Black holes,
quantum information, entanglement,
⋯ and all that

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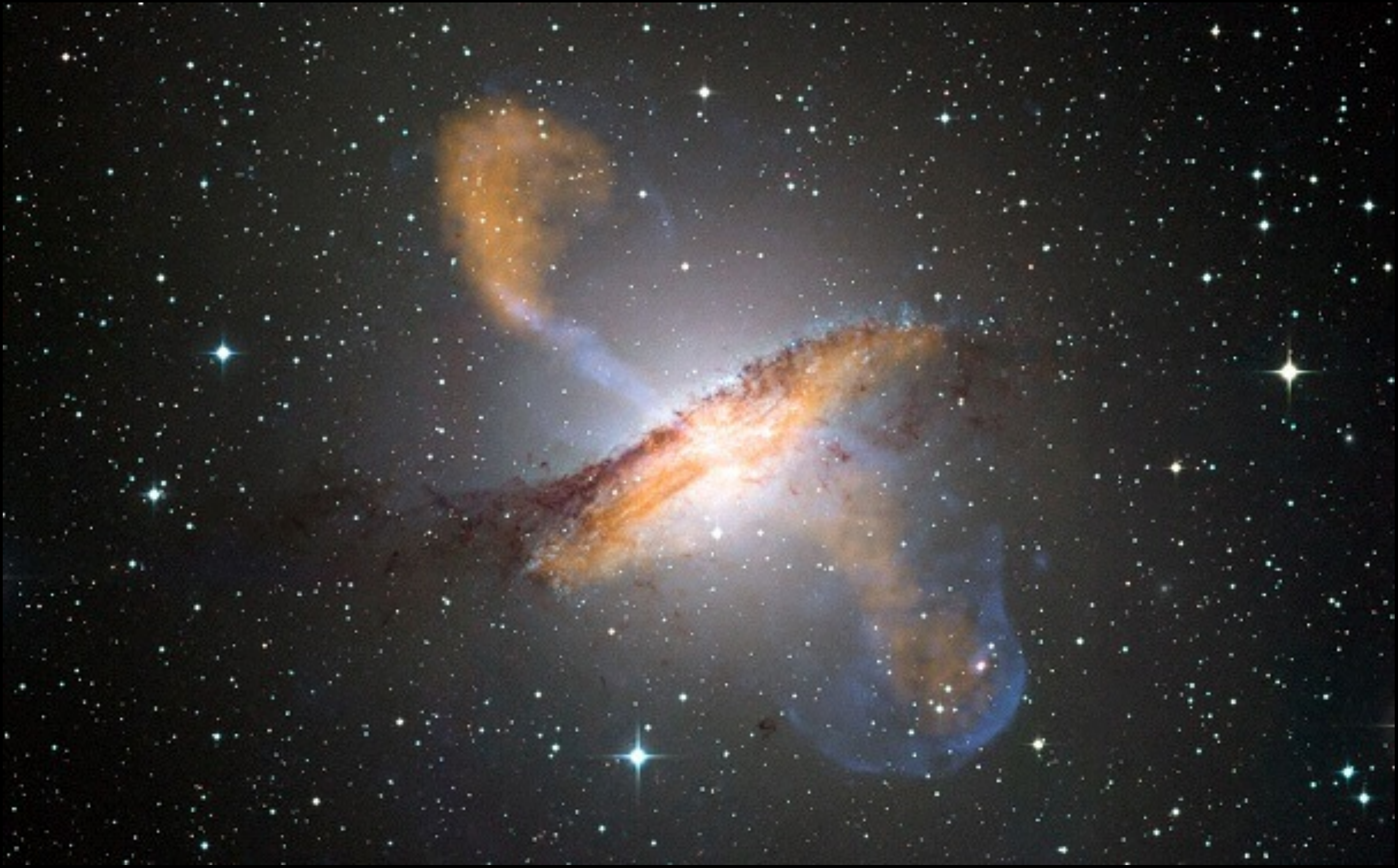
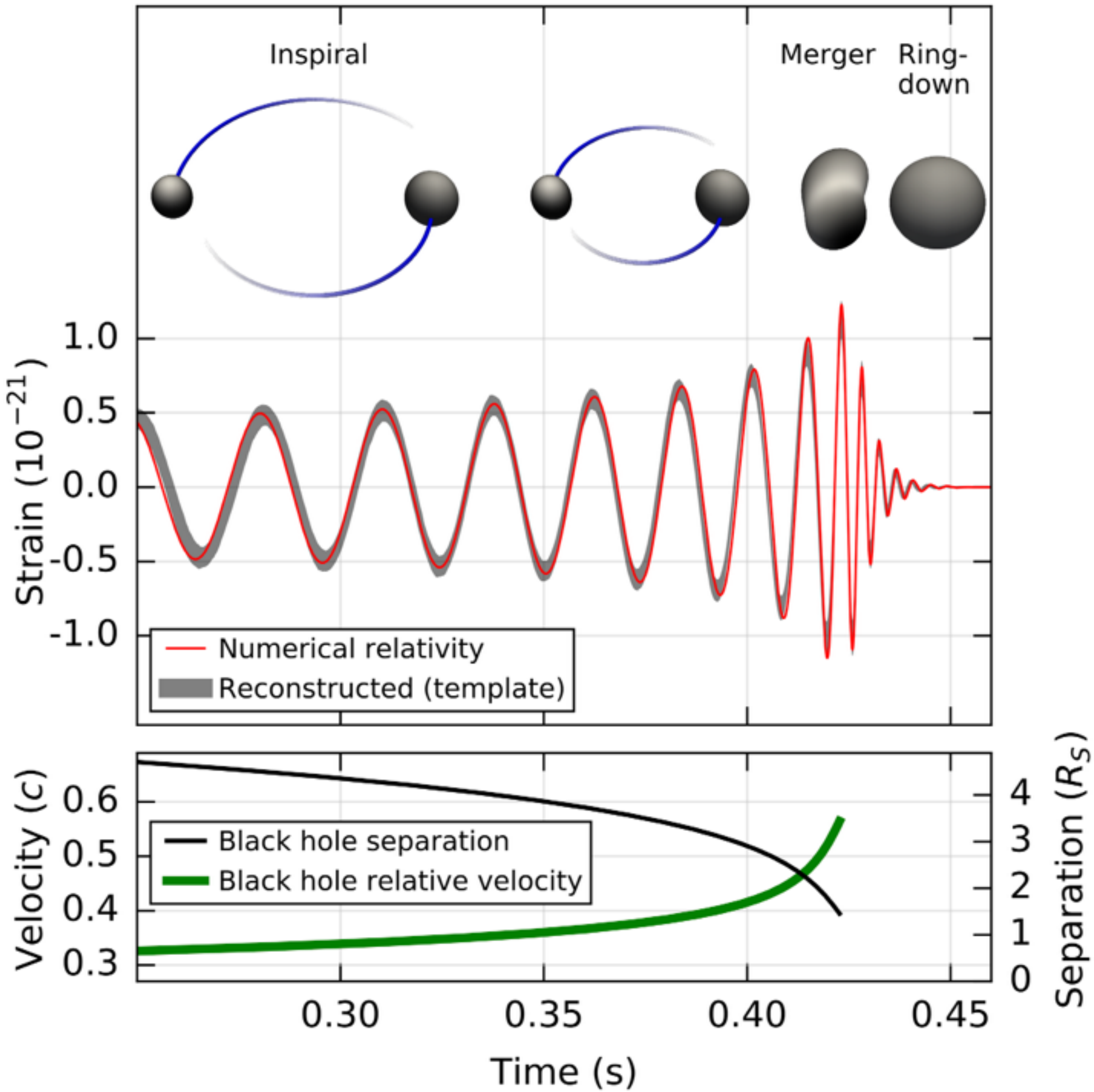
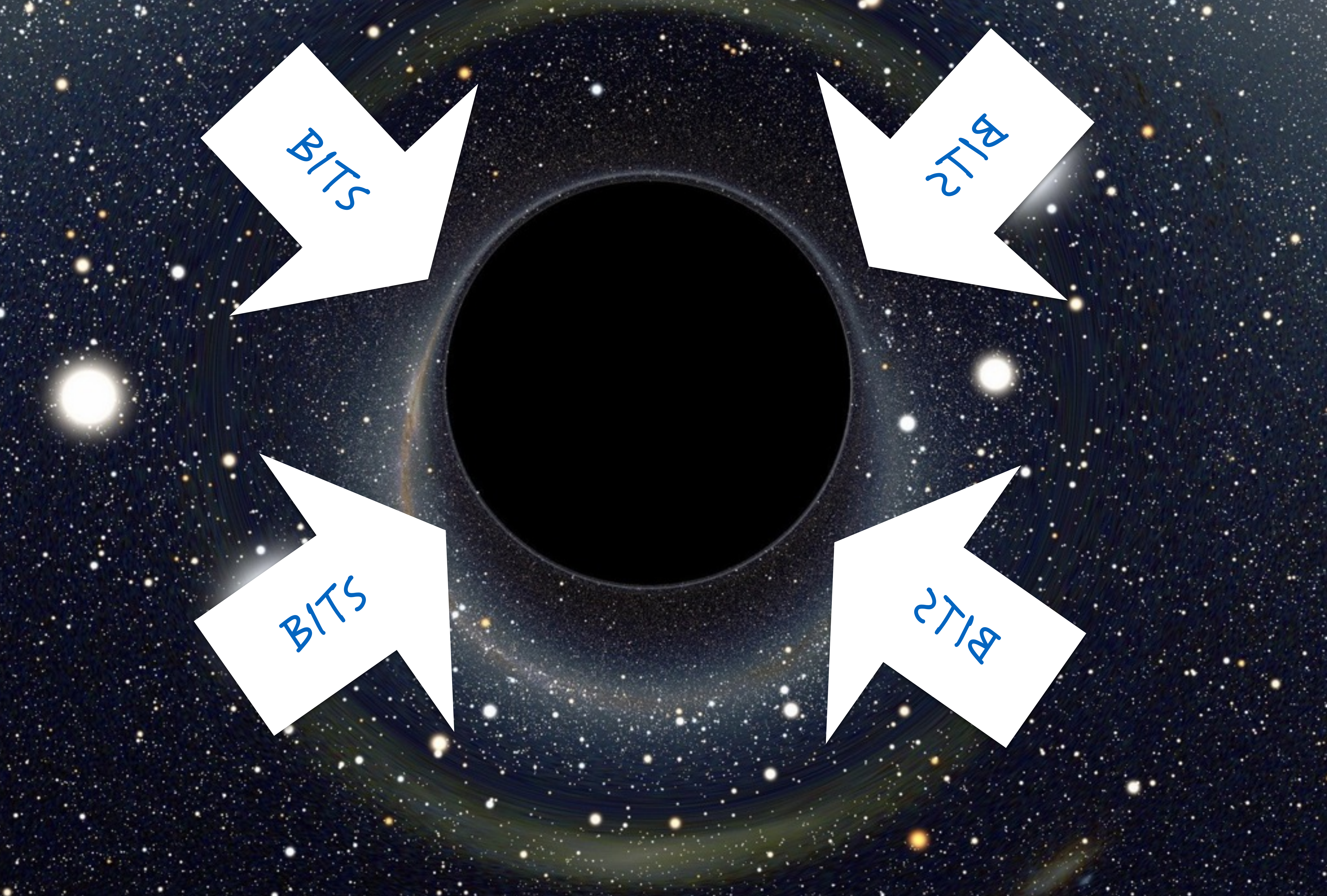




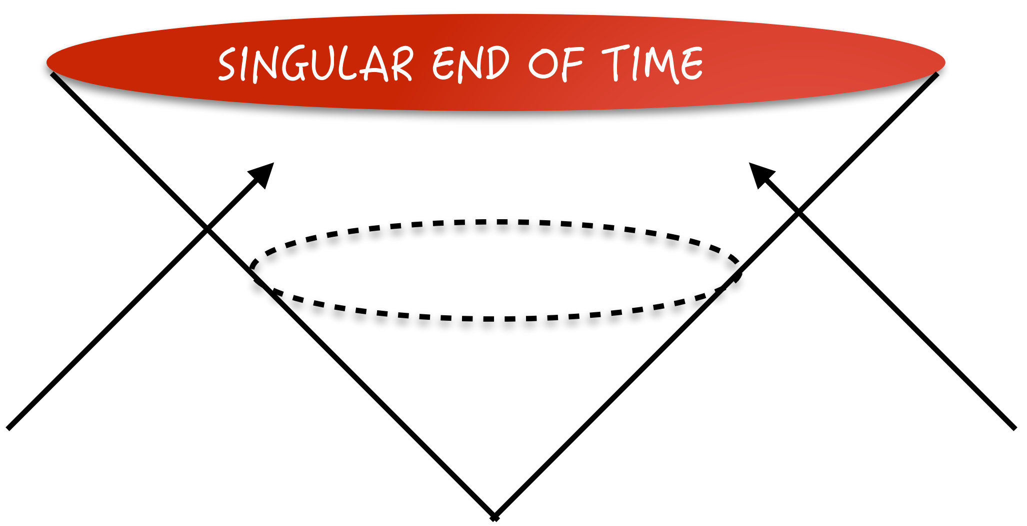
Inspiral
Merger
Ring-down
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
Numerical relativity
Reconstructed (template)
Velocity (c)
0.6
0.5
0.4
0.3
Black hole separation
Black hole relative velocity
Separation (R_S)
4
3
2
1
0
0.30
0.35
0.40
0.45
Time (s)

