Echoes of chaos from string theory black holes

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work with V. Balasubramanian, B. Czech and G. Sárosi, JHEP 2017

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Summary

The strongly coupled D1-D5 CFT is a microscopic model of black holes which is expected to have chaotic dynamics. We study its integrable weak coupling limit, in which the operators creating microstates of the lowest mass black hole are known exactly.

Time-ordered two-point function of light probes in these microstates (normalized by the same two-point function in vacuum) display a universal early-time decay followed by late-time sporadic behavior.

We show that in RMT a progressive time-average smooths the spectral form factor (a proxy for the 2-point function) in a typical draw of a random matrix, and agrees well with the ensemble average.

Employing this coarse-graining in the D1-D5 system, we find that the early-time decay is followed by a dip, a ramp and a plateau, in remarkable qualitative agreement with the SYK model. We comment on similarities and differences between our integrable model and the chaotic SYK model.

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Black holes are chaotic

Black holes are thermal and chaos underlies thermal behavior:

1) Relaxation to thermal equilibrium

2) Sensitivity to initial conditions

$$\frac{dq(t)}{dq(0)} = \{q(t), p(0)\} \sim e^{\lambda t}$$

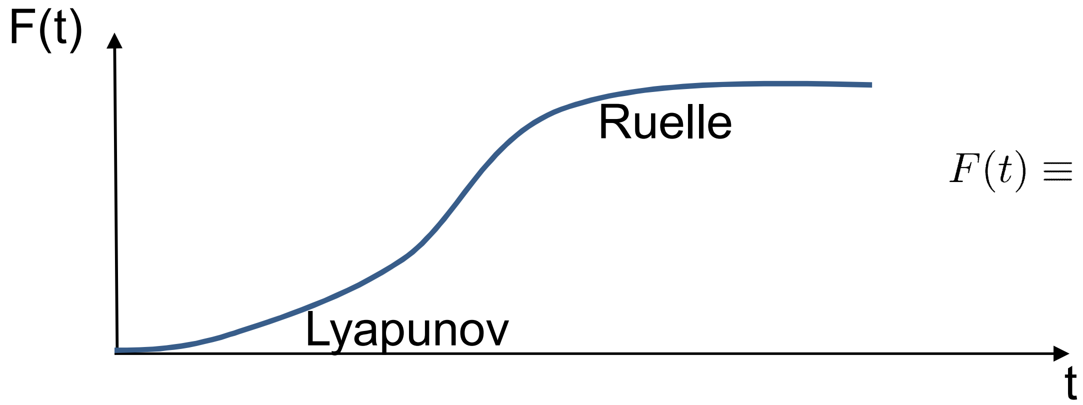
Semiclassical approximation: replace Poisson bracket by commutator and consider growth of

$$F(t) \equiv Z^{-1} \mathrm{Tr} \left(e^{-\beta H} [\hat{W}(t), \hat{V}(0)][\hat{W}(t), \hat{V}(0)]^\dagger \right)$$

[Larkin, Ovchinnikov 1969]

as $\hat{W}(t) \equiv e^{-itH} \hat{W}(0) e^{itH}$ “grows” (spreads over the system).

Black holes are chaotic



$$F(t) \equiv Z^{-1}\mathrm{Tr}\left(e^{-\beta H}[\hat{W}(t),\hat{V}(0)][\hat{W}(t),\hat{V}(0)]^{\dagger}\right)$$

Figure based on [Polchinski]

1) Exponential saturation (Ruelle) $\Longleftrightarrow$ QNM (cf. 2pt function)

[Horowitz, Hubeny 1999]

AdS/CFT

2) Transient Lyapunov growth $\Longleftrightarrow$ redshift $\frac{dt}{d\tau} \sim e^{2\pi t/\beta}$

[Kitaev] [Shenker, Stanford]

Black holes saturate a "chaos bound" [Maldacena, Shenker, Stanford]

Discrete BH microstates → Ruelle/QNM decay does not continue indefinitely! [Maldacena 2001]

Probing discrete microstates: 2pt functions

$$\frac{1}{Z(\beta)}\mathrm{Tr}\left[e^{-\beta H}\mathcal{O}(t)\mathcal{O}(0)\right] = \frac{1}{Z(\beta)}\sum_{m,n} e^{-\beta E_m}\,|\langle m|\mathcal{O}|n\rangle|^2\, e^{i(E_m - E_n)t}$$

Early times:

Can coarse grain → typically exponential decay

Late times:

Discreteness → erratic oscillations. (Also for pure states.)

[Maldacena 2001] see also [Barbon, Rabinovici 2003] [Fitzpatrick, Kaplan, Li, Wang]

Probing discrete microstates: spectral form factor

Simpler diagnostic: spectral form factor

$$|Z(\beta,t)|^2 \equiv \left|\mathrm{Tr}(e^{-\beta H - iHt})\right|^2 = \sum_{m,n} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t}$$

[Papadodimas, Raju]

Long time average: $$\lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, |Z(\beta,t)|^2 = \sum_E N_E^2 e^{-2\beta E}$$

degeneracy

If $N_E = 1$, long time average $Z(2\beta)$ is much smaller than initial value $|Z(\beta,0)|^2$.

