admit solution because the algebra $\mathcal{A}$ cannot annihilate the state $|\Psi\rangle$

Reconstructing the interior

$$\phi(t,r,\Omega)=\int_0^\infty d\omega\Big[\mathcal{O}_\omega\, f_\omega(t,\Omega,r)+\widetilde{\mathcal{O}}_\omega g_\omega(t,\Omega,r)+\mathrm{h.c.}\Big]$$

Smooth spacetime at the horizon, no firewall

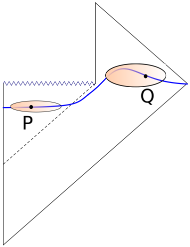
Non-locality

$\widetilde{\mathcal{O}}$ were constructed based on the fact that we restricted our attention to a "small algebra" of $\mathcal{O}$'s. The construction breaks down if the "small algebra" is enlarged to include all operators

$[\mathcal{O}, \widetilde{\mathcal{O}}] = 0$ only in simple correlators, not as operator equation, but only on "small Hilbert space" $\mathcal{H}_\Psi$

Operators $\widetilde{\mathcal{O}} =$ complicated combinations of $\mathcal{O}$.

Realization of BH complementarity



$$[\phi(P), \phi(Q)] \sim e^{-S}$$

$$[\phi(P), \Phi^{\text{complex}}(Q)] = O(1)$$

The hilbert space of Quantum Gravity does not factorize as $\mathcal{H}_{\text{inside}} \otimes \mathcal{H}_{\text{outside}}$

1) Solves problem of Monogamy of Entanglement (and avoids Mathur's theorem on small corrections)

2) Is consistent with locality in EFT

State-dependence

- Interior operators defined by

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta\omega}{2}}\mathcal{O}_\omega^\dagger|\Psi\rangle$$

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

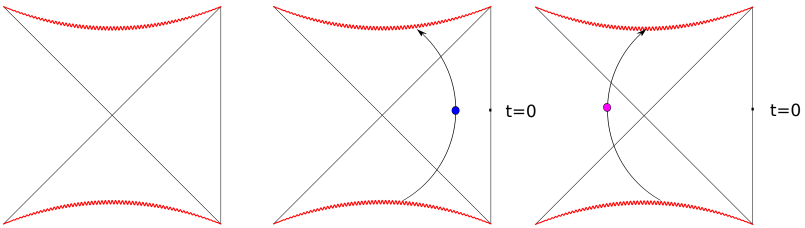
$$[H,\widetilde{\mathcal{O}}_\omega]|\Psi\rangle = \omega\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

- Solution depends on reference state $|\Psi\rangle$
- Operators cannot be upgraded to “globally defined” operators
- **State-dependence solves Entanglement vs Typicality problem naturally**
- Unusual in Quantum Mechanics, needs further study

A new class of non-equilibrium states

- The previous construction of the BH interior naturally identifies a new class of non-equilibrium states present in any chaotic statistical system. In holographic CFTs these states correspond to excitations behind the black hole horizon.
- The number of such states is in 1-1 correspondence with possible ways to excite the horizon in General Relativity
- The existence of these states is motivated by, but logically independent from state-dependent operators.
- Their existence is robust (no subtleties about "generalization of quantum mechanics")
- The CFT contains in its Hilbert space states which describe excitations of the interior $\Rightarrow$ evidence that the black hole interior is as predicted by GR

A new class of non-equilibrium states



$|\Psi\rangle$ $\qquad$ $U(\mathcal{O})|\Psi\rangle$ $\qquad$ $U(\widetilde{\mathcal{O}})|\Psi\rangle$

- $|\Psi\rangle$ = equilibrium state
- $U(\mathcal{O})|\Psi\rangle$ = standard non-equilibrium state (near equilibrium)
- $U(\widetilde{\mathcal{O}})|\Psi\rangle$ = new type of non-equilibrium state

We will argue that these states exist independent of the $\widetilde{\mathcal{O}}$ construction.

