The New Large D Limit of Matrix Models: Theory and Applications

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Plan of the talk

1. Short general introduction

2. The new large D limit of matrix models and a new large N limit of tensor models

3. On the phase diagram of planar matrix quantum mechanics

1. Short general introduction

In a series of recent developments, interesting toy models for quantum black holes have been built and studied.

The first class of models is based on large N fermionic systems with quenched disorder and was proposed by Kitaev, building on previous studies in the condensed matter literature by Sachdev, Ye, Georges, Parcollet and others.

The second class of models is based on large N tensor theories and were first proposed by Witten, building on the tensor model technology developed by Gurau and collaborators.

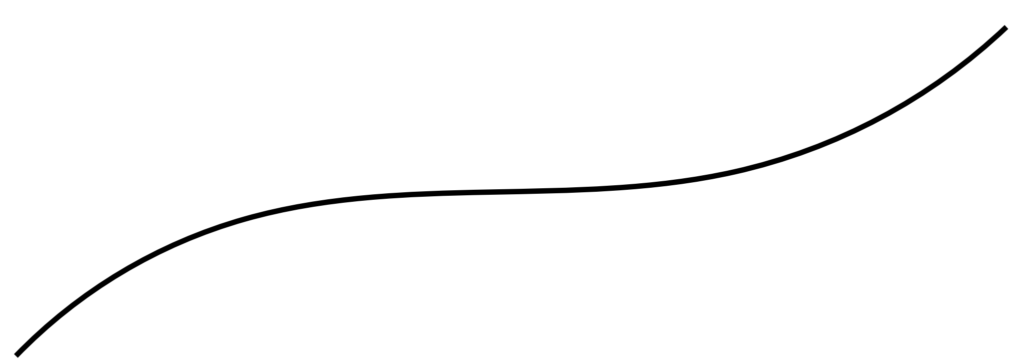
These models are able to capture very non-trivial properties of black holes, including the quasi-normal behavior and chaos.

The advantage of the tensor models over models with quenched disorder is that they are genuine quantum theories at finite N; in particular, there is no need to limit the investigations to self-averaging quantities.

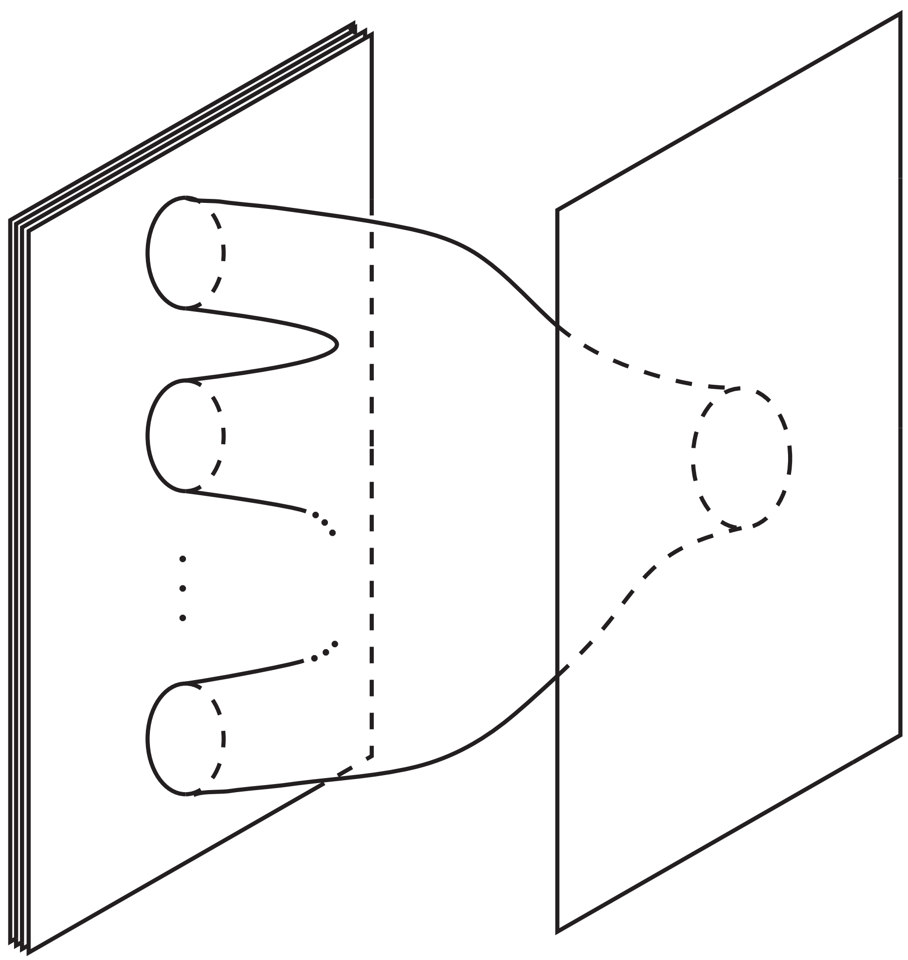
Both models with quenched disorder and tensors remain, however, rather exotic.