E.g. for CFT$_d$: $$\frac{Z(2\beta)}{Z(\beta)^2} = \exp\left[-\frac{2}{d}\left(1-\frac{1}{2^d}\right)S(\beta)\right]$$

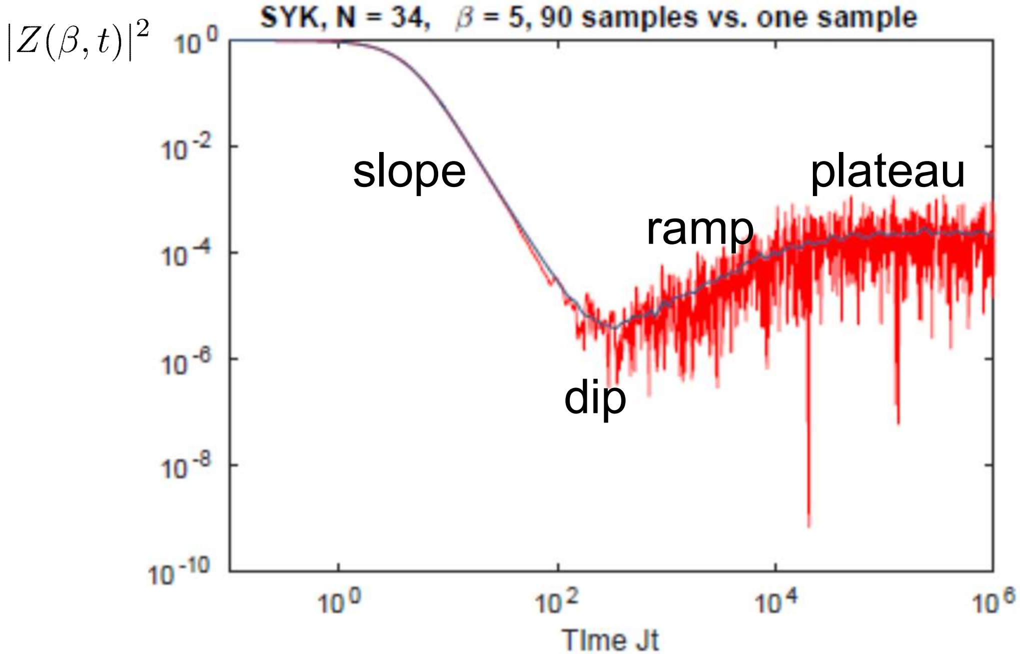
Spectral form factor in SYK

SYK model: QM with $H = \sum J_{abcd}\psi_a\psi_b\psi_c\psi_d$ [Sachdev, Ye] [Kitaev]

N Majorana fermions $\dim\mathcal{H} \equiv L = 2^{N/2}$

drawn from Gaussian distribution with $\sigma^2 \sim \frac{J^2}{N^3}$ ("disorder")

SYK model saturates chaos bound → dual to BH (?) [Kitaev]



Disorder averaging converts erratic fluctuations into smooth curve with slope, dip, ramp and plateau.

[Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, Tezuka]

RMT behavior of quantum chaotic systems

BGS conjecture: spectral statistics of quantum chaotic systems are described by Random Matrix Theory

Gaussian ensembles:
GUE, GOE, GSE

systems whose classical counterparts exhibit chaotic behavior

$$\mathcal{Z}_{GUE} = \int \prod_{i,j} dM_{ij} \exp\left(-\frac{L}{2}\mathrm{Tr}(M^2)\right)$$

