Dynamical thermalization in isolated quantum dots and black holes

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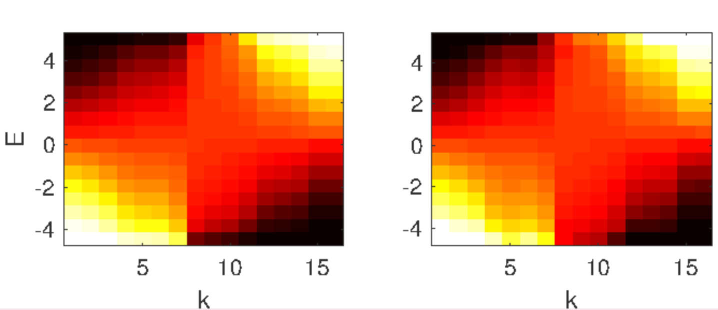
www.quantware.ups-tlse.fr/dima

with A.R.Kolovsky (RAS Krasnoyarsk) EPL **117**, 10003 (2017)



Duality relation between an isolated quantum dot with infinite-range strongly interacting fermions and a quantum Black Hole model in 1 + 1 dimensions: the Sachdev-Ye-Kitaev (SYK) model (1993-2015)

SYK model Refs

VOLUME 70, NUMBER 21 PHYSICAL REVIEW LETTERS 24 MAY 1993

Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet

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(Received 22 December 1992)

PHYSICAL REVIEW X **5,** 041025 (2015)

Bekenstein-Hawking Entropy and Strange Metals

Subir Sachdev
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Video talks: Schedule Apr 07, 2015; May 27, 2015
A simple model of quantum holography (part 1,2)
Alexei Kitaev, Caltech & KITP
http://online.kitp.ucsb.edu/online/entangled15/kitaev/

Recent SYK + quantum chaos Refs

RSYK1) J.Maldacena and D.Stanford, *Comments on the Sachdev-Ye-Kitaev model*, IAS Princeton, arXiv:1604.07818 (2016)
RSYK2) J.Polchinski, *Chaos in the black hole S-matrix*, KITP, arXiv:1505.08108 (2015)
RSYK3) Y.Gu, X.-L.Xi and D.Stanford, *Local criticality, diffusion and chaos in generalized SYK model*, Stanford-Princeton, arXiv:1609.07832 (2016)
RSYK4) D.J.Gross and V.Rosenhaus, *A generalization of Sachdev-Ye-Kitaev*, KITP, arXiv:1610.01569 (2016)
RSYK5) I. Danshita, M.Hanada and M.Tezuka, *Creating and probing the Sachdev-Ye-Kitaev model with ultracold gases: Towards experimental studies of quantum gravity*, Yukawa-Stanford, arXiv:1606.02454 (2016)
RSYK6) J.Maldacena, S.H.Shenker and D.Stanford, *A bound on chaos*, Princeton-Stanford, arXiv:1503.01409 (2015)
RSYK7) A.M.Garcia-Garcia and J.J.M. Verbaarschot, *Spectral and thermodynamic properties of the Sachdev-Ye-Kitaev model*, Cambridge UK - Stony Brook, arXiv:1610.03381 (2016)
RSYK8) A.M.Garcia-Garcia and J.J.M. Verbaarschot, *Analytical spectral density of the Sachdev-Ye-Kitaev model at finite N*, Cambridge UK - Stony Brook, arXiv:1701/06593 (2017)
...

Duality in SYK model (in short)