String theory, via the open/closed string duality, singles out unambiguously matrix models in the 't~Hooft's large N limit as being the favored candidates to describe quantum black holes.

Matrix models are ubiquitous in string theory simply because the two indices of the matrices are the Chan-Paton factors associated with the two end points of open strings. It is very difficult to find a similar interpretation for tensors of rank three or higher.



The models originating from D-brane constructions always involve several bosonic matrices X_{\mu}, which describe motion transverse to the brane worldvolume. The index \mu naturally transforms in the fundamental representation of O(D), which is the rotation group in the directions orthogonal to the branes. The full symmetry is usually U(N)XO(D), the U(N) part being gauged. These models must be studied in the planar N -> infinity limit and superficially seem to be much more difficult to solve than models with quenched disorder or tensors.



$$X^{i}_{\mu\, j}\,, \quad 1 \leq \mu \leq D = d - p - 1$$

2 The new large D limit of matrix models

Intuition: Emparan et al., also followed by Minwalla et al., have studied the large space-time dimension limit d->infinity of classical general relativity and have shown that the basic features of black holes can be studied efficiently in the large d expansion (in particular, the quasi-normal spectrum, but also non-linear effects involved in black hole collisions, etc.).

This idea provided a motivation to study the large D limit of matrix quantum mechanics.

The models

$$L = ND\Big(\mathrm{tr}\big(\dot{X}_\mu^\dagger \dot{X}_\mu + m^2 X_\mu^\dagger X_\mu\big) - \sum_B t_B I_B(X)\Big)$$

$$I_B = \mathrm{tr}\big(X_{\mu_1} X_{\mu_2}^\dagger X_{\mu_3} X_{\mu_4}^\dagger X_{\mu_5} \cdots X_{\mu_{2s}}^\dagger\big)$$

The usual large N limit is defined by keeping the couplings t_B fixed (this is the 't Hooft scaling).

The usual large D limit of vector models is defined by keeping the couplings t_B fixed as well.

The resulting double expansion has the form

$$F = \sum_{(g,L)\in\mathbb{N}^2} f_{g,L} N^{2-2g} D^{1-L}$$

The large N and large D limits commute. One gets vector model physics.

To many planar diagrams have been thrown away by taking D->infinity and the result does not capture the physics of the large N limit anymore.

$$L = ND\Big(\mathrm{tr}\big(\dot{X}_\mu^\dagger \dot{X}_\mu + m^2 X_\mu^\dagger X_\mu\big) - \sum_B t_B I_B(X)\Big)$$

Can we improve the situation by enhancing some couplings in the large D limit? That is to say, instead of taking the large D limit at fixed t_B, can we imagine a new scaling of the form

$$t_B = D^{g(B)}\lambda_B$$

where g(B) is some positive number (strictly positive at least for some interactions) and a new large D limit for which \lambda_B is kept fixed instead of t_B?

Naively this seems impossible. If one enhance a coupling, diagrams containing a large number of the associated vertices will have an arbitrary high power of D. If the highest power of D is not bounded above, the large D limit cannot exist.

In other words, the ’t Hooft’s scalings is delicate. The typical situation is that, if the couplings are diminished, the limit becomes trivial; and if they are enhanced, the limit does not exist anymore.

But it turns out that in vector-matrix model, a remarkable property holds. It turns out that the powers of D and N in a diagram are not independent.

Intuition: the power of D is related to the number of O(D) loops in the diagram. The power of N is related to the genus of the surface on which the diagram can be drawn. But on a surface of a given genus, there is a constrain on the number of loops one can draw.

Result:

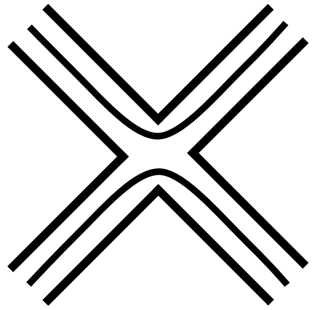
There is a precise rule to assign g(B), called the genus, to any interaction B.