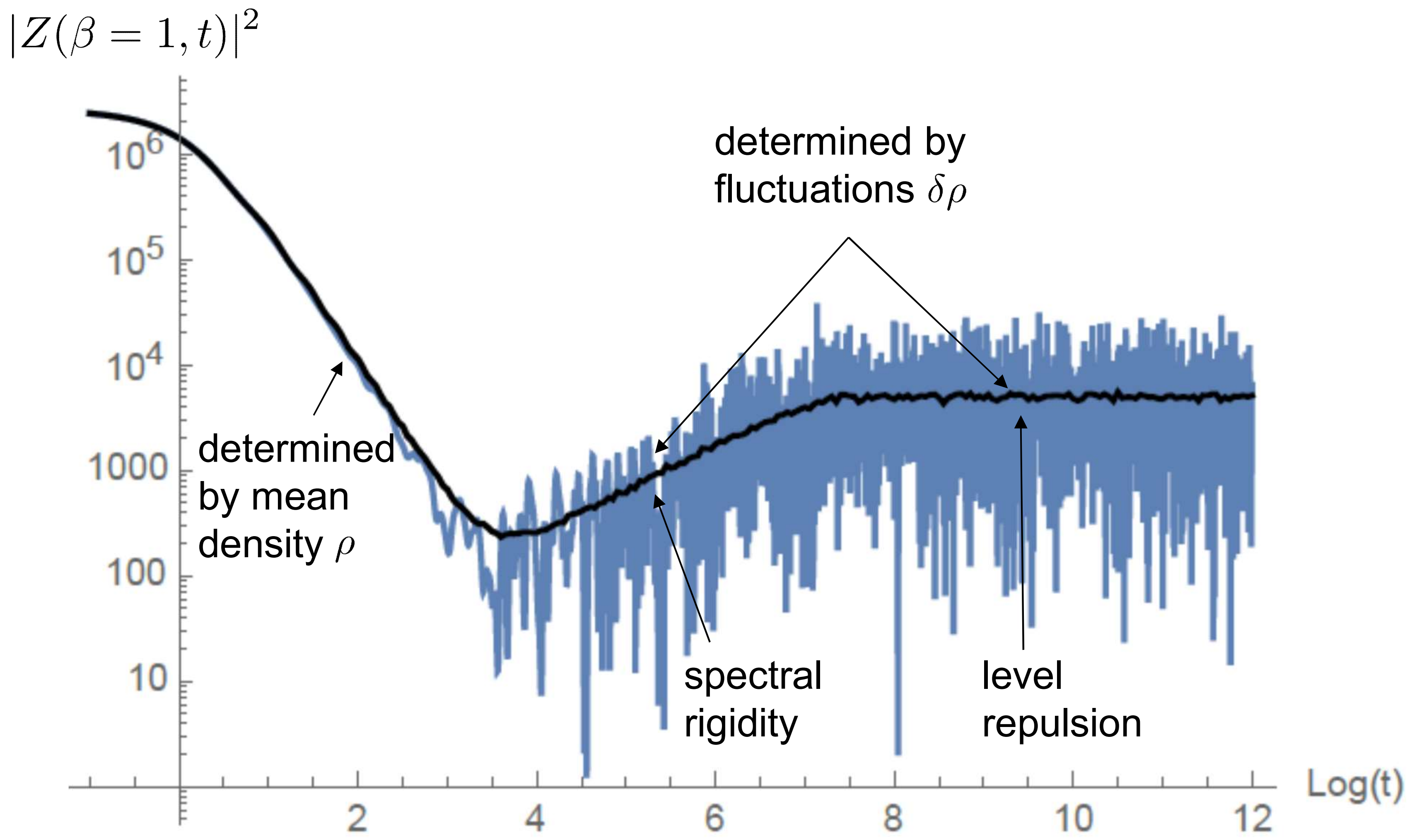
Define mean eigenvalue density $\rho(E)$ by ensemble average or coarse graining. BGS conjecture is not about mean density but about statistics of "unfolded" spectrum (with unit density).

Level repulsion: energy levels repel each other (on scales shorter than mean level spacing)

Spectral rigidity: actual number of levels in a certain energy range is close to the average number (even for ranges much larger than mean level spacing)

[Bohigas, Giannoni, Schmit 1984]

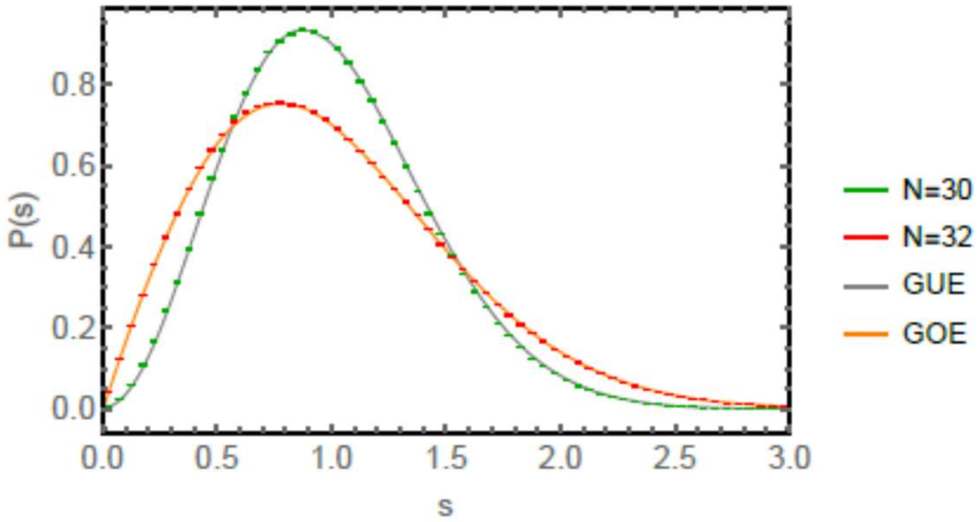
Spectral form factor in RMT



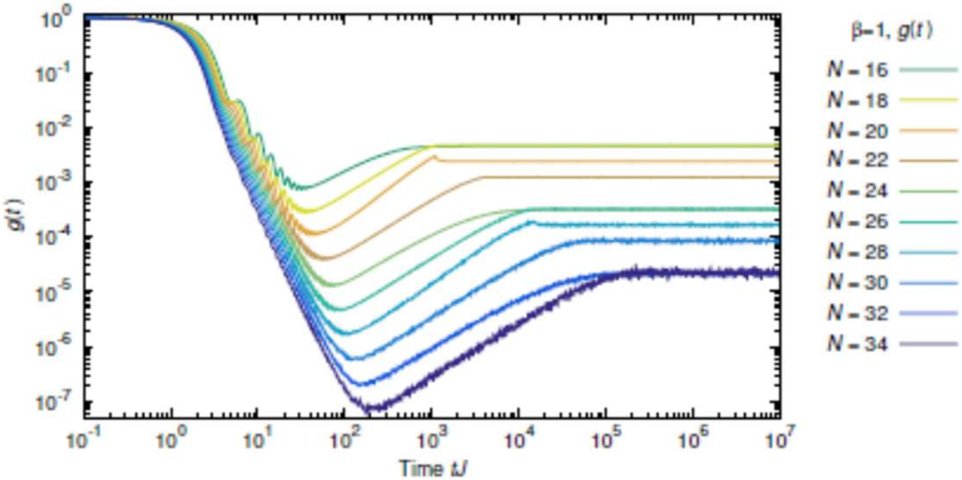
RMT behavior of SYK

GUE, GOE and GSE all realized in SYK, depending on (N mod 8).