Equilibrium states

We define them by demanding that "reasonable observables" are time independent

$$\frac{d}{dt}\langle\Psi|A(t)|\Psi\rangle = 0$$

Typical states are equilibrium states

$$|\Psi\rangle = \sum_i c_i|E_i\rangle \qquad c_i \Rightarrow \text{random coefficients with some measure}$$

Atypical states may be time-dependent, hence non-equilibrium.

Unter time evolution even these states will equilibrate, hence an atypical state will start looking like a typical state.

Typicality in large Hilbert spaces

[Lloyd], [Balasubramanian, Czech, Hubeny, Larjo, Rangamani, Simon]

More precisely: any pure state $|\Psi\rangle = \sum_i c_i |E_i\rangle$ obeys

$$\sum_i |c_i|^2 = 1$$

and defines a point on $\mathbb{S}^{2N-1}$. Define uniform measure on this sphere (Haar measure). Then

$$\overline{\langle\Psi|A|\Psi\rangle} = \mathrm{Tr}[\rho A]$$

and

$$\overline{(\langle\Psi|A|\Psi\rangle - \mathrm{Tr}[\rho A])^2} = \frac{1}{e^S + 1}\left[\mathrm{Tr}[\rho A^2] - \mathrm{Tr}[\rho A]^2\right]$$

true for **any** operator A and independent of the Hamiltonian H

Relevance for fuzzball proposal.

Eigenstate Thermalization Hypothesis (ETH)

Consider an observable A in the small algebra. Then its matrix elements on energy eigenstates are

$$\langle E_i|A|E_j\rangle = f(E_i)\delta_{ij} + e^{-S/2}g(E_i, E_j)R_{ij}$$

where f, g are smooth functions of the energy, of magnitude which is $O(S^0)$ and R_{ij} are "erratic phases".

Implications of ETH

Consider typical state $|\Psi\rangle = \sum_i c_i |E_i\rangle$. Each coefficient has $|c_i| \sim e^{-S/2}$ but phase randomly distributed. We have

$$\langle\Psi|A|\Psi\rangle = \sum_{ij} c_i^* c_j \langle E_i|A|E_j\rangle$$

$$= \sum_i |c_i|^2 f(E_i) + \sum_{i\neq j} c_i^* c_j e^{-S/2} g(E_i, E_j) R_{ij}$$

If c_i highly peaked in energy, the first term is $\mathrm{Tr}[\rho_m A]$. The second term is of order

$$e^{-3S/2} \sum (e^{2S} \text{random phases}) \sim e^{-3S/2} \sqrt{e^{2S}} \sim e^{-S/2}$$

Implications of ETH

We recover that for typical states

$$\langle\Psi|A|\Psi\rangle = \mathrm{Tr}[\rho_m A] + O(e^{-S/2})$$

Now we will show that

$$\text{typical states} \qquad \Rightarrow \qquad \text{equilibrium states}$$

Need to consider typical state and prove that

$$\frac{d}{dt}\langle\Psi|A(t)|\Psi\rangle = 0$$

Equivalently: we define the Fourier modes

$$A_\omega = \int dt e^{i\omega t} A(t)$$

and need to show

$$\langle\Psi|A_\omega|\Psi\rangle = 0 \qquad \forall\omega \neq 0$$

Implications of ETH

The operator A_ω changes the energy by ω. Hence the diagonal term is absent

$$\langle E_i | A_\omega | E_j \rangle = e^{-S/2} g(E_i, E_j) R_{ij}$$

We only get the off-diagonal terms, so

$$\langle \Psi | A_\omega | \Psi \rangle \sim O(e^{-S/2})$$

Hence typical states are time-independent and in equilibrium.

Importance of phases in QM and thermalization

Time evolution in QM

$$|\Psi(t)\rangle = \sum_i c_i e^{-iE_i t}|E_i\rangle$$

Consider atypical state of black hole with particle outside horizon

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

It is a superposition of static BH microstates. The phases of the coefficients c_i have to be fined tuned to give constructive interference to the observable which measures the particle outside horizon.