BLACK HOLES AND INFORMATION



BLACK HOLES AND CLASSICAL INFORMATION

Classically, the hole eats and destroys information



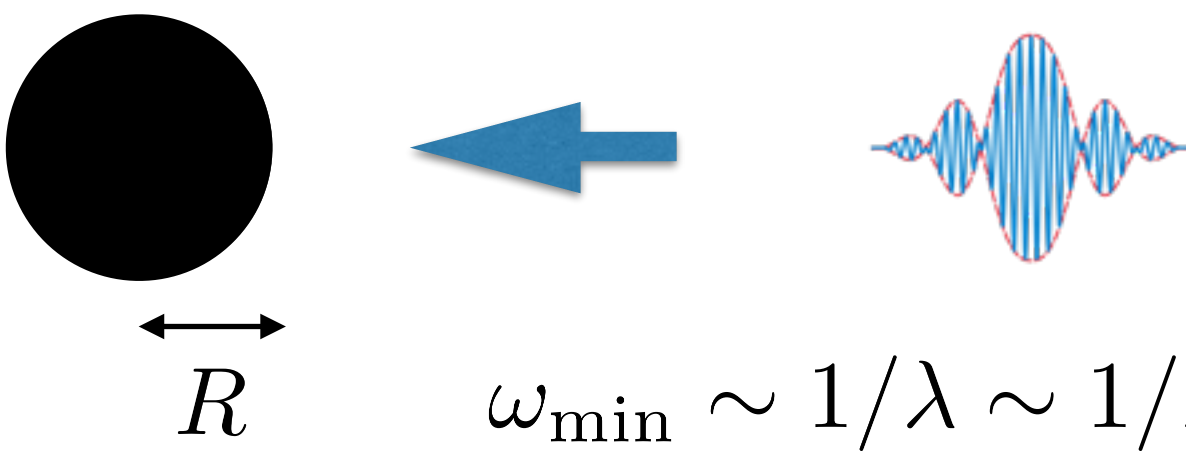
No processing. Singularity an infinite sink, so
the amount of hidden information could be infinite,

i.e.

$$S_{\text{classical}} = \infty$$

BLACK HOLES AND QUANTUM INFORMATION

BEKENSTEIN argued for a FINITE capacity because of quantum mechanics



$$\delta S_{\rm min} \sim \log 2 \sim \delta R^2/G$$

$$\omega_{\rm min} \sim 1/\lambda \sim 1/R$$

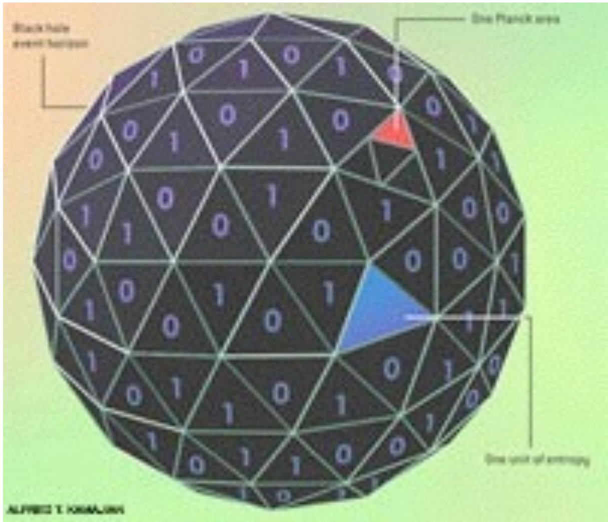
$$S \sim \frac{c^3}{G\hbar}\,(\text{area}_{\text{ horizon}})$$

STRIKING FACTS

d.o.f. not extensive in 3-space!

THE BEGINNING OF HOLOGRAPHY!

$$\log \dim \mathcal{H}_{\rm bh} \sim S \sim \frac{\rm Area}{\ell_P^2}$$



density of states REALLY strong (more than any QFT or string theory)

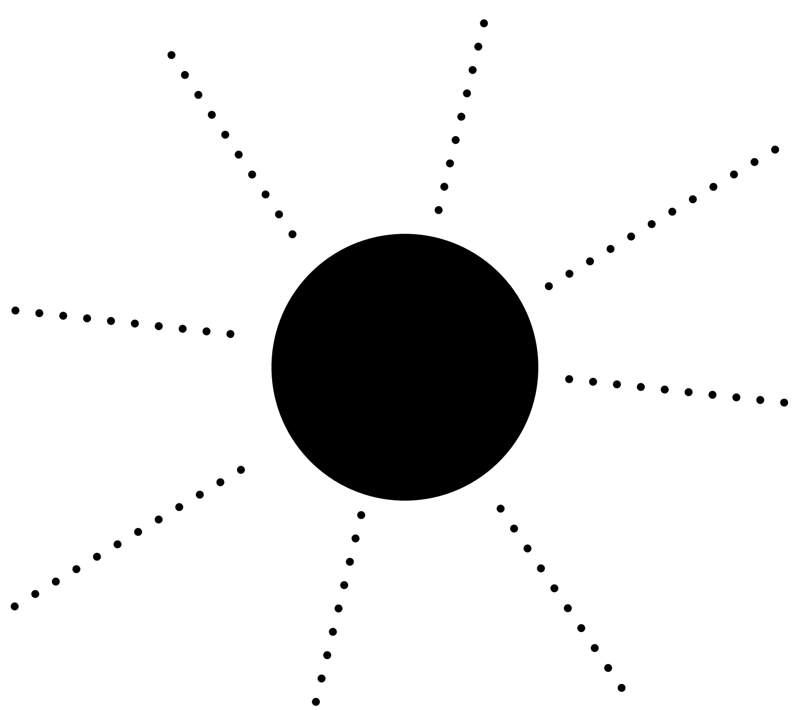
$$A_H \sim R^2 \sim G^2 M^2$$

$$\Omega(M)_{\rm bh\ band} \sim \exp\left(C G M^2\right)$$

But ··· having an entropy, we must have a TEMPERATURE

$$T = \frac{\partial M}{\partial S} \sim \frac{1}{GM} \sim \frac{1}{R}$$

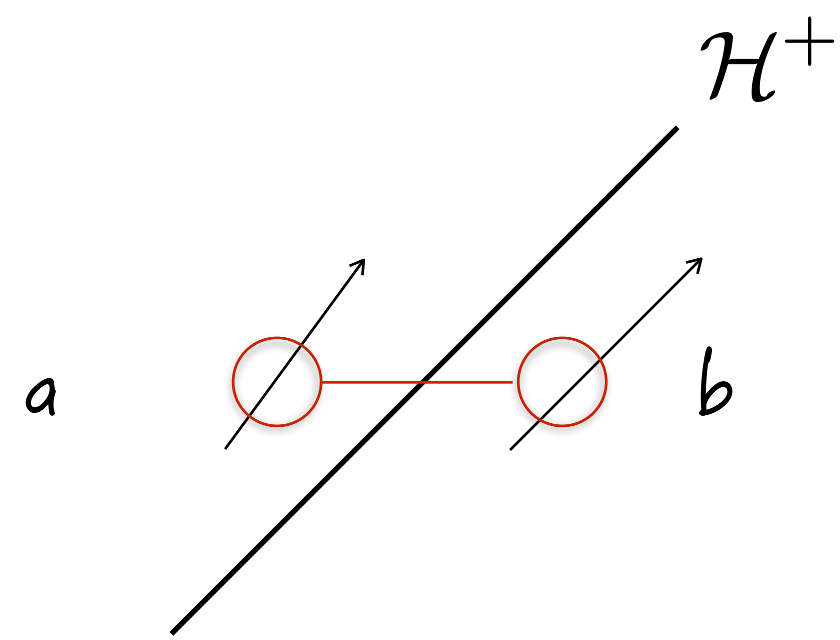
Famously found by Hawking with an entirely different argument



$$T = \frac{1}{4\pi R} = \frac{1}{8\pi GM}$$

$$S = \frac{A_H}{4G}$$

Extra assumption: horizon smothness ⟶ entanglement

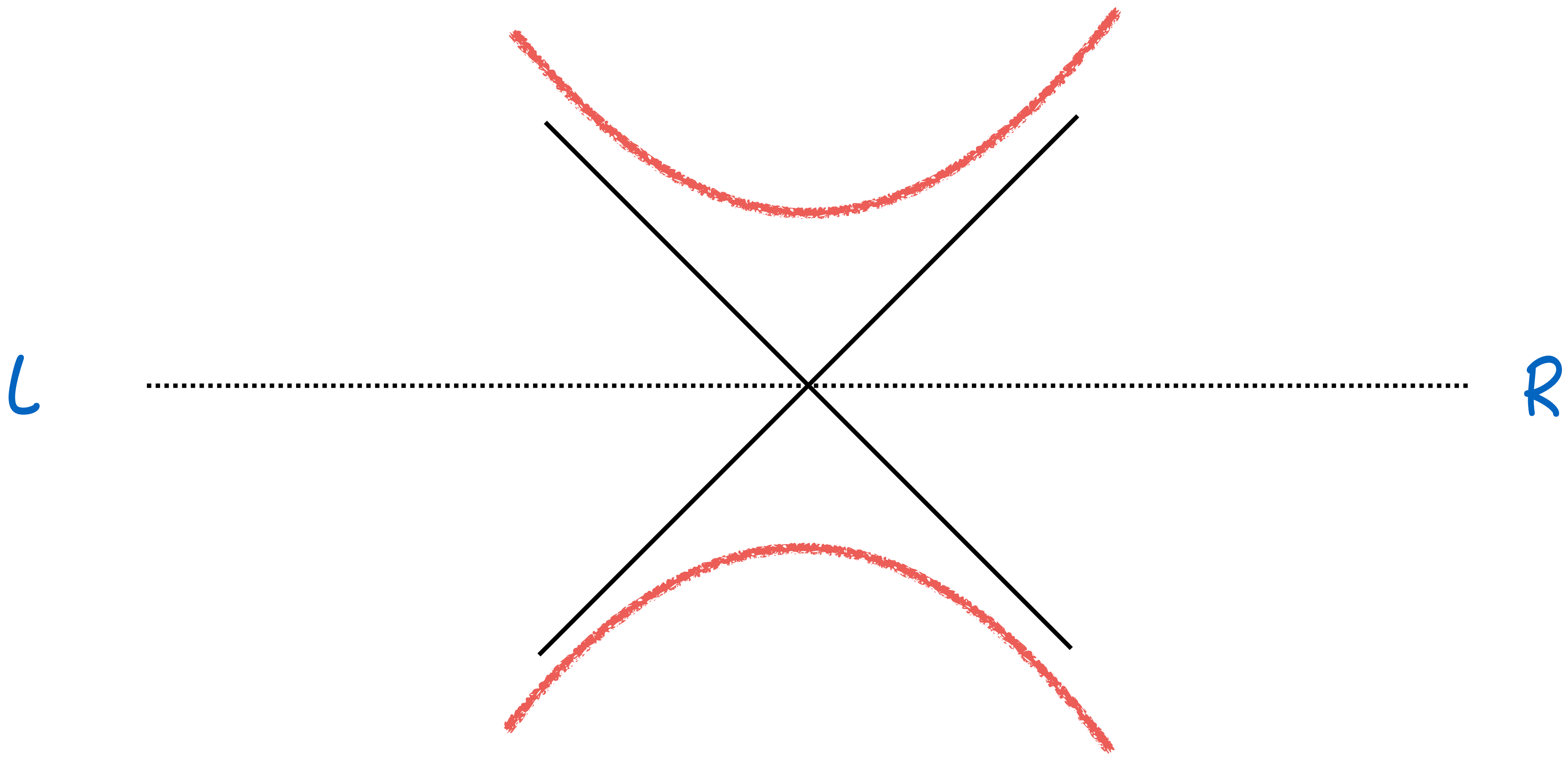


$$|a,b\rangle \propto \sum_{\omega} e^{-\omega/2T}\,|-\omega\rangle_a \otimes |\omega\rangle_b$$

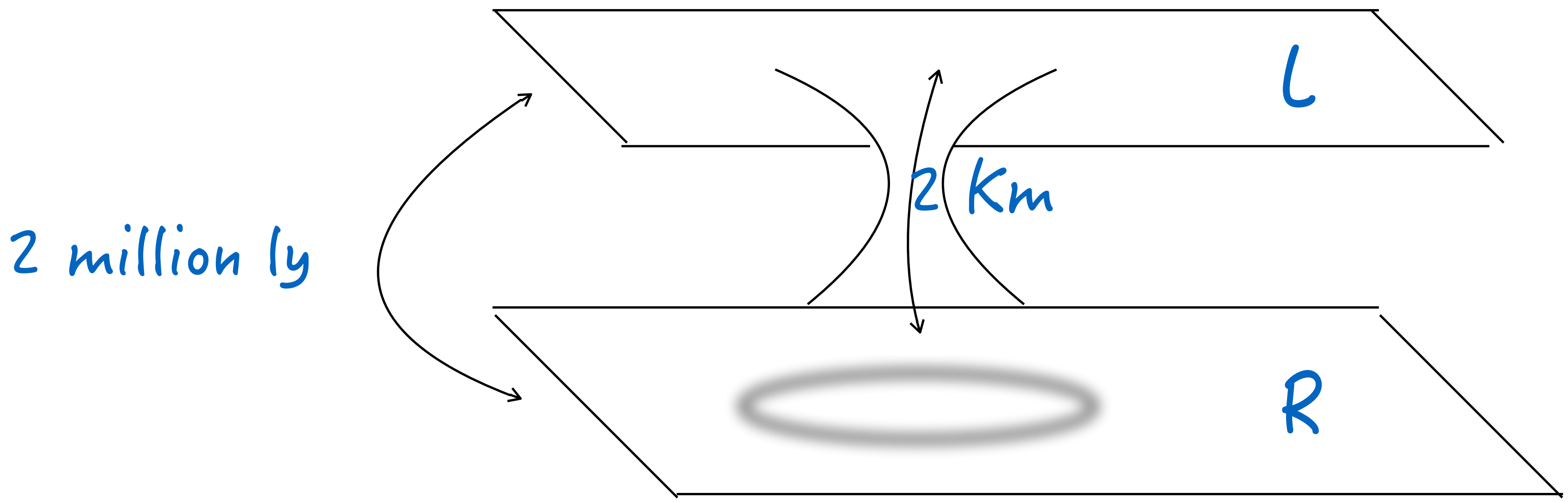
Unruh, ···

$$\rho_b \propto \sum_{\omega} |\omega\rangle_b\, e^{-\omega/T}\; {}_b\langle\omega|$$