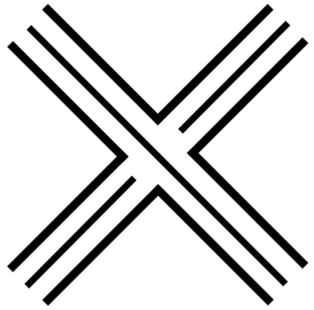
$$t_B = D^{g(B)} \lambda_B$$

Examples: $\mathrm{tr}\, X_\mu X_\mu X_\nu X_\nu$ genus zero: no enhancement

$\mathrm{tr}\, X_\mu X_\nu X_\mu X_\nu$ genus 1/2: enhancement

$\mathrm{tr}\, X_\mu X_\rho X_\mu X_\nu X_\rho X_\nu$ genus 1: enhancement





With this rule, one shows that the highest power of D of a planar diagram is D.

In models with U(N)XU(N) symmetry, this result can be extended to any genus: the highest power of D at genus g is 1+g.

This result allows to define a new large D limit in matrix-vector models, for which many more Feynman diagrams are kept than with the usual vector model large D limit.

$$F = \sum_{g \geq 0} N^{2-2g} F_g$$

$$F_g = \sum_{\ell \geq 0} N^{1+g-\ell/2} F_{g,\ell}$$

The new large D and standard large N limits do not commute with each other.

The new large D limit yields a 1/\sqrt{D} expansion of Feynman diagram of fixed genus g.

The leading order corresponds to melon-like diagrams, making the link with SYK and tensor models.

The main bonus is that we are now dealing with completely standard matrix quantum mechanical (or field theoretical) models. The relation with BH physics is then a consequence of the usual string theory arguments.

One can actually reverse the argument and say that the fact that this new large D limit exist is the underlying reason why all these result are relevant to BH physics. We are getting a new approximation to the sum over planar diagram, and one that seems to capture the essential physics of the full sum!

Another interesting bonus is that we can deal with the planar limit of Hermitian matrix models (i.e., U(N) symmetry only, not U(N)XU(N)), unlike with tensor models where imposing a symmetry constraint on the tensor typically makes the large N limit inconsistent.

The new large D limit is also consistent with linearly realised SUSY, which allows to study many interesting models is 0+1, 1+1 and 2+1 dimensions (work in progress).

A more general discussion, including multi-trace interactions and applications will appear soon (with T. Azeyanagi, P. Gregori, L. Leduc, G. Valette).

Generalisation:

The idea of defining large N or large D limits by enhancing some couplings in a particular way is very fruitful. A very basic and most important example was studied by Carrozza and Tanasa in 2016 (used by Klebanov and Tarponolsky).

It turns out that the idea is very general and yields to a new large N limit of tensor model (and a new large D and large N limit of matrix-tensor models) (with G. Valette and V. Rivasseau).

$$\mu_a = N^{\frac{2R}{(R-1)!}\,\deg \mathcal{B}_a}\,\lambda_a$$

Bonzom-Gureau-Rivasseau New scaling

The large N limit can be shown to be consistent, but the diagrams are now classified according to their index, not their degree.

Degree zero: “super-planar”, the graph is planar whatever way to draw it on a surface, by keeping all the strands.

Index zero: all the possible fat graphs embedded in the original graphs are planar, i.e. pick two indices in the tensor to define a matrix model, forget about all the other indices and draw the standard associated fat graph. It is planar.

The class of index zero graphs is much larger than the class of melonic (degree zero) graphs and the associated leading large N approximation is thus potentially interesting (able to capture more physics).

3. On the phase diagram of planar matrix quantum mechanics

Different types of models:

1) Fermionic (à la SYK-Sachdev)

$$H = ND\,\mathrm{tr}\Big(M\psi_\mu^\dagger\psi_\mu + \frac{1}{2}\Lambda\sqrt{D}\,\psi_\mu\psi_\nu^\dagger\psi_\mu\psi_\nu^\dagger\Big)$$

$$\{\psi^a_{\mu\,b}, (\psi^\dagger_\nu)^c{}_d\} = \frac{1}{ND}\delta_{\mu\nu}\delta^a_d\delta^c_b$$