Under time evolution phases decohere and particle is lost behind the horizon.

Thermalization, Poincare Recurrences

Suppose we start with an atypical state, for which at $t = 0$ we have

$$\langle \Psi | A(t = 0) | \Psi > \neq \mathrm{Tr}[\rho A]$$

This is due to fine-tuned phases in the off-diagonal terms.

However, under time evolution

$$\langle \Psi | A(t) | \Psi \rangle = \sum_i |c_i|^2 f(E_i) + \sum_{i \neq j} c_i^* c_j e^{-S/2} g(E_i, E_j) R_{ij} e^{-i(E_i - E_j)t}$$

the phases decohere and the state equilibrates.

However over very long time scales (of order e^S or e^{e^S}) the phases of the 2nd term may align to give a large contribution agan [Barbon, Rabinovici]

The state is in equilibrium/typical for most of its lifetime, but can undergo spontaneous fluctuations.

Standard non-equilibrium states of conventional kind

Take equilibrium state $|\Psi_0\rangle$ and excite it as

$$|\Psi'\rangle = U(O)|\Psi_0\rangle$$

and example might be $U(O) = e^{i\theta(O_\omega + O_\omega^\dagger)}$
(quench vs autonomous state)

Let us check that this is indeed a non-equlibrium state

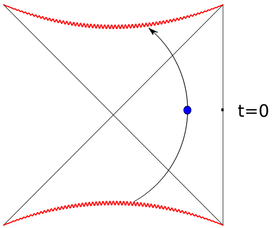
$$\langle\Psi'|O_\omega|\Psi'\rangle = \langle\Psi_0|U(O)^\dagger O_\omega U(O)|\Psi_0\rangle$$

$$= \langle\Psi_0|O_\omega|\Psi_0\rangle + i\theta\langle\Psi_0|[O_\omega, O_\omega^\dagger]|\Psi_0\rangle + \mathcal{O}(\theta^2)$$

The first term is zero, but the second is $O(1)$.

So we find that these are indeed time dependent states

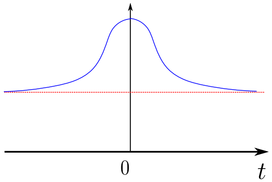
Bulk interpretation of standard non-eq states



$U(\mathcal{O})|\Psi\rangle$

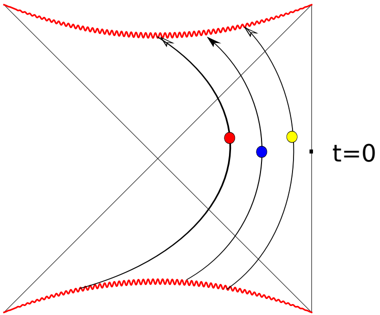
Excited state $U(\mathcal{O})|\Psi_0\rangle$

$\langle\Psi_0|U^\dagger O(t)U|\Psi_0\rangle = t$-dependent



Space of non-equilibrium states

They are in 1-1 correspondence to possible excitations in region outside the horizon



$$U_1(\mathcal{O})U_2(\mathcal{O})U_3(\mathcal{O})|\Psi_0\rangle$$

The new non-equilibrium states

Consider the states

$$|\Psi'\rangle = U(\widetilde{O})|\Psi_0\rangle$$

where $|\Psi_0\rangle$ is an equilibrium state and

$$U(\widetilde{O}) = e^{i\theta(\widetilde{O}_\omega + \widetilde{O}^\dagger_\omega)}$$

The main points about these states:

1) They seem to be equilibrium states wrt the algebra $\mathcal{A}$

2) In reality they are time-dependent, non-equilibrium states. This can be seen by correlators including H in the list of observables.

3) The existence of these states is robust and does not rely on state dependence.

4) Bulk interpretation: particles behind the horizon. The number of such state is in 1-1 correspondence with possible excitations behind the horizon.

5) We will explain the microscopic nature of these peculiar states.