The most symmetrical case: the maximal analytic extension, or two black holes sharing the interior

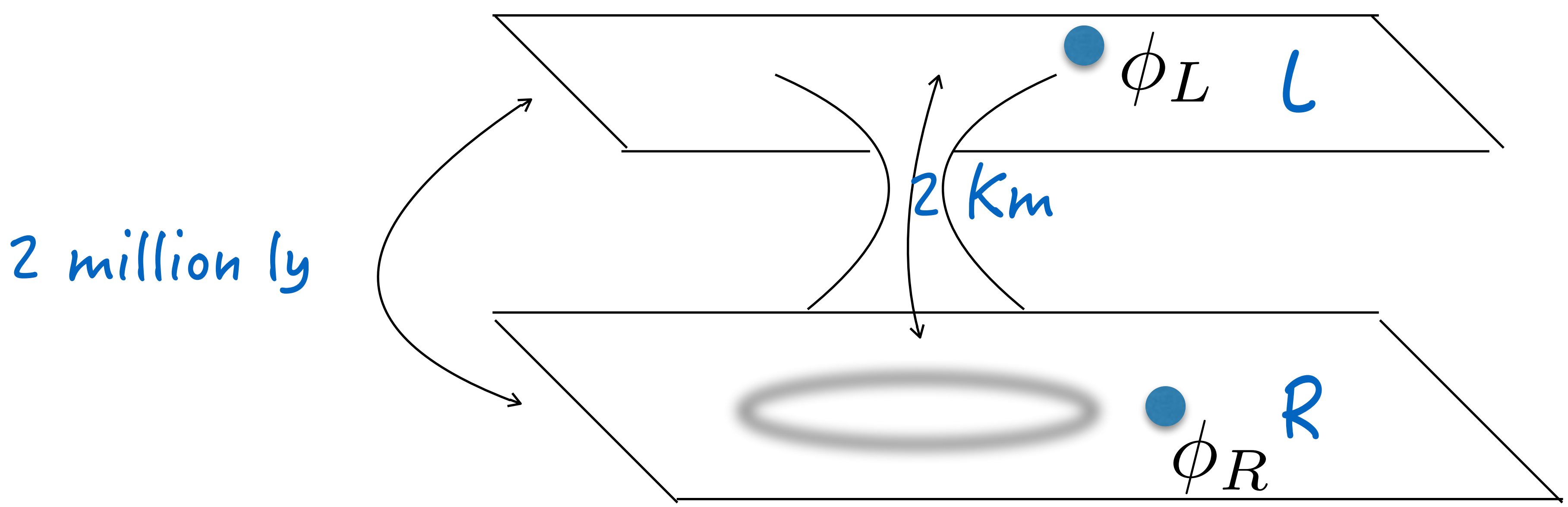


the time-symmetrical slice contains a wormhole !



Einstein-Rosen

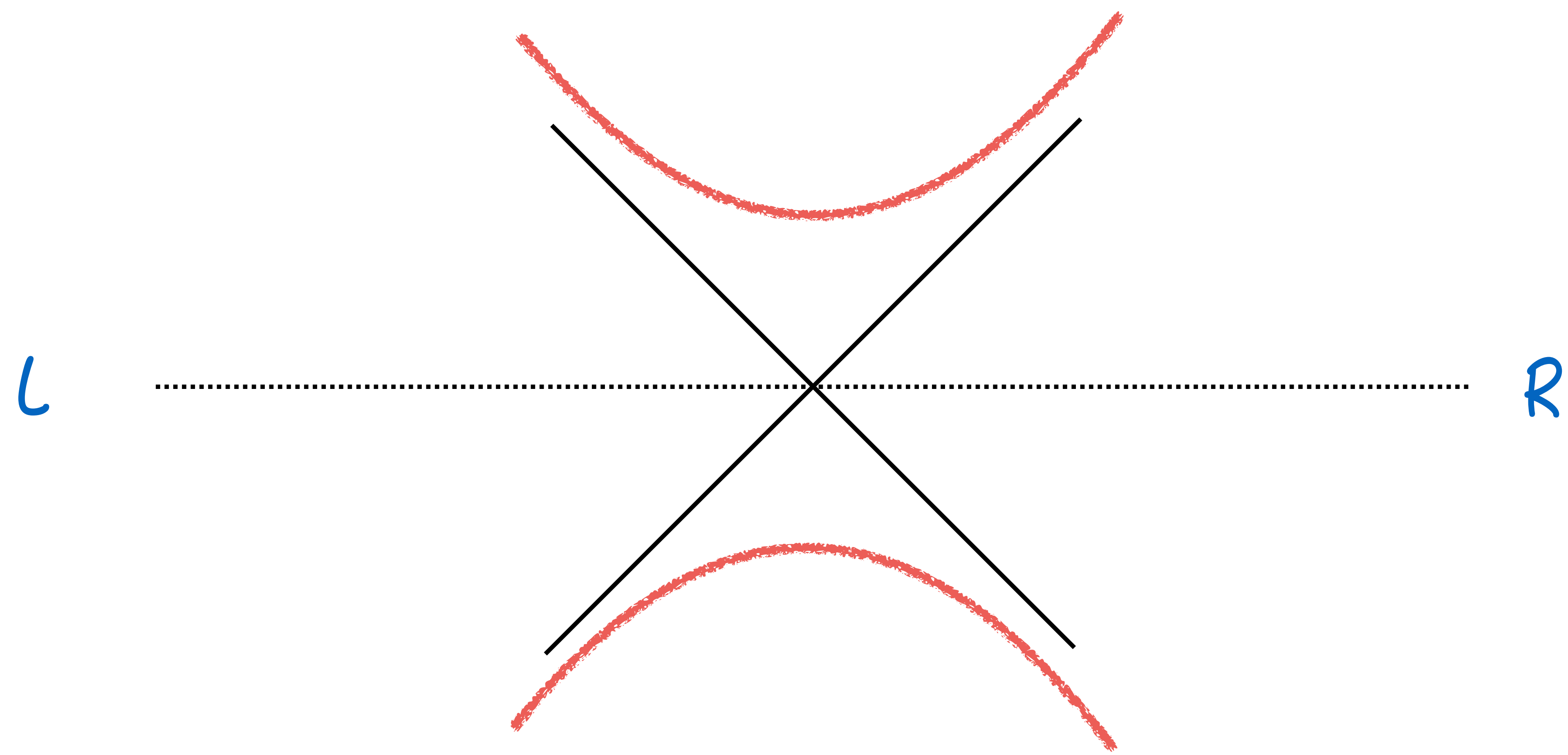
wormhole detection !



$$\langle \phi_L \, \phi_R \rangle \sim \frac{1}{(2 \text{ km})^2}$$

despite the fact that they are separated by 2 million l.y. ⋯ or more, along the "outside route"

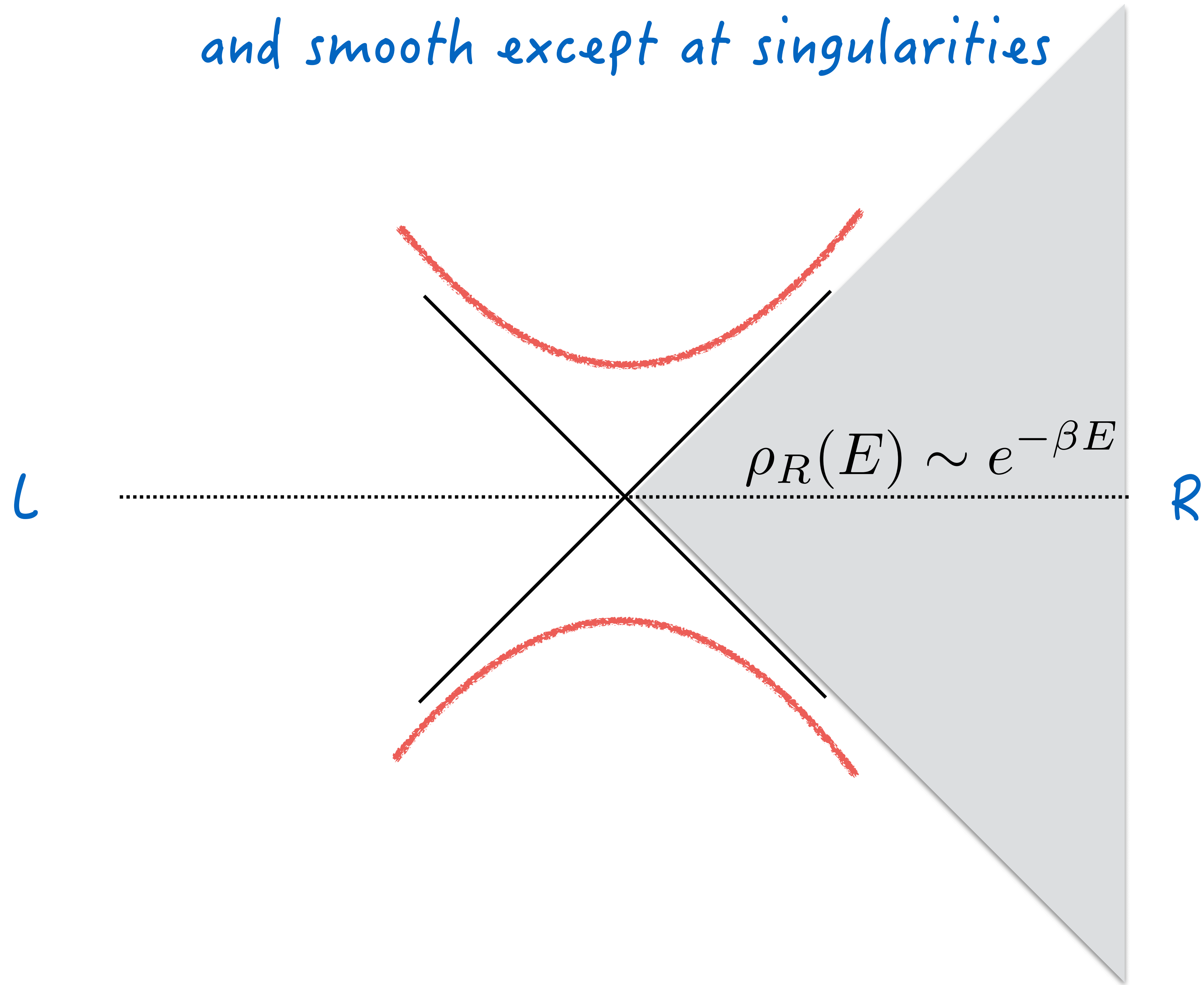
Global state of quantum fields (Hartle-Hawking) is pure and smooth except at singularities



$$|\mathrm{HH}\rangle \sim \int_E e^{-\beta E/2} |E\rangle_L \otimes |E\rangle_R$$

Israel

Global state of quantum fields (Hartle-Hawking) is pure and smooth except at singularities



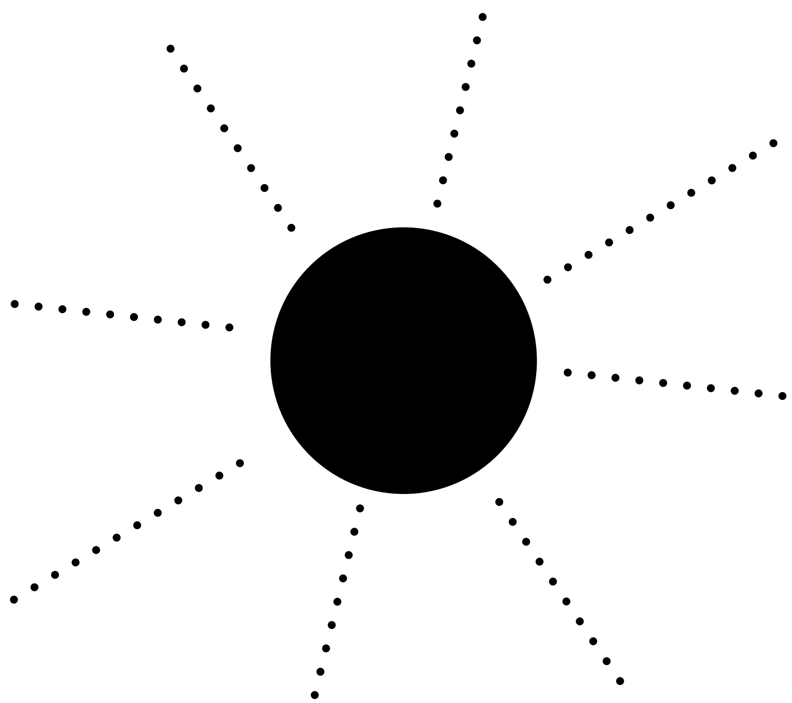
Spectrum of one-sided energies is continuous

BLACK HOLES AND QUANTUM ENTANGLEMENT

Horizon smoothness guarantees large amounts of entanglement in the local state of the near-horizon region