2) Bosonic unstable

$$H = ND\,\mathrm{tr}\Big(\frac{1}{2}\Pi_\mu\Pi_\mu + \frac{1}{2}M^2X_\mu X_\mu + \frac{1}{2}\Lambda^3\sqrt{D}\,X_\mu X_\nu X_\mu X_\nu\Big)$$

3) Bosonic stable

$$H = ND\,\mathrm{tr}\Big(\frac{1}{2}\Pi_\mu\Pi_\mu + \frac{1}{2}M^2X_\mu X_\mu + \frac{1}{2}\Lambda^4 D\,X_\mu X_\rho X_\mu X_\nu X_\rho X_\nu\Big)$$

4) SUSY (2 or 4 supercharges) $\quad W \propto \mathrm{tr}\,X_\mu X_\nu X_\mu X_\nu$

Very little is known about these modes which were believed to be totally out of analytic control six months ago!

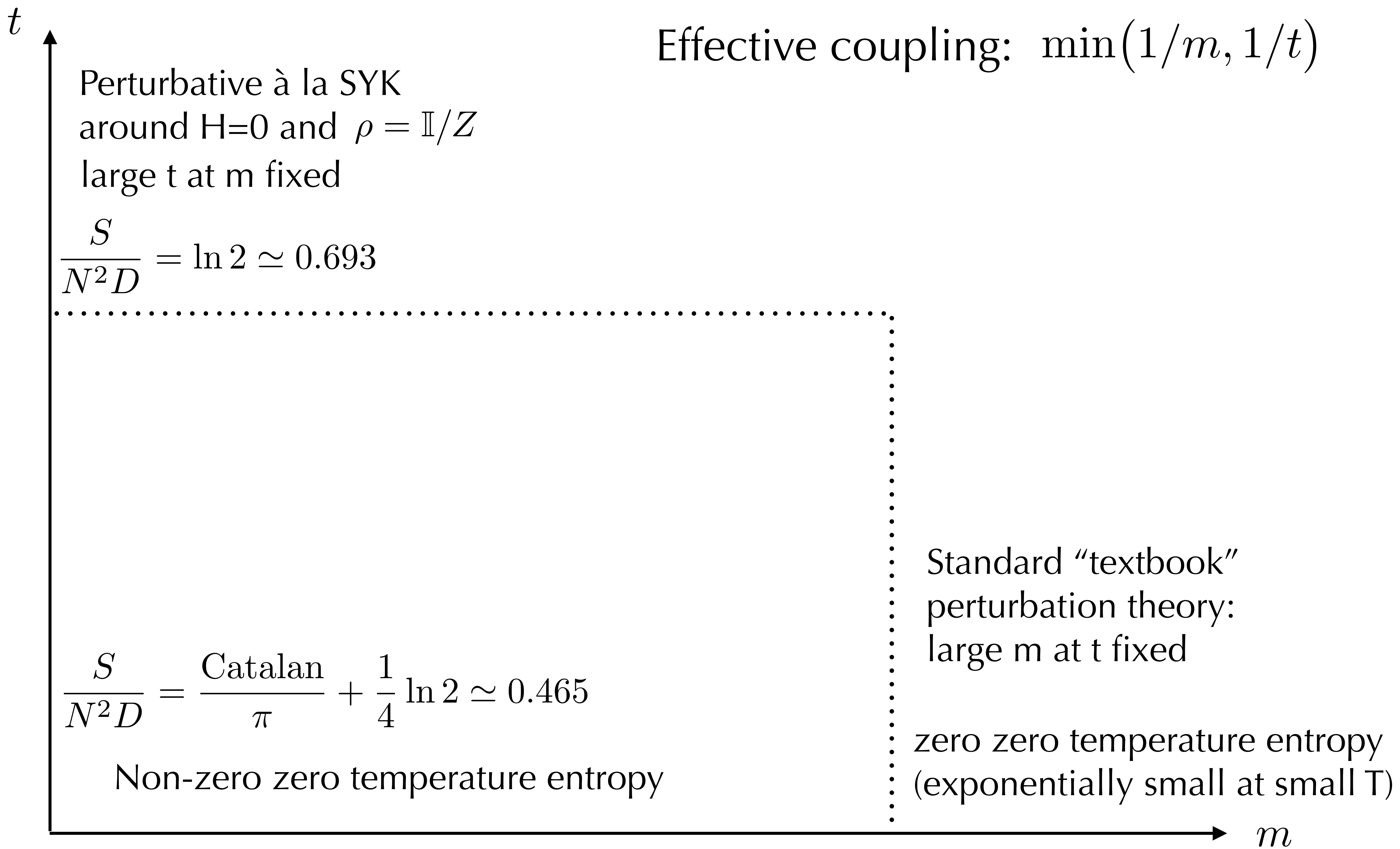
Even for the fermionic model à la SYK, it turns out that only a small part of the physics has been understood, as we shall now see.