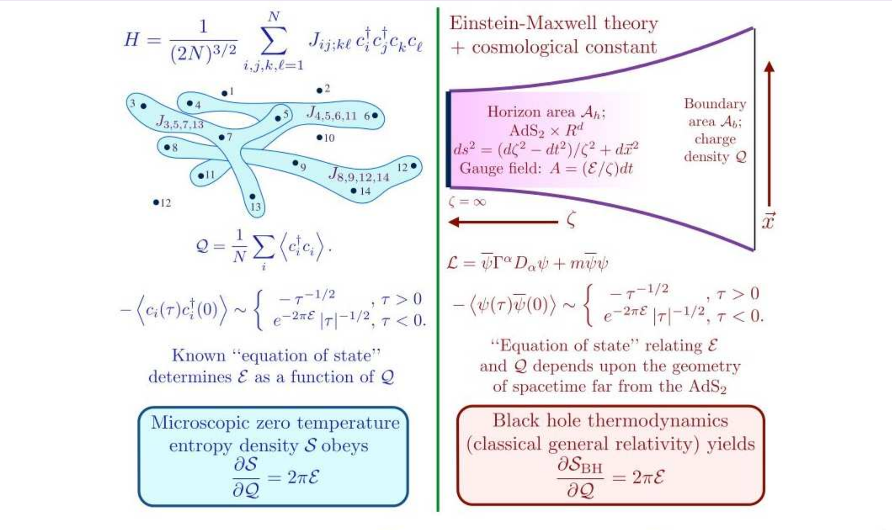


FIG. 2. Summary of the properties of the SY state (Sec. II) and planar charged black holes (Sec. III) at $T = 0$. The spatial coordinate $\vec{x}$ has d dimensions. All results also apply to spherical black holes considered in Appendix B. The $AdS_2 \times R^d$ metric has unimportant

from Sachdev PRX (2015)

GOE for TBRIM (1971)

Volume 35B, number 5 PHYSICS LETTERS 21 June 1971

SPACING AND INDIVIDUAL EIGENVALUE DISTRIBUTIONS OF TWO-BODY RANDOM HAMILTONIANS

O. BOHIGAS and J. FLORES *

Institut de Physique Nucléaire, Division de Physique Théorique **, 91 - *Orsay, France*

Received 10 April 1971

Comparison is made of the results of the Gaussian orthogonal ensemble with the ones produced with an ensemble in which the two-body character of the hamiltonian and the Pauli principle are taken into account. Significant differences arise between the two ensembles of random matrices.

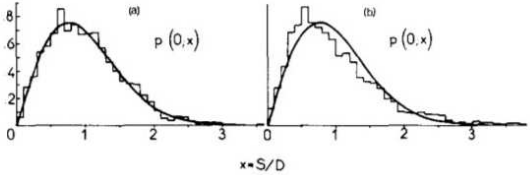


Fig.4. Nearest-neighbour spacing distributions in the ground state region (a) Gaussian orthogonal ensemble; (b) a two-body random hamiltonian ensemble. The curve is the same as in fig. 2 for $k = 0$.

Model description (TBRIM)

The model is described by the Hamiltonian for L spin-polarized fermions on M energy orbitals ϵ_k ($\epsilon_{k+1} \geq \epsilon_k$):

$$\widehat{H} = \widehat{H}_0 + \widehat{H}_{int} \ , \ \widehat{H}_0 = \frac{1}{\sqrt{M}} \sum_{k=1}^{M} v_k \hat{c}_k^\dagger \hat{c}_k \ , \widehat{H}_{int} = \frac{1}{\sqrt{2M^3}} \sum_{ijkl} J_{ij,kl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l \ ,$$

$\hat{c}_i^\dagger, \hat{c}_i$ are fermion operators; matrix elements $J_{ij,kl}$ are random complex variables (Sachdev2015) with a standard deviation J and zero average value (Kitaev2015 used Majorana fermions). In addition to the interaction Hamiltonian $\widehat{H}_{int}$, there is an unperturbed part $\widehat{H}_0$ describing one-particle orbitals $\epsilon_k = v_k/\sqrt{M}$ in a quantum dot of non-interacting fermions. The average of one-orbital energies is taken to be $\overline{v_k^2} = V^2$ with $\overline{v_k} = 0$. Thus the unperturbed one-particle energies ϵ_k are distributed in an energy band of size V and the average level spacing between them is $\Delta \approx V/M^{3/2}$ while the two-body coupling matrix element is $U \approx J/M^{3/2}$. Hence, in our model the effective dimensionless conductance is $g = \Delta/U \approx V/J$. The matrix size is $N = M!/L!(M-L)!$ and each multi-particle state is coupled with $K = 1 + L(M-L) + L(L-1)(M-L)(M-L-1)/4$ states. We consider an approximate half filling $L \approx M/2$.