Whether the converse is also true is subject of current heated debate

Evaporation phenomenology



$$\omega_q \sim \frac{1}{R}$$

$$\lambda_q \sim R$$

$$\tau_q \sim R$$

$$\#_q \sim \frac{M}{\omega_q} \sim M\,R \sim S$$

$$\tau_{\rm ev} \sim \tau_q \cdot \#_q \sim R\,S$$

So, a black hole is a long-lived "resonance" with a very narrow width and a huge density of states

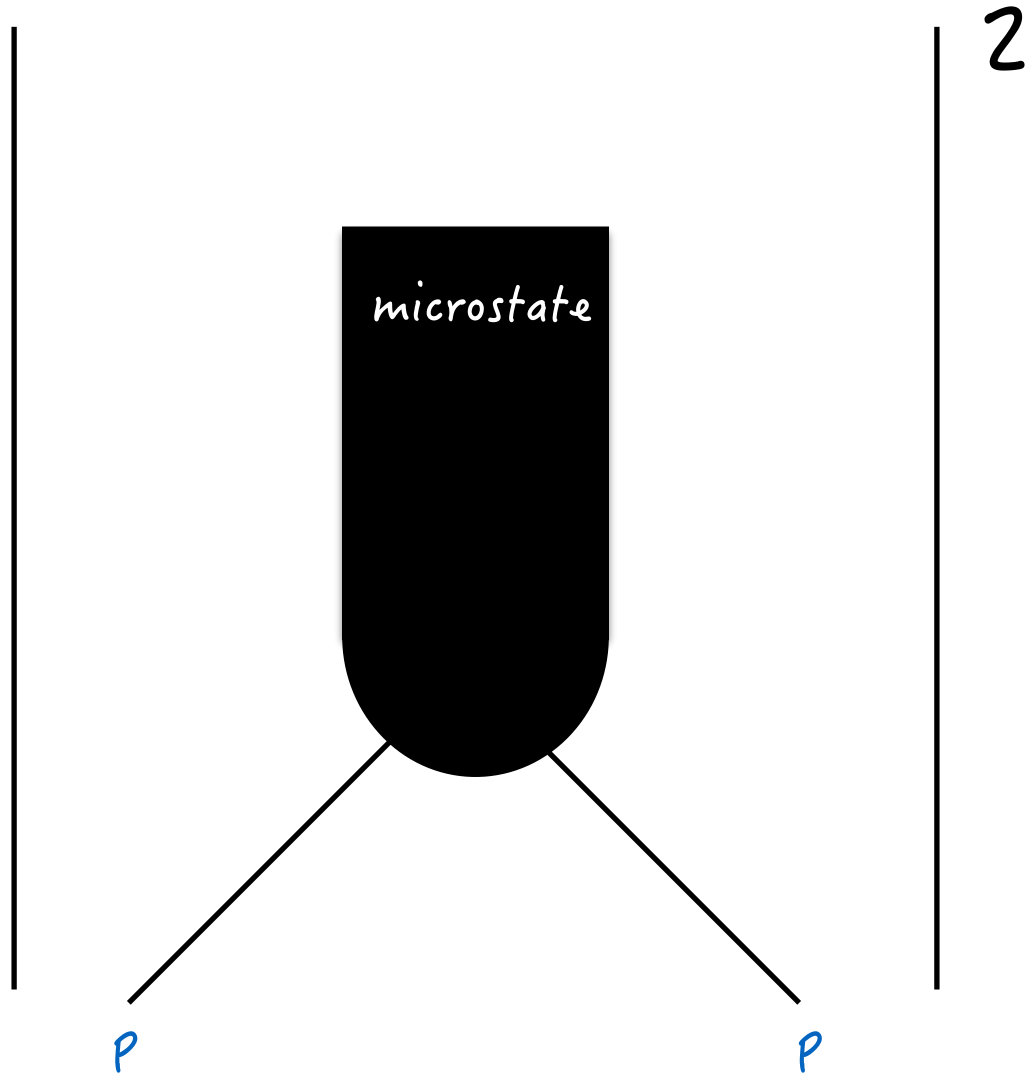
$$\Gamma \sim \frac{1}{\tau_{\rm ev}} \sim \frac{1}{RS} \ll M$$

$$\Omega(M) \propto e^{S} \sim \exp\left(4\pi^2 G M^2\right)$$

It is natural to treat the formation/evaporation as any other S-matrix process

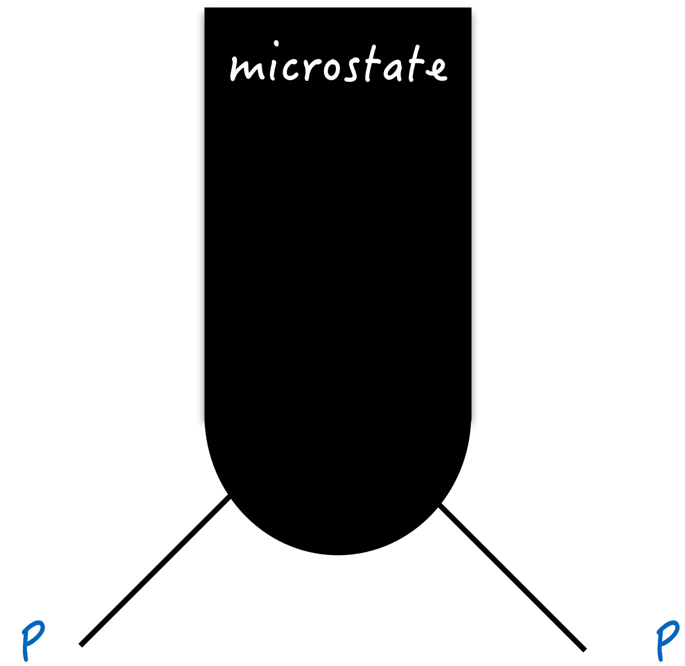
Constraints from UNITARITY

$$\sum_{e^S \text{ states}}$$



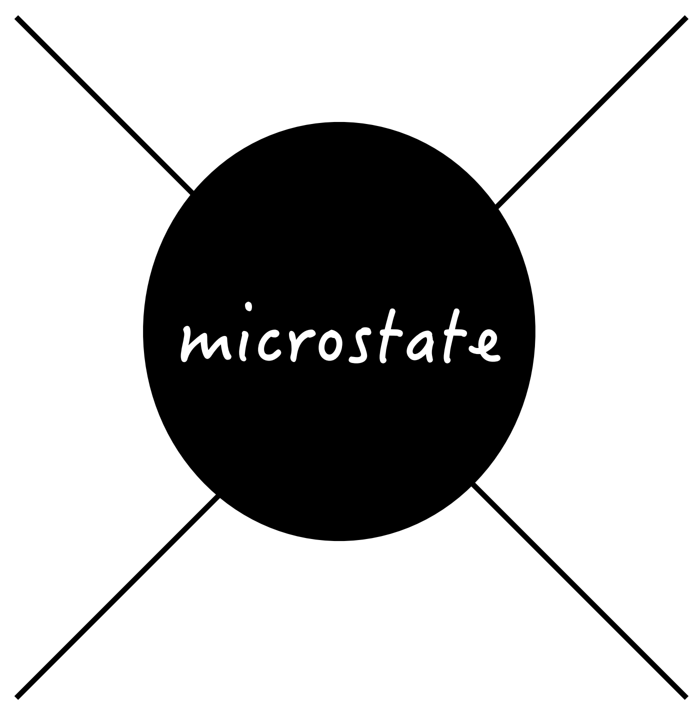
$$= \mathcal{O}(1)$$

proton-proton-bh microstate vertex



$$\sim e^{-S/2}$$

2 ⟶ 2 scattering amplitude



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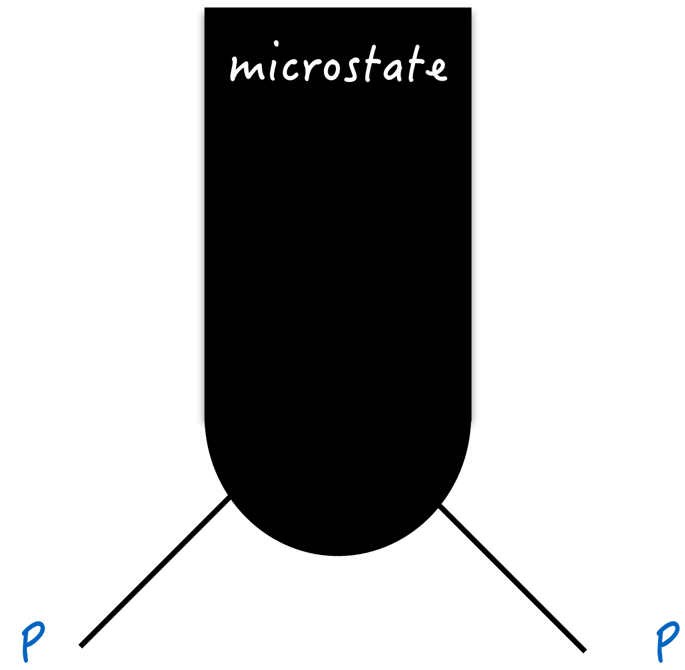
proton-proton-bh microstate vertex



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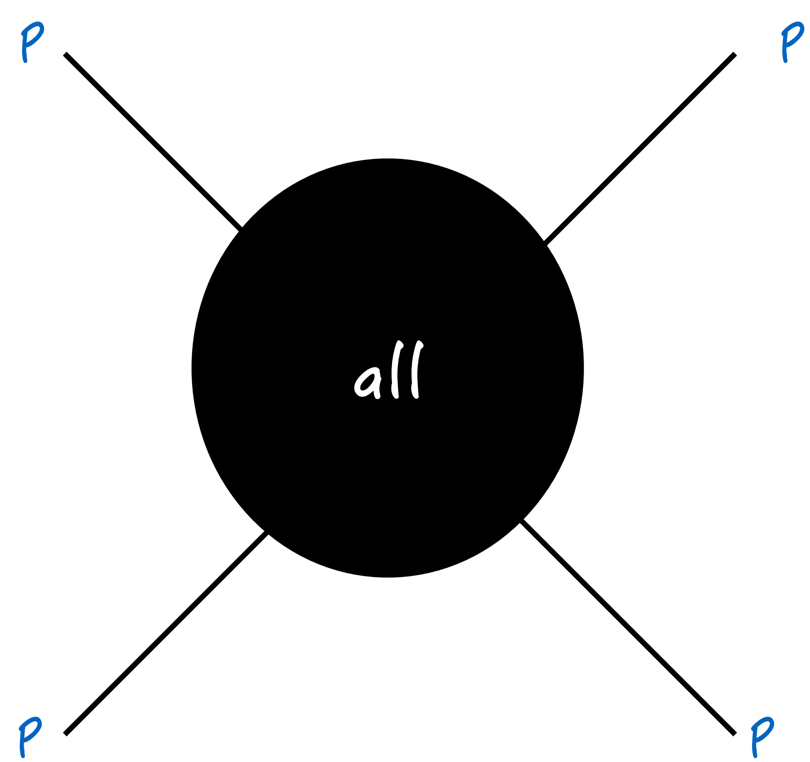


$$\sim e^{-S/2}$$

Scattering amplitudes of order e^{-M^2/M_P^2} are much softer than anything QFT or even string theory can produce

Both the very strong density of states and the extremely soft high energy scattering suggest that the fundamental theory of quantum gravity is not local $\cdots$ holographic?

In what follows, we will try to "follow the money" in the form of these elusive e^{-S} effects

Any effect of order e^{-S} is non-perturbative in gravity and beyond low-energy effective field theory

The effective expansion parameter for graviton loops is

$$\alpha_{\rm eff} = G\,\omega_q^2 \sim \frac{G}{R^2} \sim \frac{1}{S}$$

$$e^{-S} \sim e^{-1/\alpha_{\rm eff}}$$

··· but such effects are precisely those we need to control to avoid the infamous "information paradox"

In Hawking's picture, each rad quantum is well approximated by a member of an EPR pair: radiation comes out in a highly mixed state