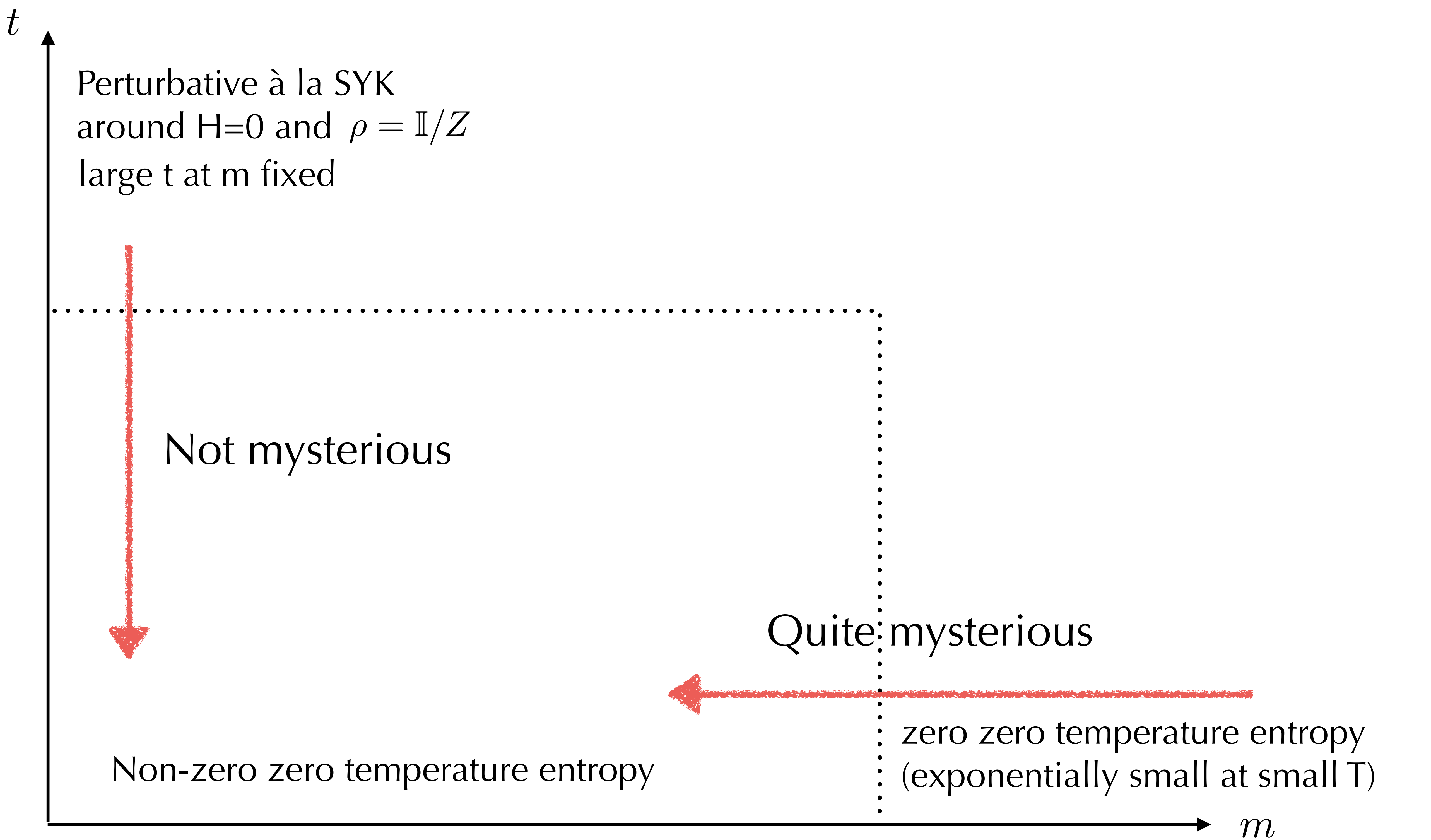
The phase diagrams of all these models turn out to be very interesting - to appear soon (work with T. Azeyanagi and F. Schaposnik).