Emergence of quantum ergodicity

At $g \gg 1$ the RMT statistics appears only for relatively high excitation above the quantum dot Fermi energy E_F:

$$\delta E = E - E_F > \delta E_{ch} \approx g^{2/3}\Delta \; ; \quad g = \Delta/U \approx V/J \gg 1 \; .$$

This border is in a good agreement with the spectroscopy experiments of individual mesoscopic quantum dots (Sivan1994).

This is the Åberg criterion (PRL1990): coupling matrix elements are comparable with the energy spacing between directly coupled states

(also Sushkov, DS EPL1997, Jacquod, DS PRL1997).

Related Eigenstate Thermalization Hypothesis (ETH),
Many-Body Localization (MBL).

At $g = 0$ TBRIN or SYK model => Wigner-Dyson level spacing statistics $P(s)$:
Bohigas, Flores PLB1970-71; French, Wong PLB1970-71

Dynamical thermalization ansatz

At $g \gg 1$ => Fermi-Dirac thermal distribution of M one-particle orbitals:

$$n_k = \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1} \; ; \;\; \beta = 1/T \; ,$$

with the chemical potential μ determined by the conservation of number of fermions $\sum_{k=1}^{M} n_k = L$.
At a given temperature T, the system energy E and von Neumann entropy S are

$$E(T) = \sum_{k=1}^{M} \epsilon_k n_k \; , \quad S(T) = -\sum_{k=1}^{M} n_k \ln n_k \; .$$

Fermi gas entropy is $S_F = -\sum_{k=1}^{M} (n_k \ln n_k + (1 - n_k) \ln(1 - n_k))$.
S and E are obtained from eigenstates ψ_m and eigenenergies E_m of H via $n_k(m) = < \psi_m | \hat{c}_k^+ \hat{c}_k | \psi_m >$.

$S(T)$ and $E(T)$ are extensive and self-averaging.

This gives the implicit dependence S(E).

Dynamical thermalization in quantum computer

$H = \sum_i \Gamma_i \sigma_i^z + \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x$, nearest-neighbor qubit pairs on 2D lattice (central band); $\Gamma_i = \Delta_0 + \delta_i$, $|J_{ij}| \leq J$; $J_c \approx 4\delta/n$ - chaos border

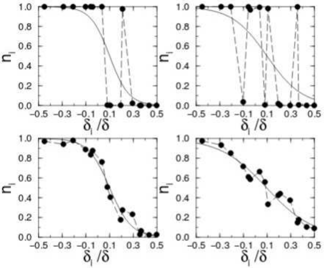


Fig. 5. Same as in Figure 4 but for a given random realization and a single eigenstate for $n = 16$ qubits (left: $m = 5$; right: $m = 100$). Top left: $J = 0.03\delta$, $T_{\rm FD} = 0.08\delta$, $\delta E = 0.25\delta$, $S_{\rm q} = 0.23$; top right: $J = 0.03\delta$, $T_{\rm FD} = 0.15\delta$, $\delta E = 0.97\delta$, $S_{\rm q} = 0.49$; bottom left: $J = 0.3\delta$, $T_{\rm FD} = 0.09\delta$, $\delta E = 0.28\delta$, $S_{\rm q} = 5.85$; bottom right: $J = 0.3\delta$, $T_{\rm FD} = 0.20\delta$, $\delta E = 1.19\delta$, $S_{\rm q} = 8.41$.