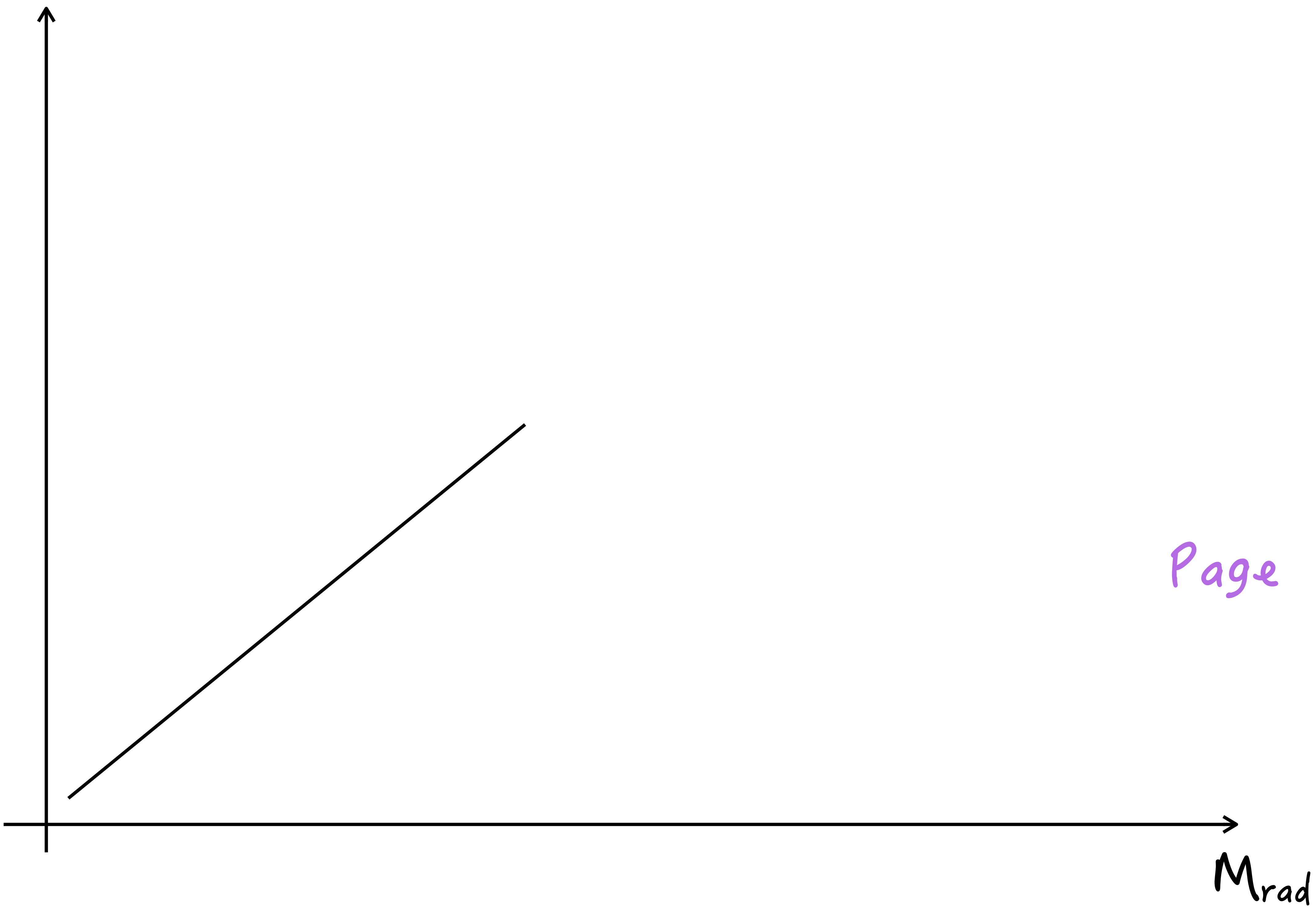
$$\Delta S_{\rm rad}\Big|_{\rm emission} \sim \log 2 = O(1)$$

entropy production is roughly maximal

$$S_{\rm rad} \sim \#_q \sim R\, M_{\rm rad}$$

$$S_{\rm rad} = -{\rm Tr}\, \rho_{\rm rad} \log \rho_{\rm rad}$$

$$S_{\mathrm{rad}} = -\mathrm{Tr}\,\rho_{\mathrm{rad}}\,\log\,\rho_{\mathrm{rad}}$$



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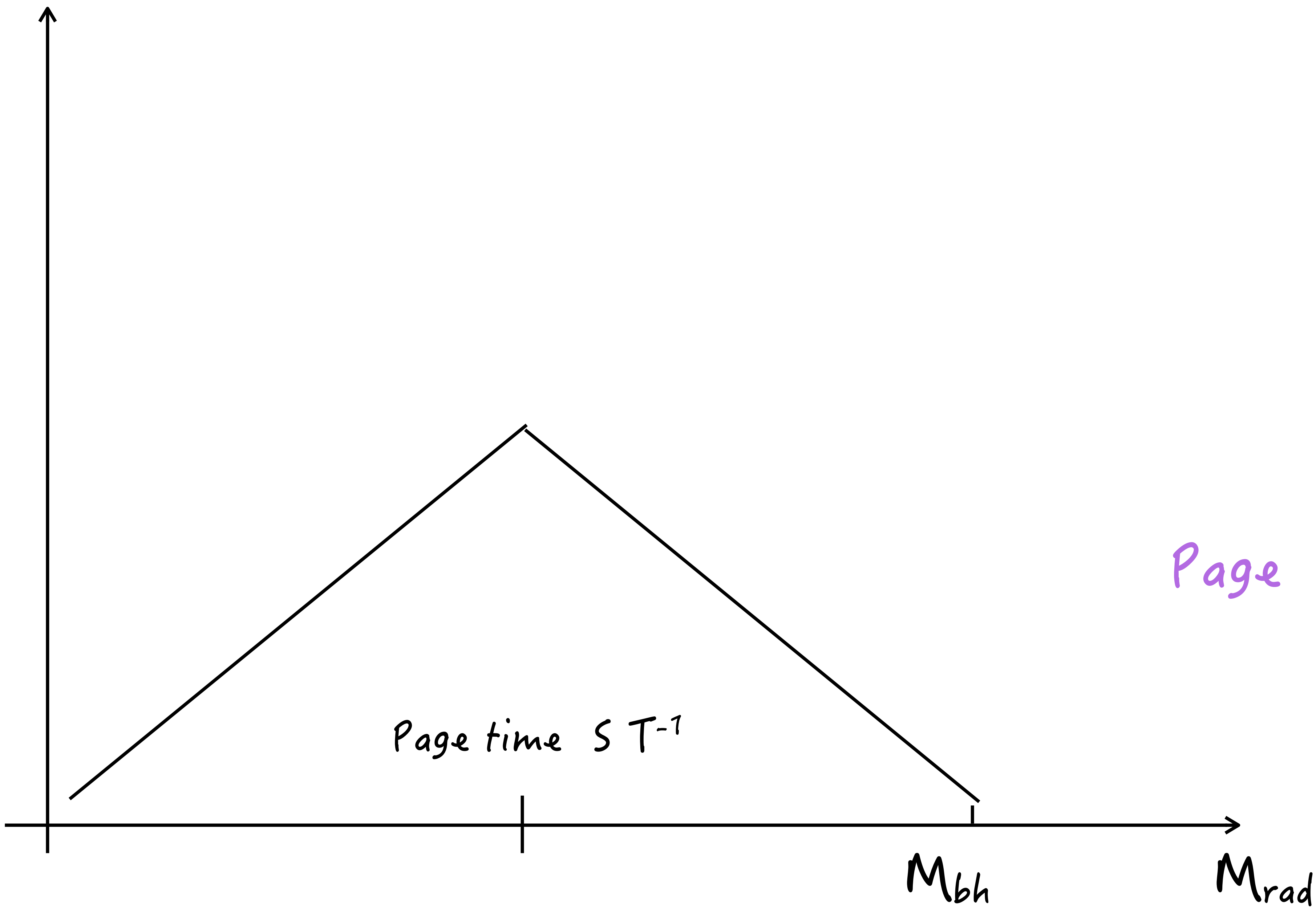
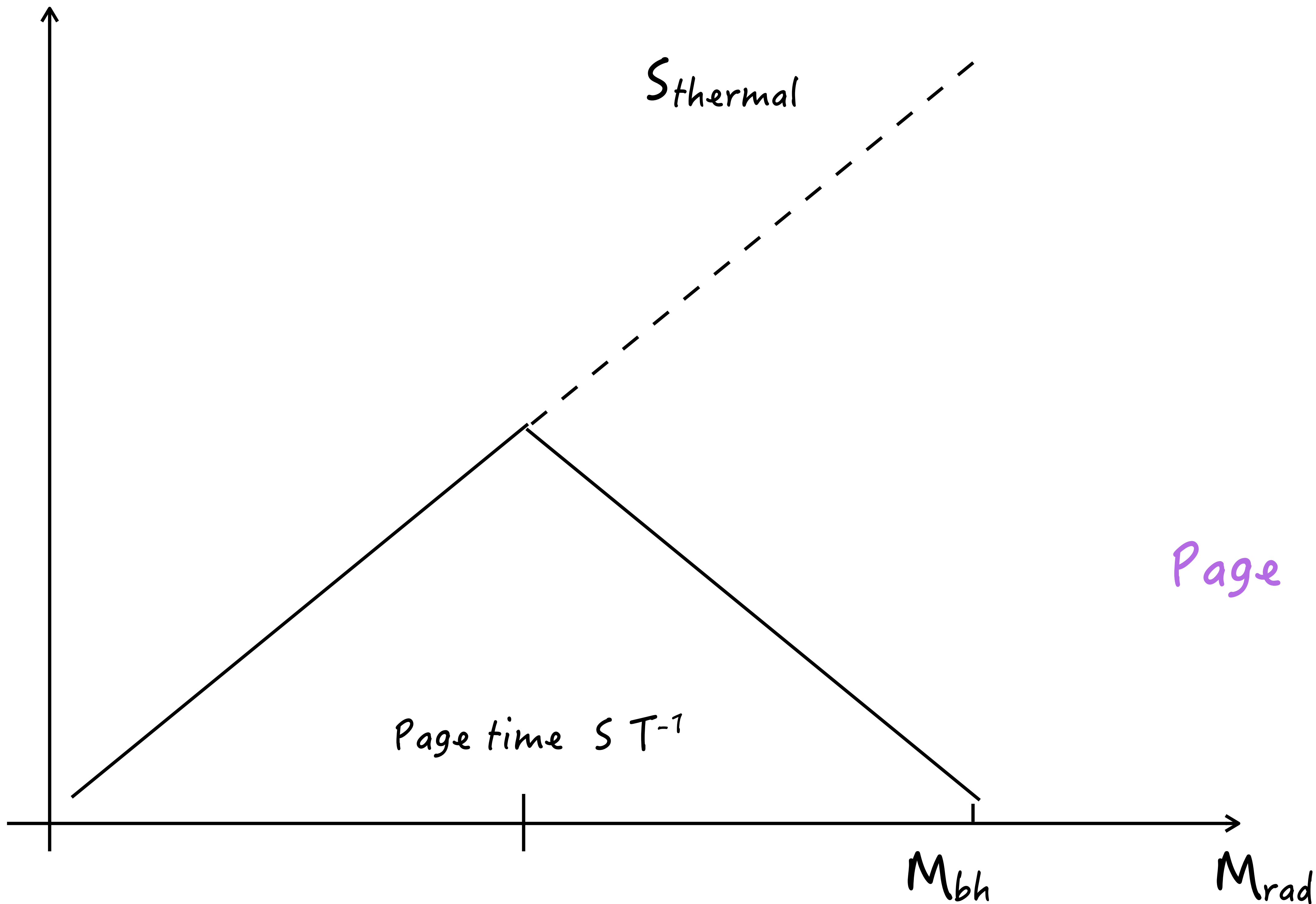


Page

$$S_{\mathrm{rad}} = -\mathrm{Tr}\,\rho_{\mathrm{rad}}\,\log\,\rho_{\mathrm{rad}}$$



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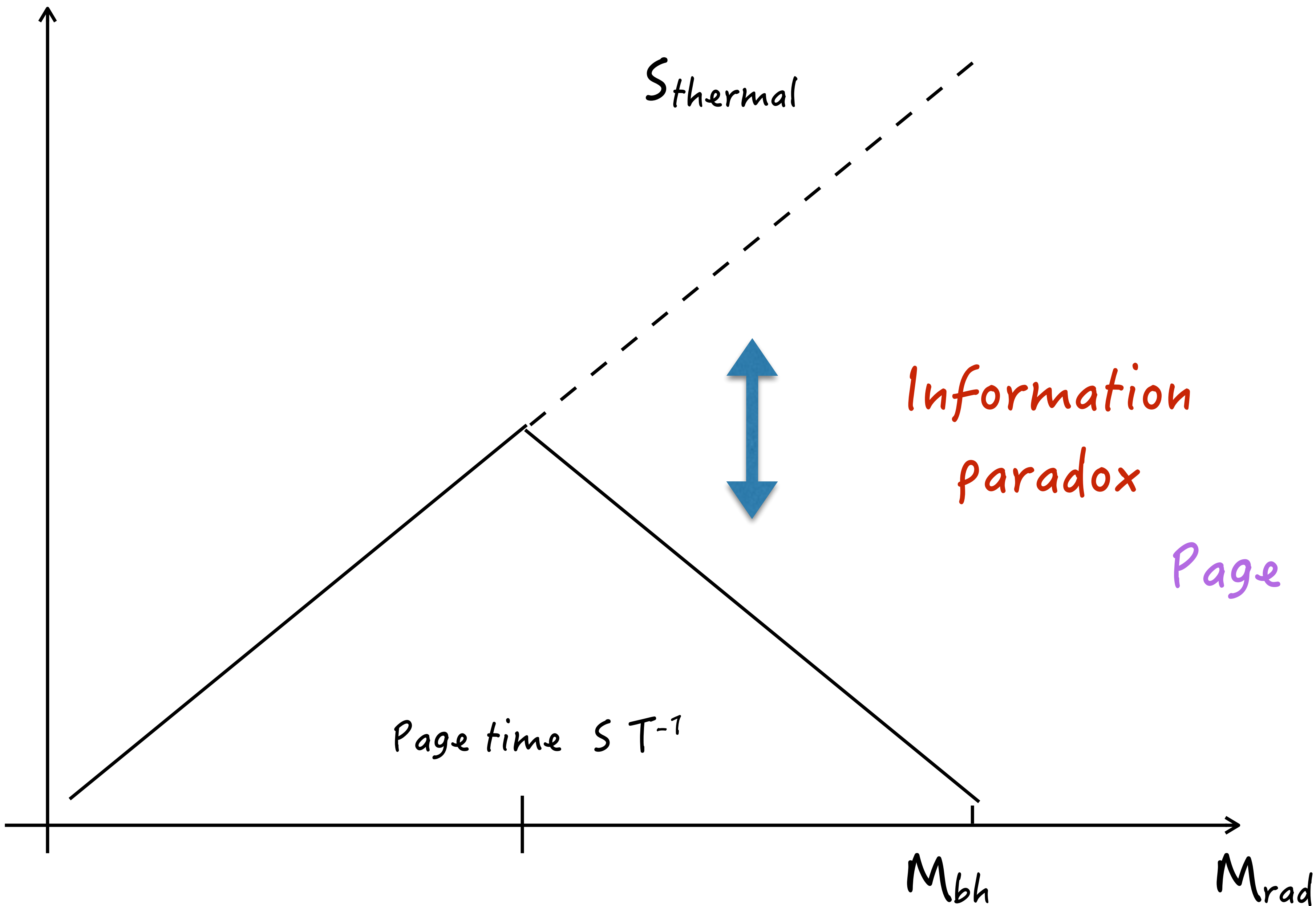