New non-equilibrium states

Consider the new state

$$|\Psi'\rangle = U(\widetilde{O})|\Psi_0\rangle$$

For operators O in the small algebra $\mathcal{A}$ we have

$$\langle\Psi'|OO...O|\Psi'\rangle = \langle\Psi_0|U(\widetilde{O})^\dagger OO...OU(\widetilde{O})|\Psi_0\rangle$$

$$= \langle\Psi_0|OO...O|\Psi_0\rangle$$

so the state seems to be in equilibrium.

In particular

$$\frac{d}{dt}\langle\Psi'|A(t)|\Psi'\rangle = 0$$

New non-equilibrium states

On the other hand, if we include H then

$$\langle \Psi' | HOO...O | \Psi' \rangle \neq \langle \Psi_0 | HOO...O | \Psi_0 \rangle$$

so the state seems not to be in equilibrium. Here we used $[H, \widetilde{O}] \neq 0$.

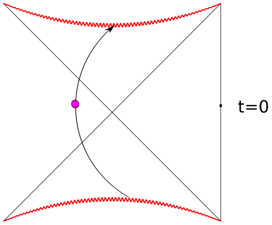
In particular **it is possible** to find observables A in the small algebra, such that

$$\frac{d}{dt} \langle \Psi' | HA(t) | \Psi' \rangle \neq 0$$

and the RHS is O(1). So these states are genuinely time-dependent.

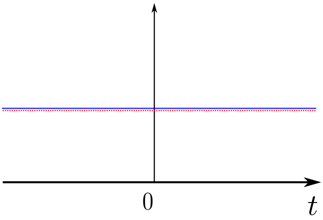
Bulk interpretation of new non-eq states

Excited state $|\Psi'\rangle = U(\widetilde{\mathcal{O}})|\Psi_0\rangle$

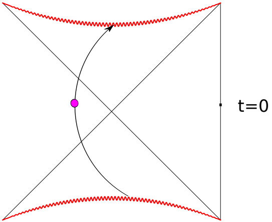


$U(\widetilde{\mathcal{O}})|\Psi\rangle$

$\langle\Psi'|A(t)|\Psi'\rangle$

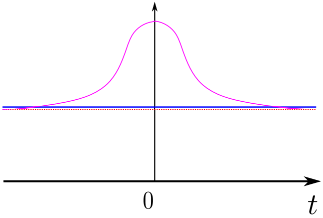


Bulk interpretation of new non-eq states



$U(\widetilde{\mathcal{O}})|\Psi\rangle$

$\langle\Psi'|A(t)H|\Psi'\rangle$



New non-equilibrium states

The new states that we defined seem to depend on the the state-dependent operators $\widetilde{O}$.

$$|\Psi'\rangle = U(\widetilde{O})|\Psi_0\rangle$$

However, we notice that they can be rewritten as

$$\boxed{|\Psi'\rangle = e^{-\frac{\beta H}{2}} U(O)^\dagger e^{\frac{\beta H}{2}} |\Psi_0\rangle}$$

All previous claims can be rederived without any reference to state-dependent operators

i) Such states appear to be equilibrium when probed by the O's
ii) It can be seen that they are non-equilibrium by including H in correlators

The Hilbert space of the $\mathcal{N}=4$ SYM contains states which can be naturally identified with black holes with excitations behind the horizon.

Notice that operator

$$e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}}$$

is not a unitary. However the state

$$|\Psi'\rangle = e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}} |\Psi_0\rangle$$

has unit norm. Indeed

$$\langle \Psi' | \Psi' \rangle = \langle \Psi_0 | e^{\frac{\beta H}{2}} U(O)^\dagger e^{-\frac{\beta H}{2}} e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}} |\Psi_0\rangle$$

$$= \frac{1}{Z} \mathrm{Tr}[e^{-\beta H} e^{\frac{\beta H}{2}} U(O)^\dagger e^{-\frac{\beta H}{2}} e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}}] + O(1/S)$$

$$= 1 + O(1/S)$$

Alternatively we can produce the same state via state-dependent unitary operator

$$U(\widetilde{O}) |\Psi_0\rangle = e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}} |\Psi_0\rangle$$