Unfolded nearest-neighbor level spacing:



Spectral form factor:



[You, Ludwig, Xu]
[Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, Tezuka]

String theory black holes: D1-D5 CFT

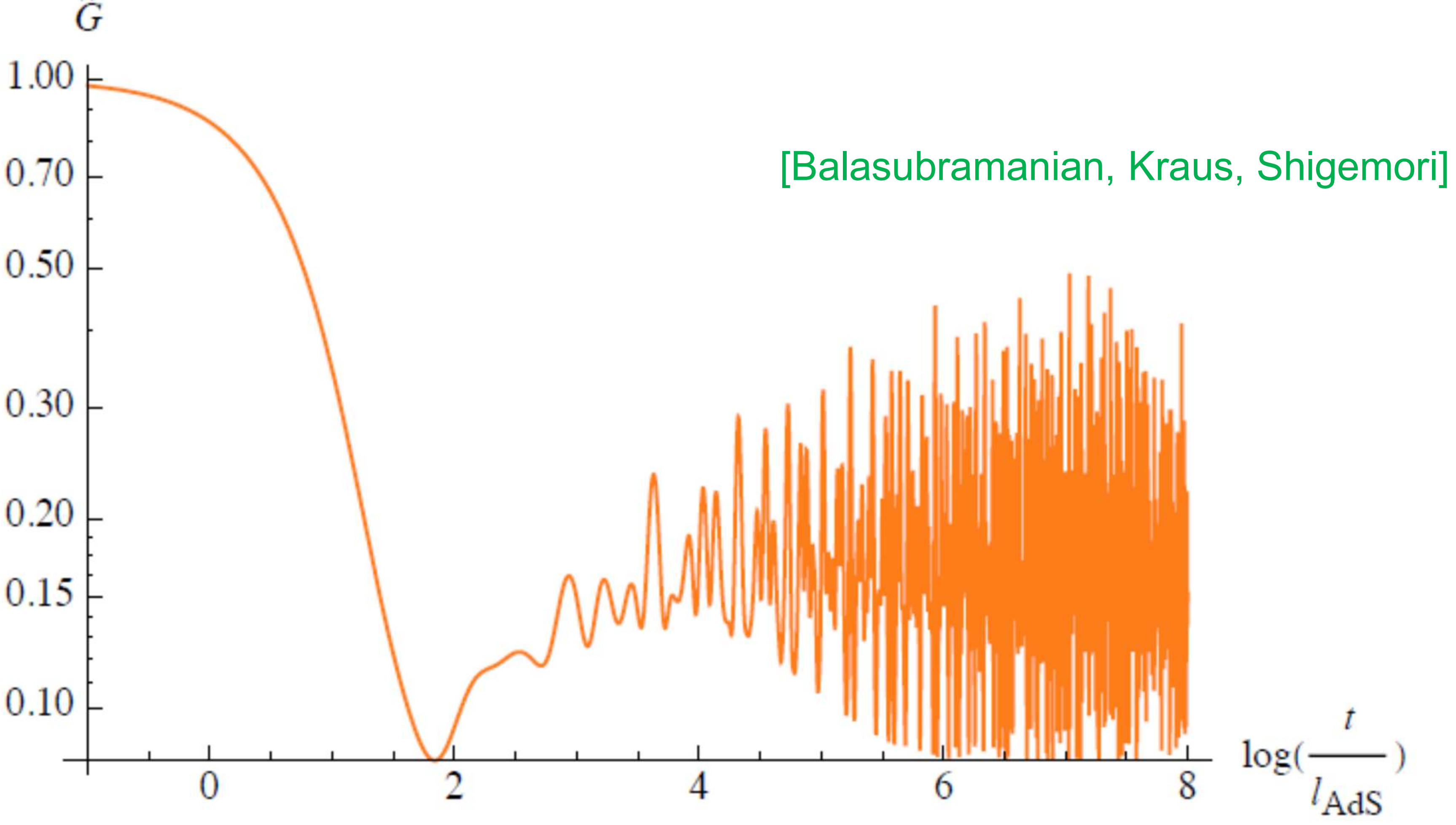
D1-D5 CFT: marginal deformation of σ-model on $(T^4)^N/S_N$

- undeformed: orbifold limit → integrable
- strongly deformed: BH in gravity → chaotic

Consider integrable limit and microstates of lightest “black holes” (RR ground states).

Study 2pt function G(t) of “graviton operators”, normalized by the same 2pt function in vacuum.