At the Page time, memory effects from initial state on each rad quantum are of order

$$e^{-T t_{\mathrm{Page}}} \sim e^{-S}$$

But the fine-grained von Neumann entropy of the radiation involves e^S measurements of $O(S)$ degrees of freedom, so we can get an $O(1)$ deviation for S_{rad}

This is actually "natural" ...

$$\begin{pmatrix} 1 & & & & \mathbf{0} \\ & 0 & & & \\ & & 0 & & \\ & & & \ddots & \\ \mathbf{0} & & & & 0 \end{pmatrix}$$

$e^S \times e^S$ density matrix of pure state

This is actually "natural" ...

$$U^{\dagger} \begin{pmatrix} 1 & & & & 0 \\ & 0 & & & \\ & & 0 & & \\ & & & \ddots & \\ 0 & & & & 0 \end{pmatrix} U$$

$e^S \times e^S$ density matrix of pure state

random $e^S \times e^S$ unitary

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$$U^{+}\begin{pmatrix}1 & & & & 0\\ & 0 & & & \\ & & 0 & & \\ & & & \ddots & \\ 0 & & & & 0\end{pmatrix}U = e^{-S}\begin{pmatrix}\text{Random}\\ O(1)_{ij}\end{pmatrix}$$

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$$\rho_{ij} = e^{-S}\delta_{ij} + O(e^{-S})_{ij}$$

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Hawking estimate

Nonperturbative correction of order $\exp(-1/\alpha_{eff})$

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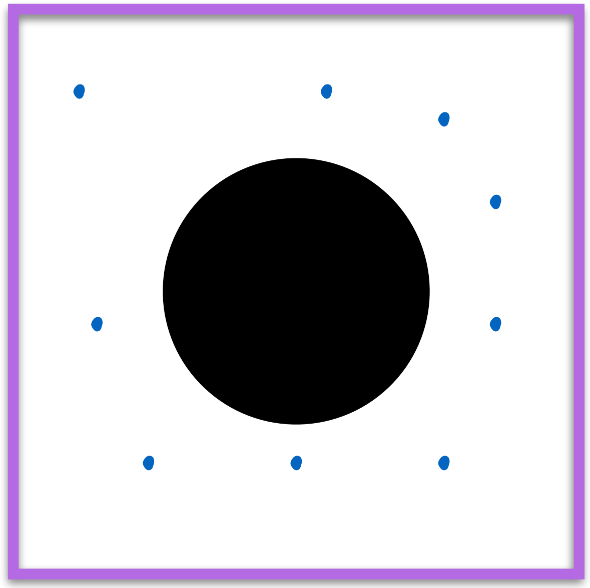
Hawking estimate

Nonperturbative correction of order $\exp(-1/\alpha_{eff})$

PURIFICATION AT PAGE TIME NOT EXPENSIVE!

A sharper way of getting the same insight is to box the black hole and have it equilibrate at fixed box temperature



Gibbons-Hawking

What is a good box for gravity?

According to Hawking & Page ⋯ AdS!

MOREOVER …

In this case there is a candidate for the exact Hamiltonian with a black hole spectrum

$$S_{\mathrm{AdSbh}_{d+1}} \sim N_*^{1/d} (M\ell)^{\frac{d-1}{d}} \sim S_{\mathrm{CFT}_d}$$

$$N_* \sim \frac{\ell^{d-1}}{G}$$

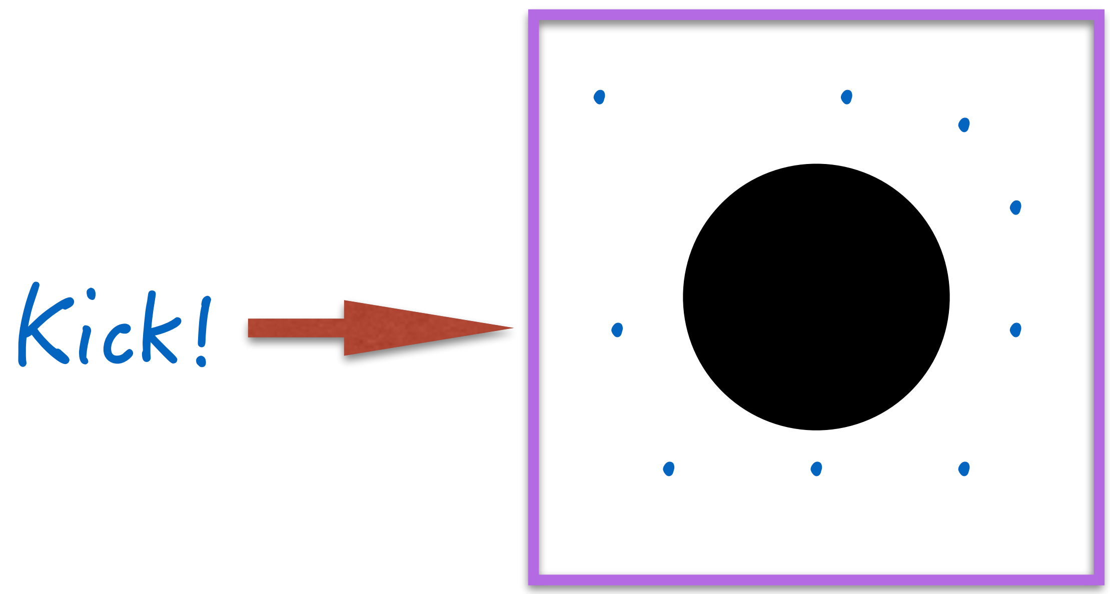
ℓ = curvature radius of AdS

Enter AdS/CFT !

Maldacena

BOX and everything inside = CFT

The decay of local box kicks reveals the absorption of information by the black hole



$$\langle B(t)\, B(0) \rangle \sim e^{-\Gamma t}$$

If the spectrum of bh microstates is approximated as continuous (neglecting spacings of order e^{-S}) then the correlation will continue to decay forever

But the long-time average must be bounded-below in any system with discrete spectrum

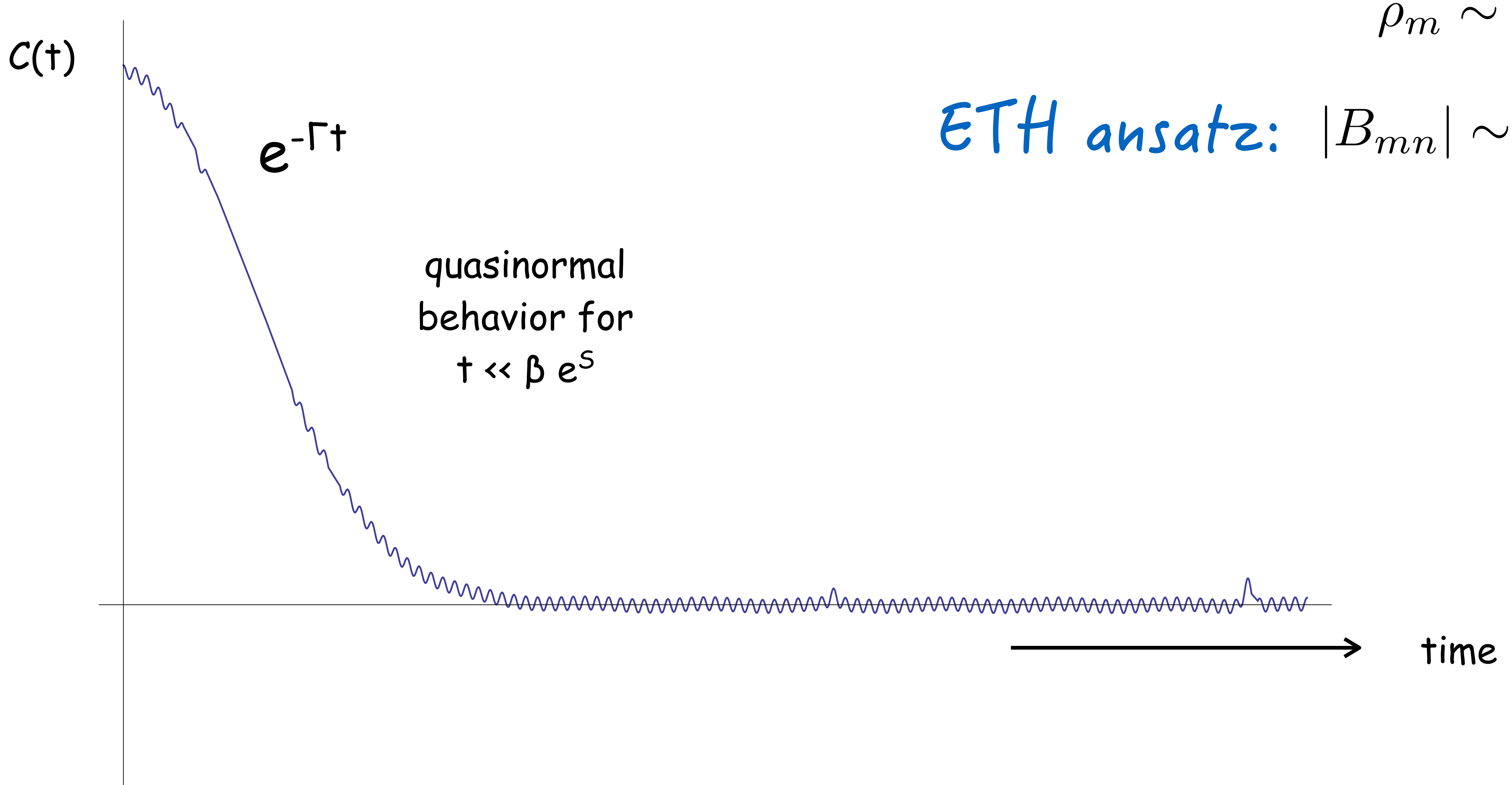
$$C(t) = \mathrm{Tr}\left[\rho\, B(t)\, B(0)\right] = \mathrm{Tr}\left[\rho\, B\, e^{itH}\, B\, e^{-itH}\right]$$

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

ETH ansatz: $|B_{mn}| \sim e^{-S/2}$



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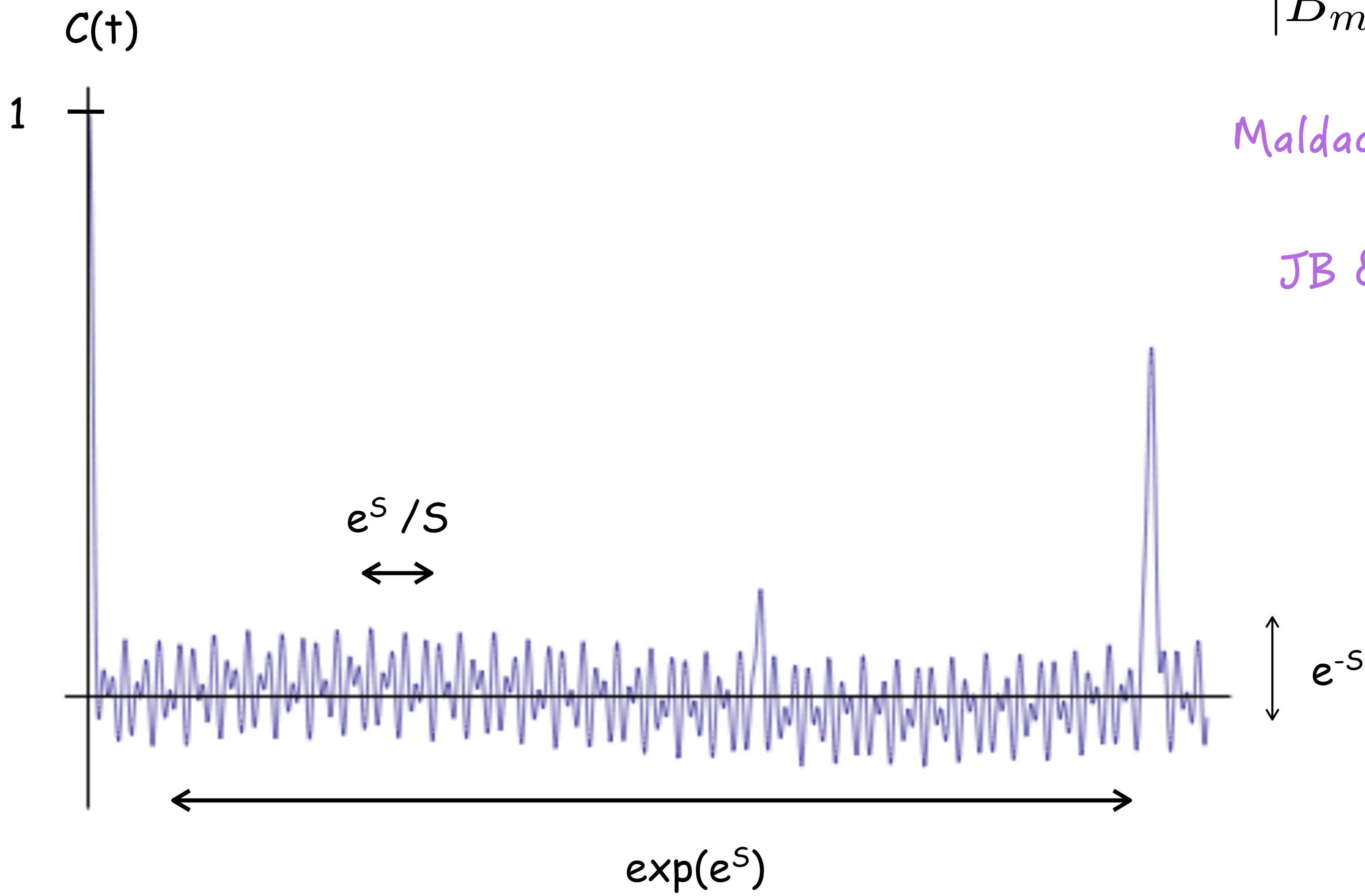
Maldacena, Susskind,

...