Similarly, consider

$$\langle\Psi'|A|\Psi'\rangle = e^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}\langle\Psi_0|e^{\frac{\beta H}{2}}U(O)^\dagger e^{-\frac{\beta H}{2}}Ae^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}|\Psi_0\rangle$$

$$= \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}e^{\frac{\beta H}{2}}U(O)^\dagger e^{-\frac{\beta H}{2}}Ae^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}] + O(1/S)$$

$$= \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}A] + O(1/S)$$

$$\langle\Psi_0|A|\Psi_0\rangle + O(1/S)$$

Similarly, consider

$$\langle\Psi'|A|\Psi'\rangle = e^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}\langle\Psi_0|e^{\frac{\beta H}{2}}U(O)^\dagger e^{-\frac{\beta H}{2}}Ae^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}|\Psi_0\rangle$$

$$= \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}e^{\frac{\beta H}{2}}U(O)^\dagger e^{-\frac{\beta H}{2}}Ae^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}] + O(1/S)$$

$$= \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}A] + O(1/S)$$

$$\langle\Psi_0|A|\Psi_0\rangle + O(1/S)$$

On the other hand, to see that the state is non-equilibrium we can consider $\hat{H} = H - E_0$ and then

$$\langle \Psi' | A\hat{H} | \Psi' \rangle = \langle \Psi_0 | V^\dagger A\hat{H} V | \Psi_0 \rangle \approx \langle \Psi_0 | V^\dagger A [\hat{H}, V] | \Psi_0 \rangle$$

$$= \frac{1}{Z} \mathrm{Tr}[e^{-\beta H} V^\dagger A [\hat{H}, V]] + O(1/S)$$

In general the first term is different from the expectation value of $A\hat{H}$ by an $O(1)$ amount.

We select $U = \exp\left[i\theta(A_\omega + A_\omega^\dagger)\right]$, and we measure the operator $A_\omega^\dagger \hat{H}$. In an honest equilibrium state we have

$$\langle\Psi_0|A_\omega^\dagger \hat{H}|\Psi_0\rangle = O(e^{-S/2})$$

On the other hand, in the new states we have

$$\langle\Psi'|A_\omega^\dagger \hat{H}|\Psi'\rangle = \langle\Psi_0|A_\omega^\dagger \hat{H}|\Psi_0\rangle$$

$$+i\theta\langle\Psi_0|\left[A_\omega^\dagger \hat{H}(e^{\frac{\beta\omega}{2}}A_\omega + e^{-\frac{\beta\omega}{2}}A_\omega^\dagger) - (e^{\frac{\beta\omega}{2}}A_\omega^\dagger + e^{-\frac{\beta\omega}{2}}A_\omega)A_\omega^\dagger \hat{H}\right]|\Psi_0\rangle$$

The linear term is:

$$i\theta\langle\Psi_0|\left[e^{\frac{\beta\omega}{2}}A_\omega^\dagger \hat{H} A_\omega - e^{-\frac{\beta\omega}{2}}A_\omega A_\omega^\dagger \hat{H}\right]|\Psi_0\rangle$$

we use the commutator $[H, A_\omega] = -\omega A_\omega$ to get

$$i\theta\langle\Psi_0|\left[e^{\frac{\beta\omega}{2}}A_\omega^\dagger A_\omega - e^{-\frac{\beta\omega}{2}}A_\omega A_\omega^\dagger\right]\hat{H}|\Psi_0\rangle - i\omega\theta e^{\frac{\beta\omega}{2}}\langle\Psi_0|A_\omega^\dagger A_\omega|\Psi_0\rangle \tag{1}$$

The second term is $O(1)$. The first term, using KMS, is almost zero.