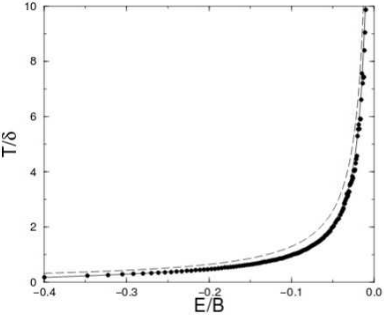
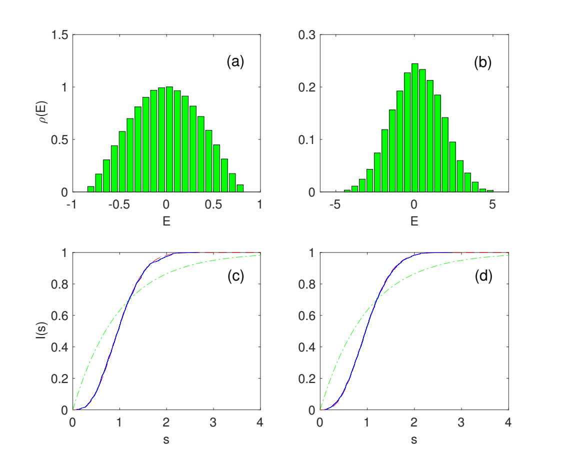


Fig. 8. Dependence of different definitions of temperature T on the scaled energy E/B, for $n = 16$, $J = 0.3\delta$, $N_{\rm D} = 2$: $T_{\rm FD}$ (circles), $T_{\rm can}$ (full curve), and $T_{\rm th}$ (dashed curve).

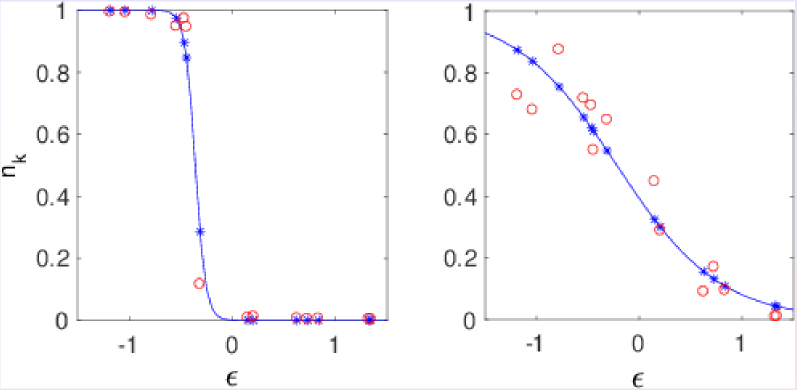
Benenti et al. EPJD 17, 265 (2001) [now 10 years later ETH]

Wigner-Dyson (RMT) level spacing statistics



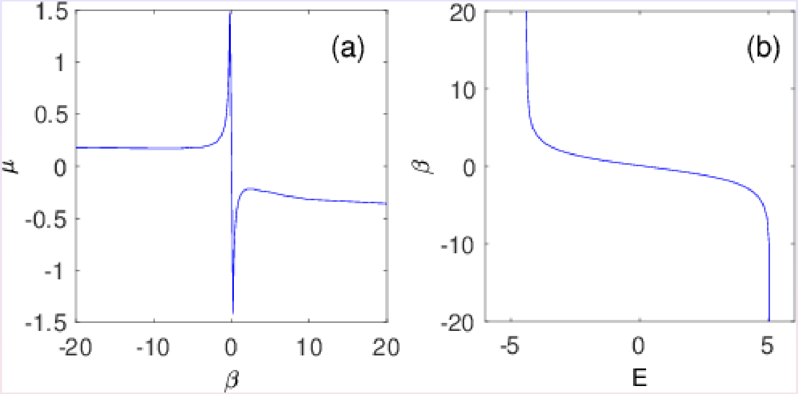
Top row: density of states $\rho(E) = dN(E)/dE$. Bottom row (c,d): integrated statistics $I(s) = \int_0^s ds' P(s')$; Poisson case $P_P(s)$ (green), Wigner surmise $P_W(s) = 32s^2 \exp(-4s^2/\pi)/\pi^2$ (red) and numerics $P(s)$ for central energy region with 80% of states (blue); $M = 14, L = 6, N = 3003$, and $J = 1, V = 0, g = 0$ (a,c) and $J = 1, V = \sqrt{14}, g = \sqrt{14}$ (b,d).