JB & Rabinovici



Poincaré time scale

The church of the doubled Hilbert space

Any thermal box can be obtained by tracing over a second identical copy, if appropriately entangled into a global pure state

$$\rho_R = \mathrm{Tr}_{\mathrm{L}} \sum_n C_n \, \Psi_n^L \otimes \Psi_n^R$$

$$(C_n)_{\mathrm{thermal}} = \left[\frac{e^{-\beta E_n}}{\sum_m e^{-\beta E_m}} \right]^{1/2}$$

BUT!!

If the entanglement basis is taken to be the high-energy band of two "entangled" CFTs ⋯

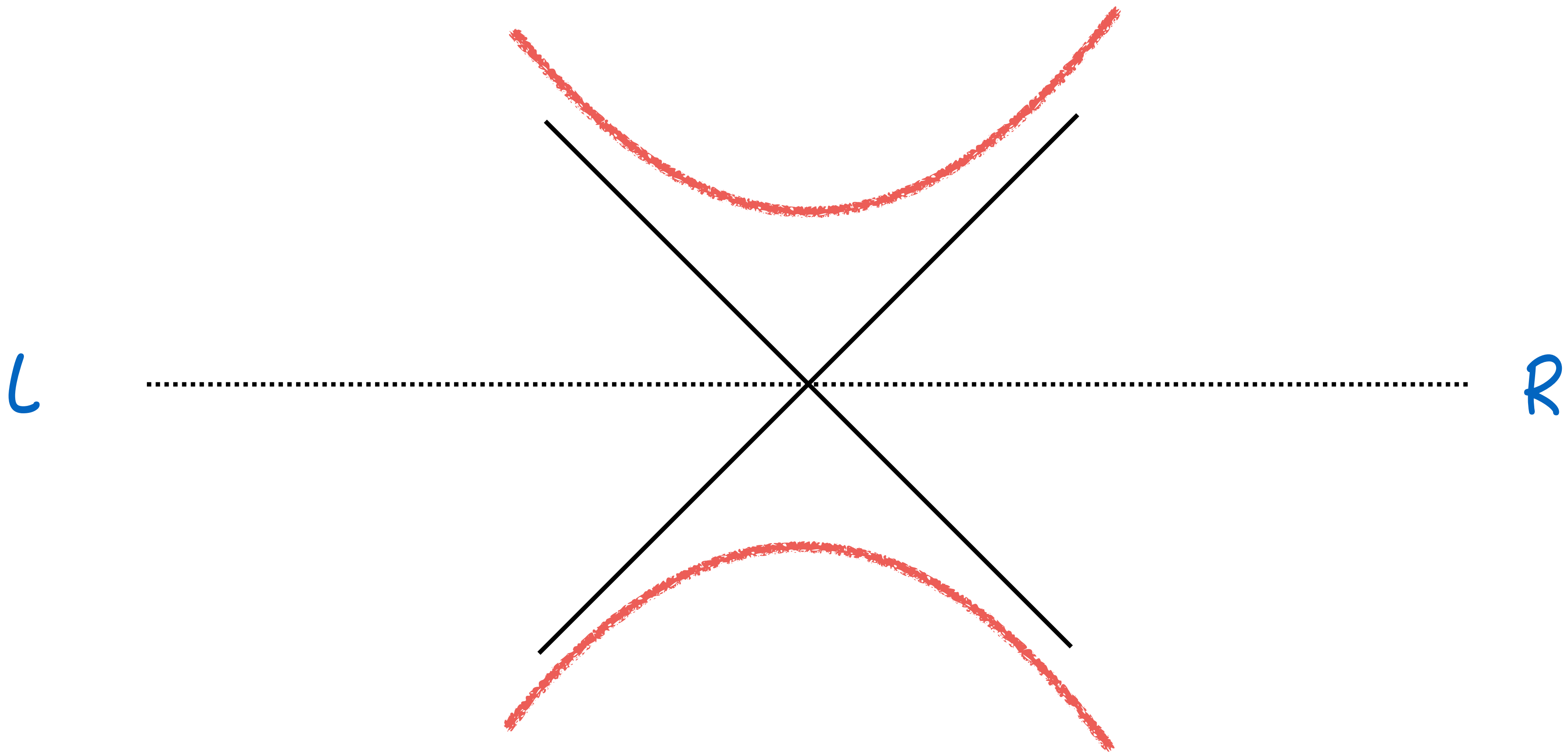
$$|TFD\rangle \sim \sum_{E_n} e^{-\beta E_n/2} |E_n\rangle_L \otimes |E_n\rangle_R$$

neglecting the tiny e^{-S} spacings,
we can approximate by continuous spectrum of fields in the background of an AdS black hole, to get ⋯

$$\int_E e^{-\beta E/2} |E\rangle_L \otimes |E\rangle_R$$

The HH state of the bulk fields !

Two black holes sharing an interior



$$|\mathrm{HH}\rangle \sim \int_E e^{-\beta E/2} |E\rangle_L \otimes |E\rangle_R$$

So, by bringing in the e^{-S} details, AdS/CFT implies that the background geometry itself is a result of entanglement of the two disconnected CFTs

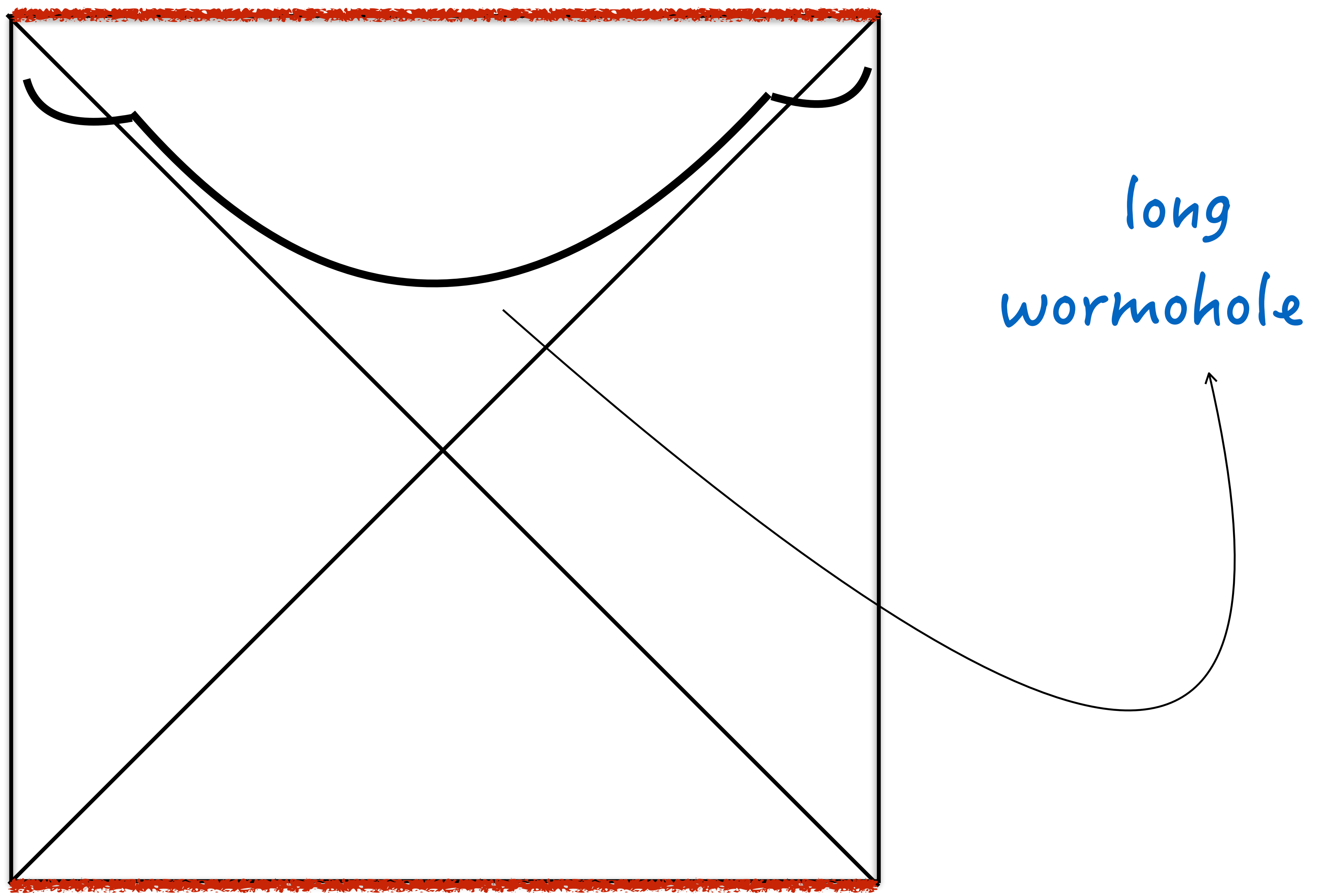
SLOGAN: EPR = ER

Maldacena-Susskind

Accumulating a density of entanglement of $S >> 1$ well-separated Bell pairs within a transversal size of order $(G\,S)^{1/2}$ seems to generate a geometrical bridge of area $G\,S$

What about dynamics?

$$|TFD\rangle_t \sim \sum_n e^{-\beta E_n/2+2iE_n t}|E_n\rangle_L \otimes |E_n\rangle_R$$



What is the entanglement interpretation of the growing wormhole?

A hint is obtained by looking at a system of 2S entangled Qbits

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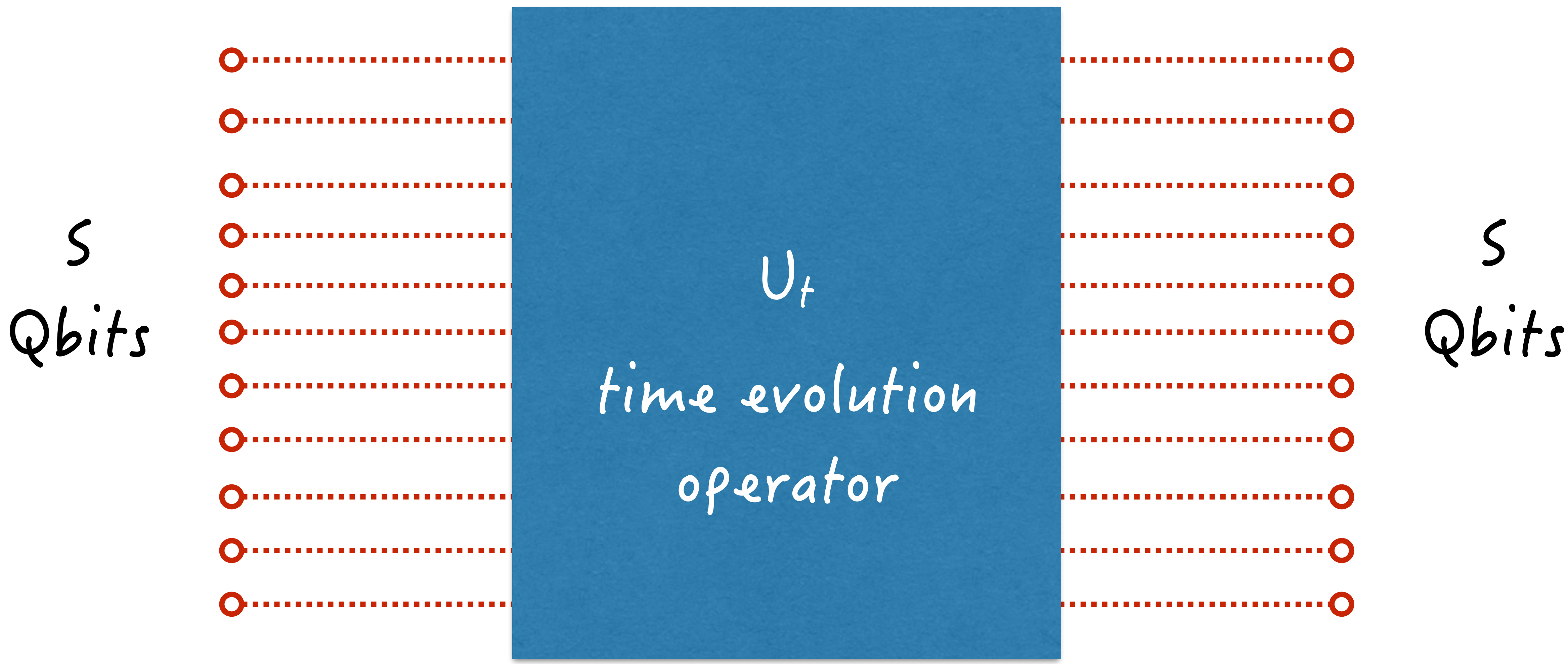