Microscopic description

When we excite the state as

$$|\Psi'\rangle = U(O)|\Psi_0\rangle$$

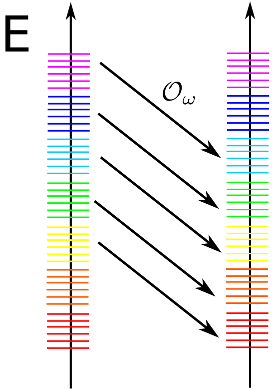
we have

$$\langle\Psi'|\mathcal{O}_\omega|\Psi'\rangle \sim O(1)$$

How can it be consistent with ETH?
Write $|\Psi'\rangle = U(O)|\Psi_0\rangle = \sum_i c_i|E_i\rangle$

$$\langle\Psi'|\ \mathcal{O}_\omega|\Psi'\rangle = \sum_{ij} c_i^* c_j f(E_i, E_j) R_{ij} e^{-S/2}$$

The reason that this is not $O(e^{-S/2})$ is that the coefficients c_i, R_{ij} are now correlated. Notice that under time evolution we have $c_i \to c_i e^{-iE_i t}$ and after a while the phases randomize again and state $|\Psi'\rangle$ equilibrates (particle falls into horizon).



Define projection operators P_E on states between E and $E + \delta E$. We can decompose an excited state as

$$|\Psi'\rangle \equiv U(\mathcal{O})|\Psi\rangle = \sum_E P_E|\Psi'\rangle = \sum_E |\Psi'_E\rangle$$

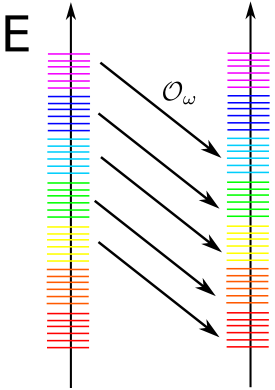
then we have

$$\langle\Psi'|\mathcal{O}_\omega|\Psi'\rangle = \sum_E \langle\Psi'_{E-\omega}|\mathcal{O}_\omega|\Psi'_E\rangle = \sum_E g(E)$$

with each $g(E) \sim \mathcal{O}(1)$

The microscopic phases in the component of the state $|\Psi'\rangle_E$ are correlated with those of $|\Psi'\rangle_{E-\omega}$

Each "energy bin" is out of equilibrium



Consider the new excited states

$$|\Psi''\rangle = e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}} |\Psi_0\rangle = e^{-\beta \frac{H-E_0}{2}} U(\mathcal{O}) |\Psi\rangle$$

The microscopic phases are the same as in $U(\mathcal{O})|\Psi\rangle$, but the prefactor $e^{-\beta \frac{H-E_0}{2}}$ enhances lower energy vs higher energy states.

Inserting again the projection operators P_E on energy bins we find

$$\langle \Psi'' | \mathcal{O}_\omega | \Psi'' \rangle = \sum_E e^{-\beta \frac{E-E_0}{2}} g(E) = 0$$

where the $g(E) \sim O(1)$ and nonzero!

Each energy bins is out of equilibrium, however the time dependence from different bins cancels out (due to $e^{-\beta \frac{E-E_0}{2}}$) and the state seems to be equilibrium wrt the algebra $\mathcal{A}$.
Cancellations spoiled when inserting $H \Rightarrow$ can detect excitation

Summary

We argued that there is a canonical class of non-equilibrium states, of the form

$$e^{-\beta(H-E_0)/2}U(O)|\Psi_0\rangle$$

which are parametrized in a similar way as perturbations outside horizon (i.e. by unitaries $U(O)$) — yet the perturbations are undetectable by single trace operators.

This indicates the existence of a seemingly causally disconnected region of spacetime in the bulk, whose natural interpretation is the region behind the horizon.

The tilde operators cause transitions between these states.

But the existence of these states is rather robust (no need to use state-dependent operators)

They exist in any chaotic statistical system. What is their meaning for the strongly coupled QGP?

Additional evidence that large AdS black holes have a smooth

Thank you