2pt functions in string theory “black holes”



How to obtain smooth curve? Idea: progressive time-averaging!
Test first in RMT. $\Delta t \sim t$

Spectral form factor in RMT: single realization

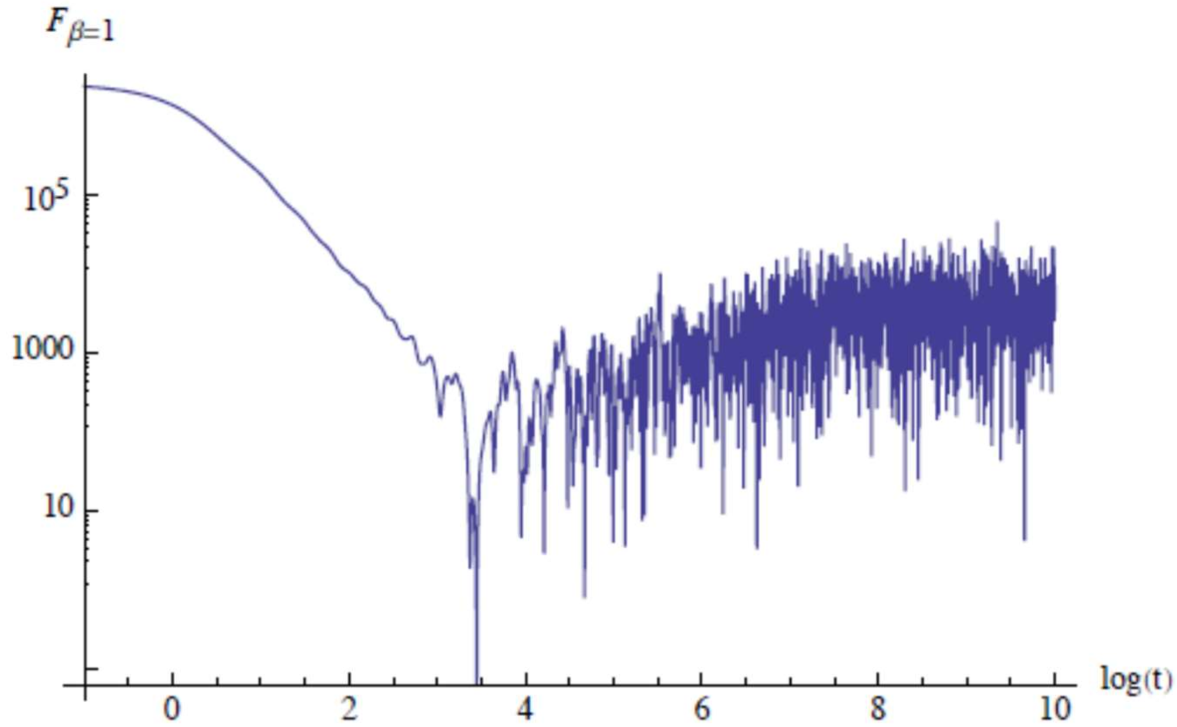


Figure 2: Log-log plot of the spectral form factor (3.1) with $\beta = 1$ for a single 1024×1024 matrix drawn from the Gaussian Unitary Ensemble (GUE). The early part is self-averaging but the late part is superseded by noise.