A notion of "size" for U is that of quantum complexity theory

Complexity (U) = minimal size of quantum circuit

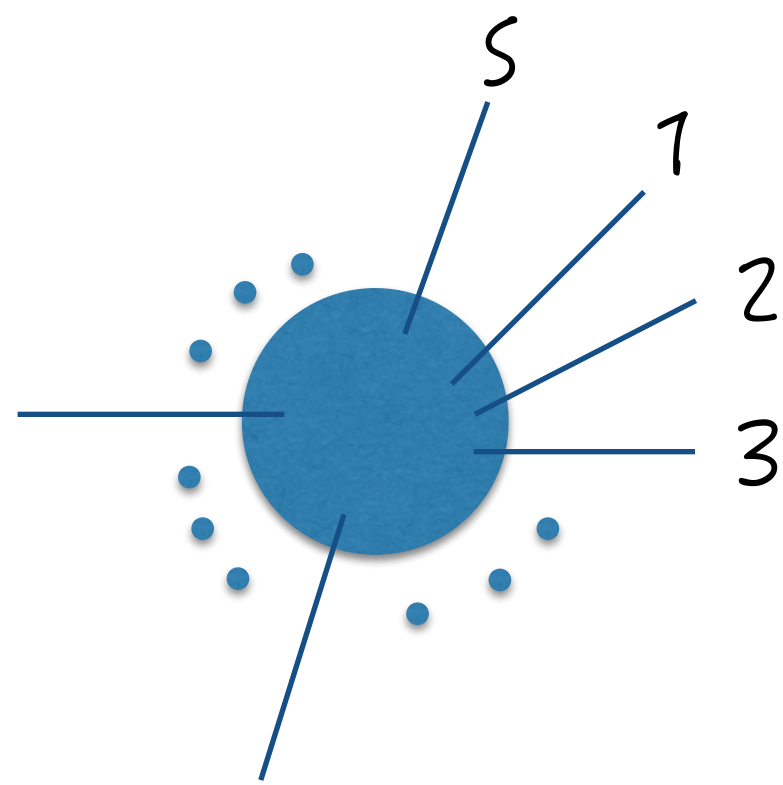
Hartman-Maldacena, Susskind, ···

Parametrizing complexity of entanglement

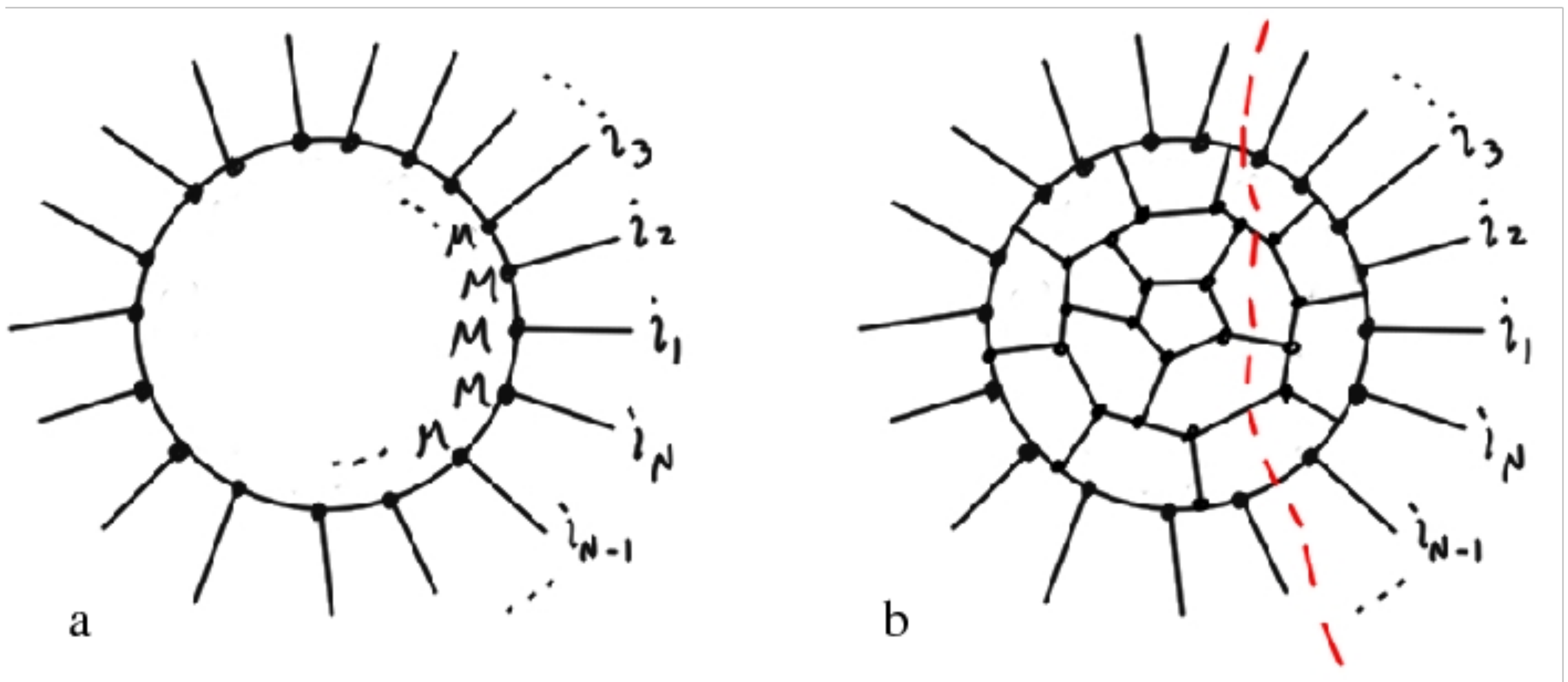
Pick a tensor decomposition of Hilbert space of dimension exp(S) into S factors of O(1) dimension

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_S$$

A tensor of **S** indices gives a generic state



The decomposition of the big tensor in **small** building blocks gives a notion of "complexity of entanglement"

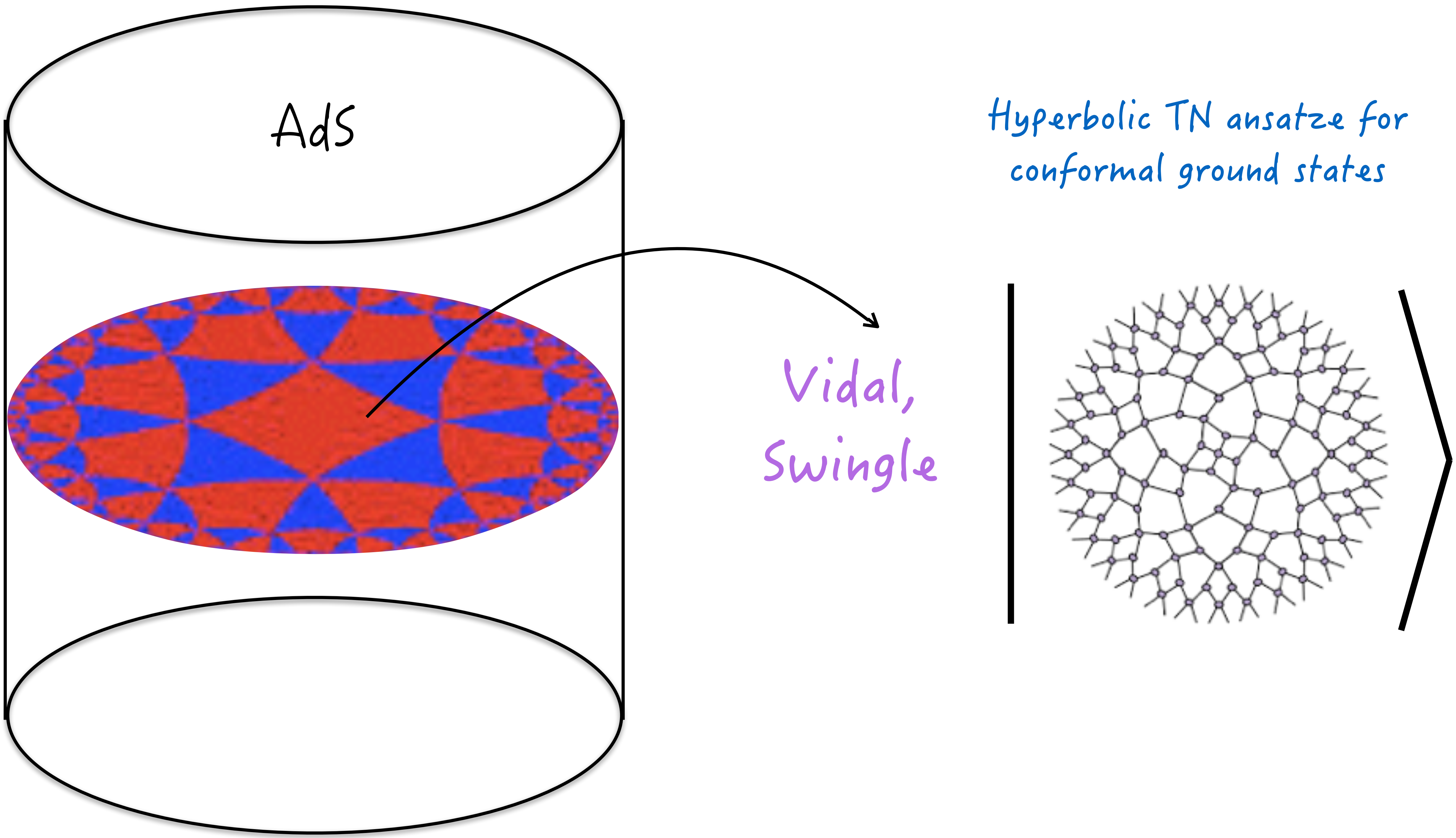


rather simple entanglement pattern

somewhat more complex entanglement pattern

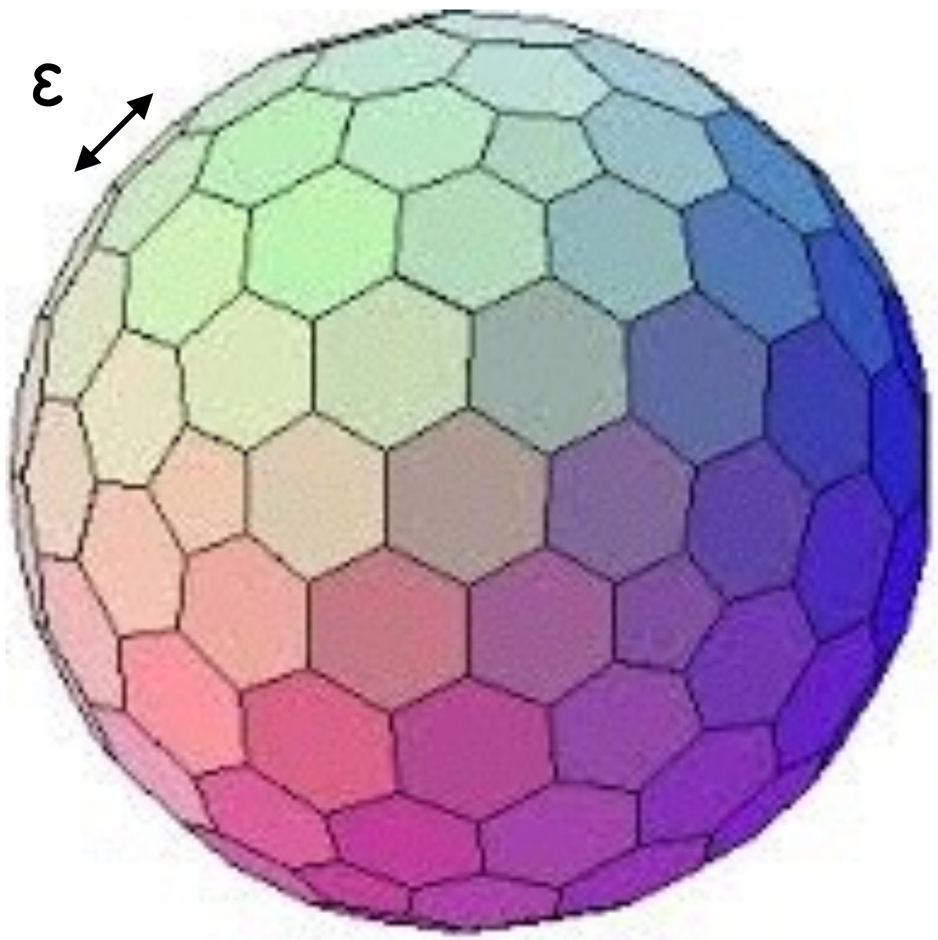
picture from M van Raamsdonk's review

The key example



Size of TN ⟷ Volume of spatial slice

In quantum complexity theory, we try to map out the quantum Hilbert space as if it was a classical phase space



$$\frac{U(e^S)}{U(e^S - 1) \times U(1)}$$

$$\# \text{ cells} \sim \exp\left(e^S \log(1/\epsilon)\right)$$

Just like classical entropy is bounded by the coarse-grained volume of phase space, quantum complexity is bounded by the number of ε-cells

Being serious about e^{-S} effects …

we came to the astonishing suggestion that the very fabric of space could be a "condensate" of pure quantum entanglement

Plenty of circumstantial evidence from AdS/CFT …

ENTANGLEMENT ⟷ BULK CONNECTIVITY

COMPLEXITY ⟷ BULK VOLUME

A list of open questions & problems

- Need exactly calculable toy models of AdS/CFT along the lines of SYK model
- Give a "renormalized" definition of quantum complexity for continuum CFTs
- Can tensor networks describe bulk gravitons?
- What is the space-time meaning of quantum complexity saturation?
- Can we define approximate local observables for black hole interiors?
- Are there obstructions related to firewalls and/or fuzzballs?

THANK YOU