Quantum dot regime ($g \gg 1$)



Dependence of filling factors n_k on energy ϵ for individual eigenstates obtained from exact diagonatization of (red circles) and from Fermi-Dirac ansatz with one-particle energy ϵ (blue curve); blue stars are shown at one-particle energy positions $\epsilon = \epsilon_k$). Here $M = 14$, $L = 6$, $N = 3003$, $J = 1$, $V = \sqrt{14}$ and eigenenergies are $E = -4.4160$ (left), -3.0744 (right); the theory (blue) is drown for the temperatures corresponding to these energies $\beta = 1/T = 20$ (left), 2 (right).

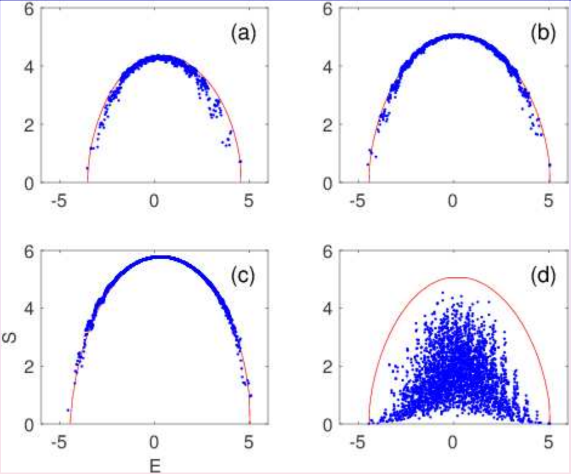
Quantum dot regime $\mu(T)$, $E(T)$



Dependence of inverse temperature $\beta = 1/T$ on energy E (right) and chemical potential μ on β (left) given by the Fermi-Dirac ansatz for the set of one-particle energies ϵ_k as in above Fig.

Negative temperatures $T < 0$.

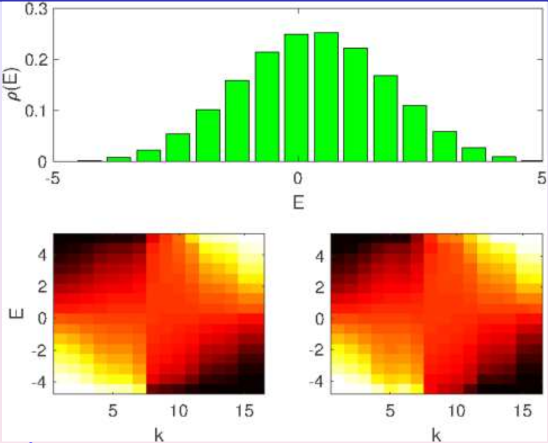
Quantum dot regime $S(E)$



(a) $M = 12$, $L = 5$, $N = 792$, $J = 1$; (b) $M = 16$, $L = 7$, $N = 3003$, $J = 1$; (c) $M = 14$, $L = 6$, $N = 11440$, $J = 1$; (d) $M = 16$, $L = 7$, $N = 3003$, $J = 0.1$. Blue points show the numerical data E_m, S_m for all eigenstates, red curves show the Fermi-Dirac thermal distribution; $V = \sqrt{14}$.

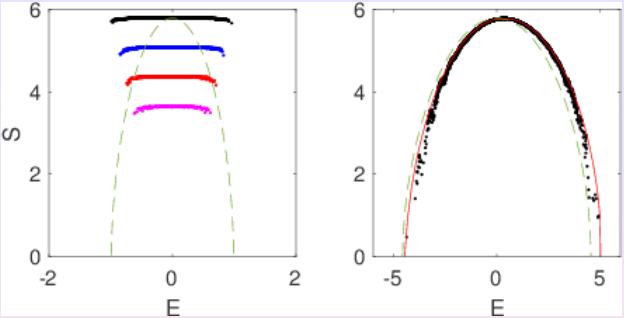
$S(E = 0) = -L \ln(L/M)$ (equipartition).