Parameters:

$$M\,,\quad \Lambda\,,\quad T = 1/\beta$$

$$m = M/\Lambda\,,\quad t = T/\Lambda$$

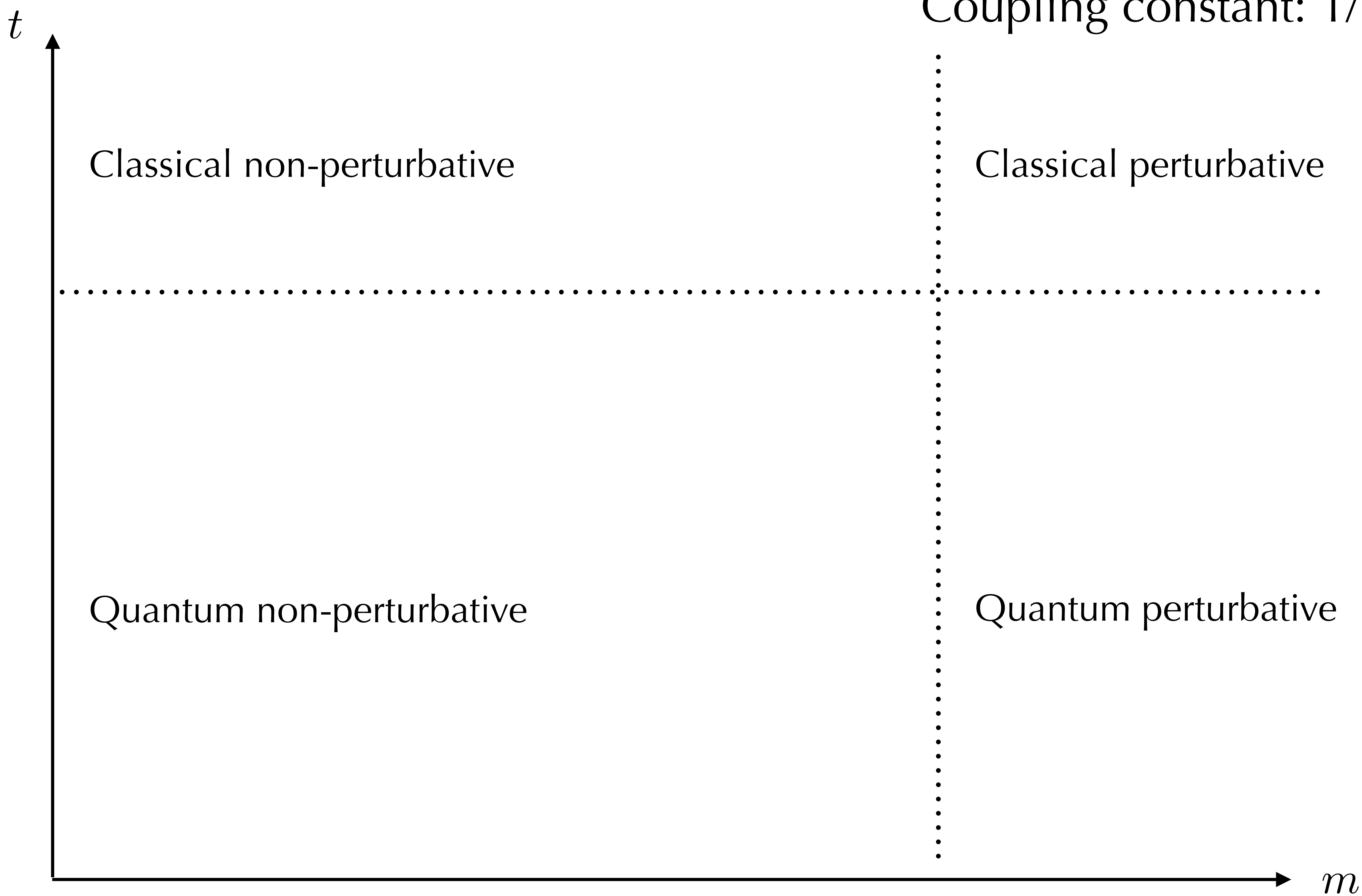




t
Perturbative à la SYK
around H=0 and $\rho = \mathbb{I}/Z$
large t at m fixed
Not mysterious
Quite mysterious
zero zero temperature entropy
(exponentially small at small T)
Non-zero zero temperature entropy
m

Understanding what is going on is particularly important in the case of the bosonic systems, because then the "SYK-like" perturbation theory does not exist at all.

Coupling constant: 1/m



The rest of the talk will be on the blackboard