Averaging with fixed time window in RMT

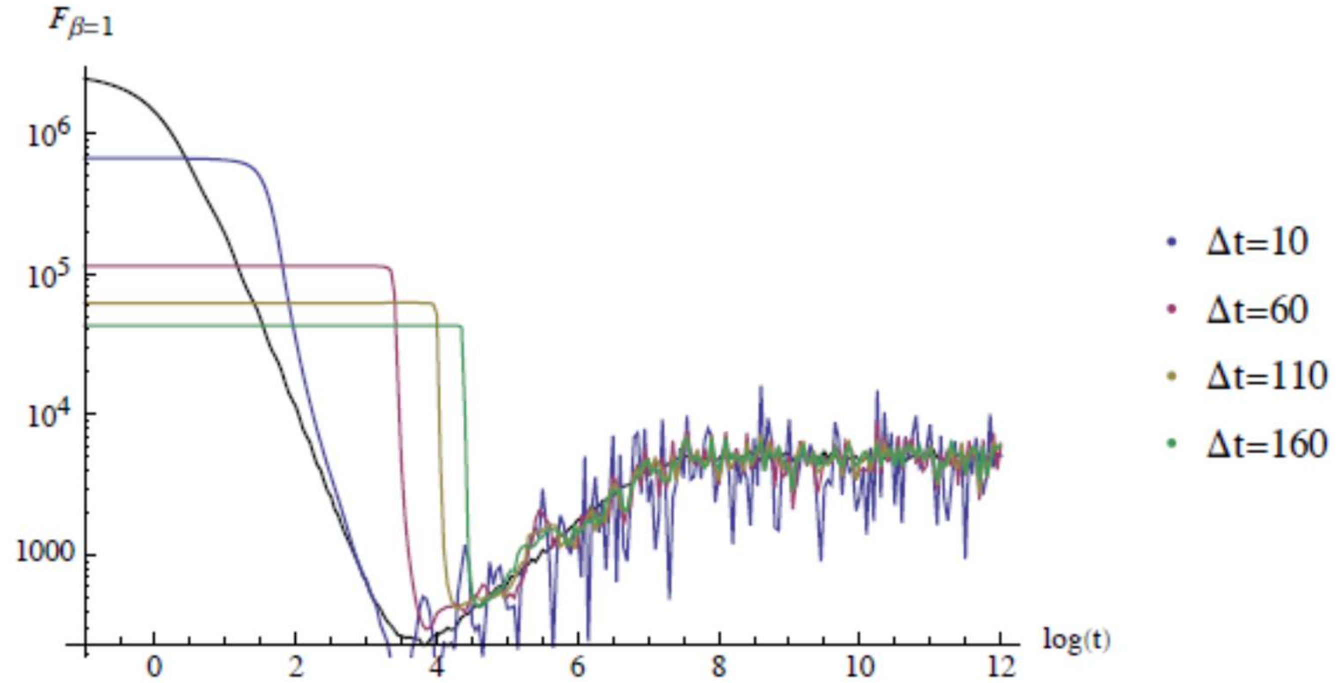


Figure 3: Log-log plot of the average spectral form factor with $\beta = 1$ for five hundred 1024×1024 matrices drawn from the Gaussian unitary ensemble (GUE) (black), and the sliding window average (3.9) with fixed time windows $\Delta t = 10, 60, 110, 160$ (color) for a single instance of a random matrix. Notice that for averaging with a fixed time window there is tension between preserving the dip and having a sufficiently smooth ramp and plateau.

Progressive time-averaging in RMT

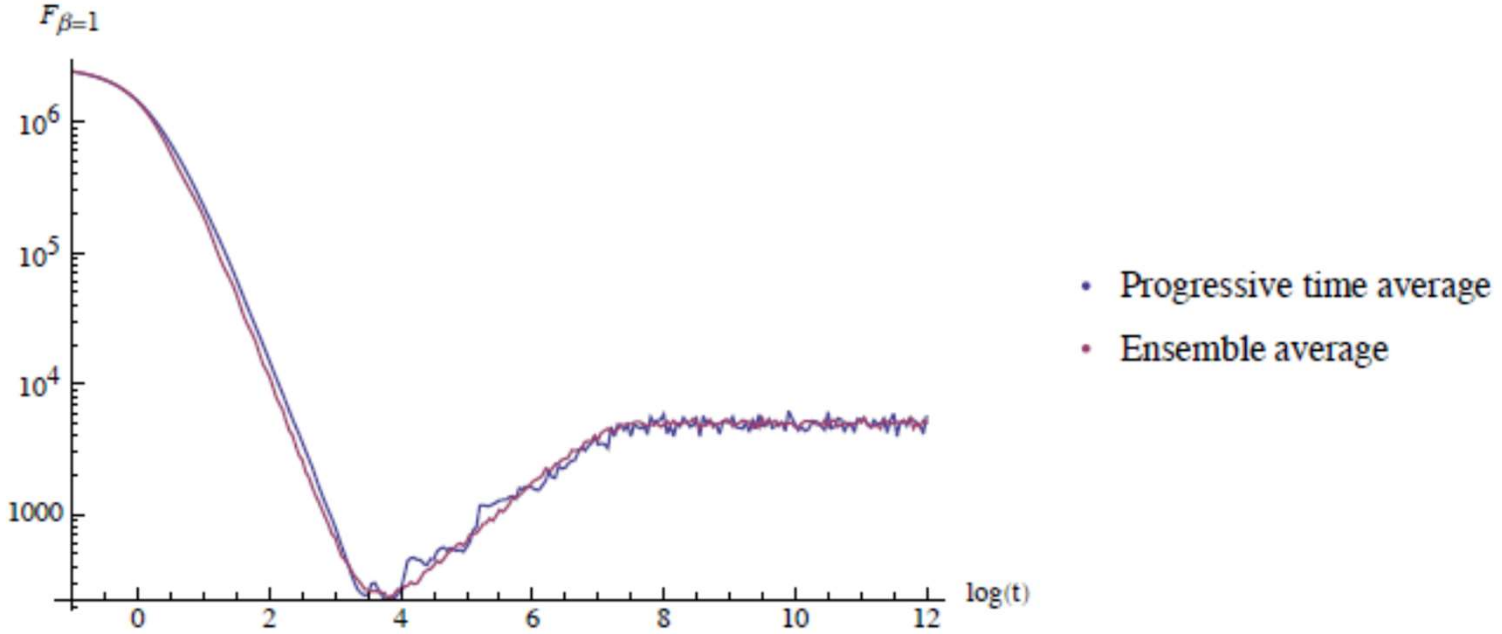


Figure 4: Log-log plot of the average spectral form factor with $\beta = 1$ for five hundred 1024×1024 matrices drawn from the Gaussian unitary ensemble (GUE) (purple), and the sliding window average (3.9) for a progressive time window $\Delta t = 0.8t$ (blue) for a single instance of a random matrix. The progressive window captures the behavior of the ensemble average, in particular the dip, the ramp and the plateau.