Fermi-Dirac distribution for quantum dot



Top: $\rho(E)$ vs. E ($\int \rho(E)dE = 1$). Bottom: occupations $n_k(E)$ of one-particle orbitals ϵ_k given by the Fermi-Dirac distribution (left), and by their numerical values obtained by exact diagonalization (right); n_k are averaged over all eigenstates in a given cell. Colors: from black for $n_k = 0$ via red, yellow to white for $n_k = 1$; orbital number k and eigenenergy E are shown on x and y axes respectively; $M = 16, L = 7, N = 11440, V = 4, J = 1$.

SYK black hole regime $S(E)$



$S(E)$ for SYK black hole at $V = 0$ (left) and quantum dot regime $V = \sqrt{14}$ (right); $M = 16, L = 7, N = 11440$ (black), $M = 14, L = 6, N = 3003$ (blue), $M = 12, L = 5, N = 792$ (red), $M = 10, L = 4, N = 210$ (magenta); here $J = 1$. Points show numerical data E_m, S_m for all eigenstates, the full red curve shows FD-distribution (right). Dashed gray curves in both panels show FD-distribution for a semi-empirical model of non-interacting quasi-particles for black points case. Here $S(E = 0) \approx L \ln 2$; $L \approx M/2$.

Semi-empirical model: non-interacting particles on orbital energies ϵ_k reproducing many-body density of states

Low energy excitations: quantum dot vs. SYK

Quantum dot: $\Delta E \propto 1/L^{3/2}$; SYK black hole: $\Delta E \propto \exp(-cL)$

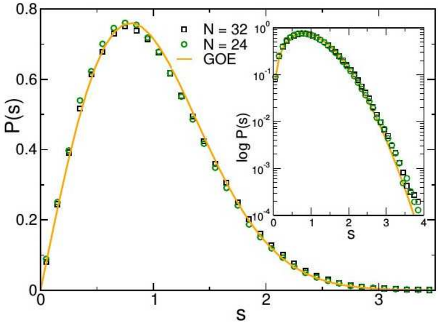


FIG. 5. Level spacing distribution $P(s)$ resulting from exact diagonalization of the SYK Hamiltonian Eq. (1) for $N = 32$ and 400 realizations (squares) and $N = 24$ and 10000 realizations (circles). We only consider the infrared part of the spectrum, about 1.5%, which is related to the gravity-dual of the model. As in the bulk of the spectrum [40, 41], we observe excellent agreement with the Wigner surmise for the Gaussian Orthogonal Ensemble (GOE). This strongly suggests that full ergodicity, typical of quantum systems described by random matrix theory, is also a universal feature of quantum black holes.

from Garcia-Garcia, Verbaarschot RSYK8 (2017)

Models with low energy chaos

Classical color dynamics of homogeneous Yang-Mills fields:

$H = (p_x^2 + p_y^2 + p_z^2 + x^2y^2 + x^2z^2 + y^2z^2)/2$

Lyapunov exponent $\Lambda \approx 0.4H^{1/4}$ - Chirikov, DS JETP Lett. 34, 183 (1981)
(also Matinyan, Savvidi ZhETF 80, 830 (1981))

Quantum compacton vacuum (quantum Newton's cradle):

$H = \sum_l p_l^2/2 + \alpha(x_l - x_{l-1})^n/n$

for $n > 2$ classical dynamics is chaotic at $H \to 0$;
classical Newton's cradle $n = 2.5$;
quantum case $n = 4 \to$ phonon-like excitations above quantum vacuum
Zhirov, Pikovsky, DS PRE 83, 016202 (2011);
cold atoms experiment Kinoshita, Wenger, Weiss Nature 440, 900 (2006)

Discussion

SYK black hole:
interesting model without evident quasi-particles,
strongly interacting many-body system

Possible experiments:
quantum dots at $g \ll 1$ (Kvon et al. IFP RAS 1998);
ions in optical lattices (Vuletic MIT 2016)

Possible extentions to higher dimensions...

Isolated black holes:
no heat bath, only dynamical thermalization is possible