Progressive time-averaging: string theory "black holes"

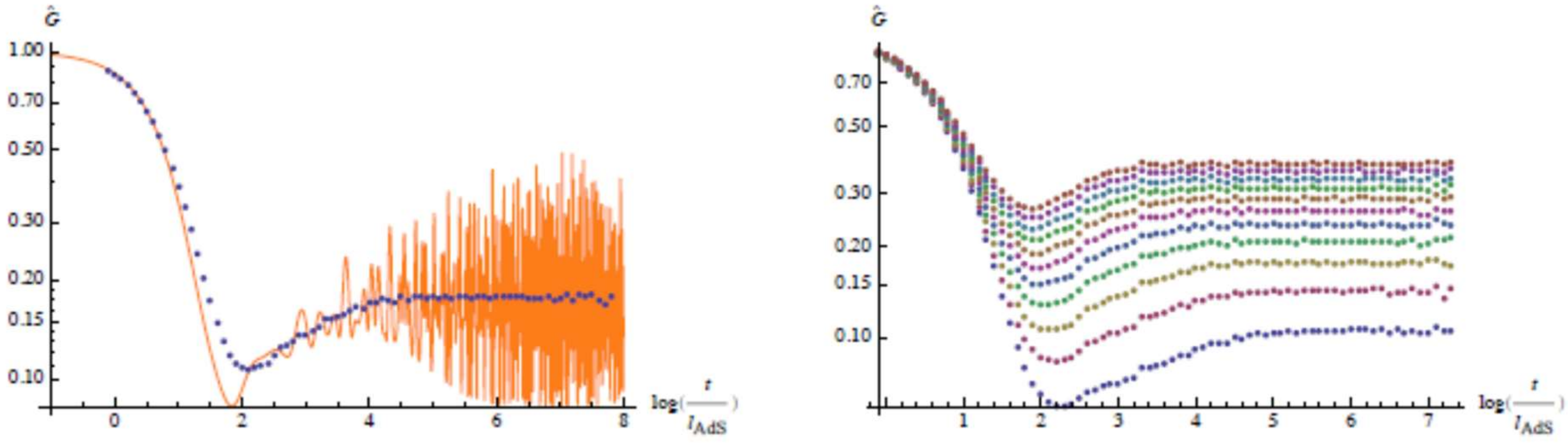


Figure 5: Left: The continuous orange line represents the regularized two-point function (4.1). The blue dotted line is its progressive time-average. Right: The progressive time-average of (4.1) for $\eta = 0.05 + 0.025j, \quad j = 0, ..., 10$. Smaller values of η correspond to larger N and smaller plateau height.

Dip and ramp structure is present despite absence of chaos in integrable limit!

Quantitative differences, e.g. plateau is higher and forms earlier.

Ramp can be approximated analytically

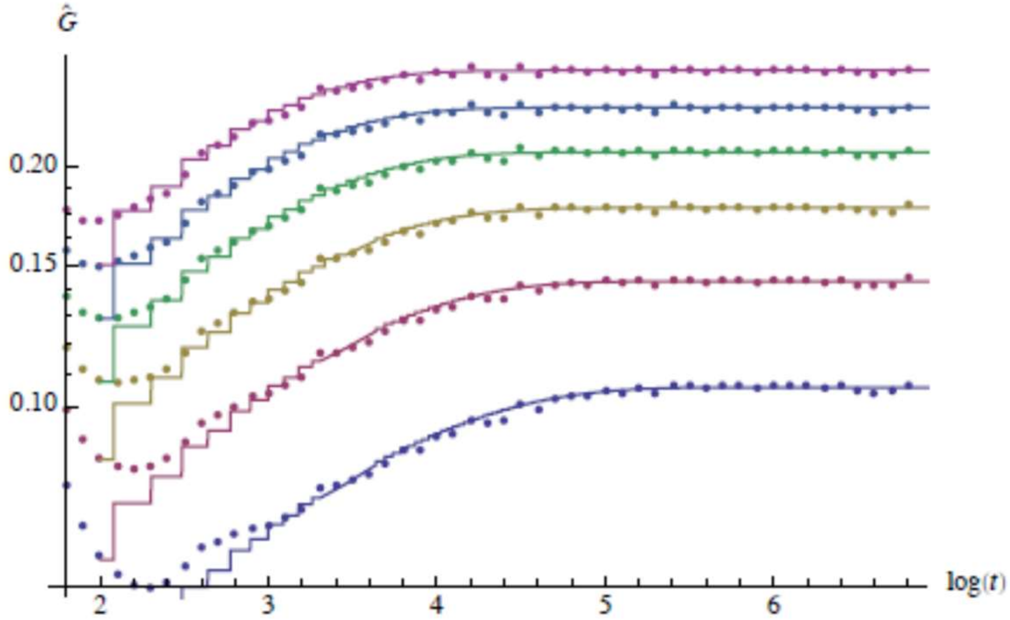


Figure 7: The estimate (4.18) for the ramp and the plateau with $\gamma = 2$ (solid lines) versus the numerically evaluated progressive time averaged regularized two point function (dots) for $\eta = 0.05, 0.075, 0.1, 0.125, 0.15, 0.175$ (from bottom to top).

Dip can be approximated analytically

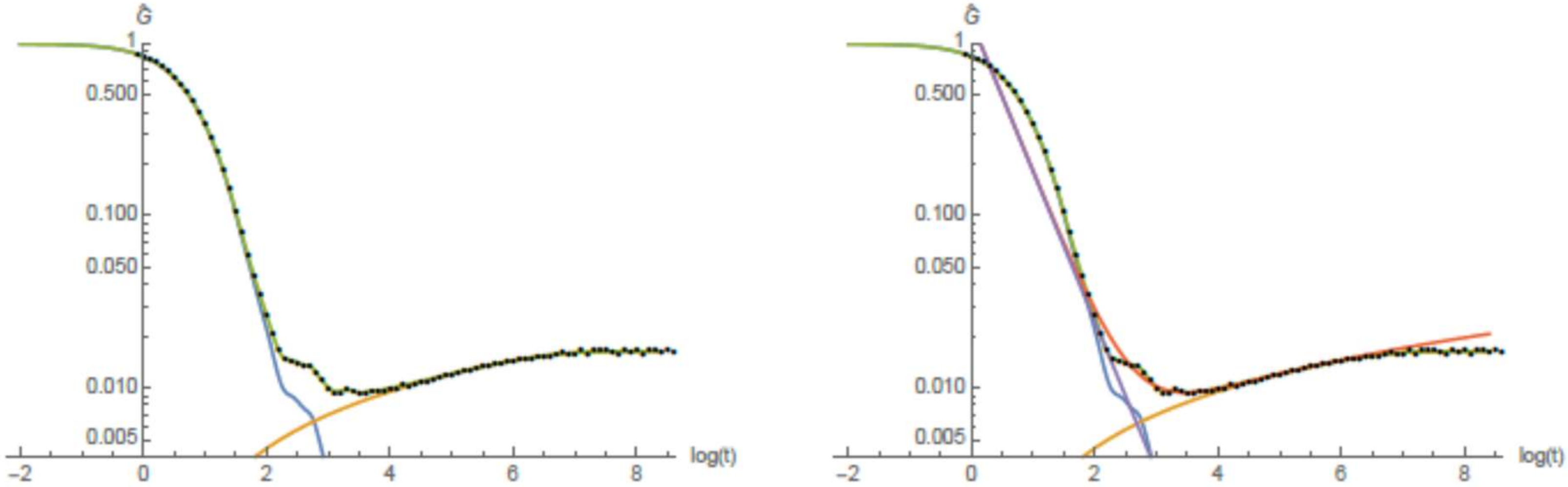


Figure 8: Left: The black dots represent the progressive time-average of the regularized two point function (4.1), the blue line is the progressive time-average of the BTZ two point function (4.25), the yellow line is the ramp estimate (4.18), while the green line is the sum of the latter two, i.e. the function (4.26). Right: Same as the left, with the addition of: purple line is the BTZ asymptote (4.27); the dark orange line is the curve (4.28). Both figures are for $\eta = 0.005$ and use $\gamma = 2$, $\delta = 0.55$ and $a = 1$.

D1-D5 CFT describes long strings

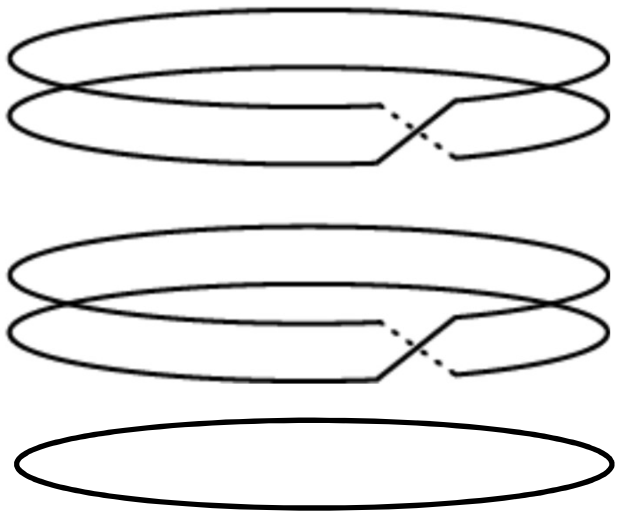
Near-horizon limit of Type IIB string theory on $S^1 \times T^4$

N_{D1} D1 $\qquad$ N_{D5} D5

String theory on $AdS_3 \times S^3 \times T^4$ $\Longleftrightarrow$ Orbifold CFT $(T^4)^N / S_N$ $\qquad (N = N_{D1} N_{D5})$

Twisted sectors: long strings

$$\sum_m m N_m = N$$



$m = 2$

$N_2 = 2$

$m = 2$

$m = 1 \quad N_1 = 1$

D1-D5 2pt functions in R ground states

Ramond ground states are created by twist operators

$$\sigma = \prod_{n,\mu} (\sigma_n^\mu)^{N_{n\mu}} (\tau_n^\mu)^{N'_{n\mu}} \qquad \sum_{n\mu} n(N_{n\mu} + N'_{n\mu}) = N$$

$$N_{n\mu} = 0, 1, 2, \ldots, \qquad N'_{n\mu} = 0, 1$$

$\mu = 1, \ldots 8$ labels polarizations

n labels lengths of strings

There are $O(e^{2\sqrt{2\pi N}})$ Ramond ground states. They have same energy but different excitation spectra.

We studied 2pt function of bosonic non-twist operator in "typical" R ground state. [Balasubramanian, Kraus, Shigemori]

Contributions from energy differences m/n with $1 \leq n \leq N$. In chaotic system, degeneracies would be broken → exponentially small energy spacings → much lower plateau, reached much later.

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