

# Algebra, Geometry and Physics: a conference in honour of Maxim Kontsevich

## programme

### Lundi 23 juin 2014

- 09:00 *Registration and Welcome coffee*  
09:25 *Welcome message from the IHES Director*  
09:30 **Yuri MANIN**  
Big Bang, Blow Up, and Modular Curves: Algebraic Geometry of Cyclic Cosmology"  
10:30 *Coffee break*  
10:45 **Yuri TSCHINKEL**  
On the arithmetic of K3 surfaces  
12:00 **Nikita NEKRASOV**  
Non-perturbative Dyson-Schwinger equations and novel symmetries  
of quantum field theory  
13:00 *Buffet lunch*  
14:30 **Mikhail KAPRANOV**  
Algebra of the infrared and secondary polytopes  
15:45 **Sergei GUKOV**  
Fivebranes and 4-manifolds  
16:45 *Coffee break*  
17:15 **Emanuel DIACONESCU**  
Donaldson-Thomas invariants and character varieties

### Mardi 24 juin 2014

- 09:00 *Welcome coffee*  
09:30 **Yakov ELIASHBERG**  
Limits of Symplectic Topology  
10:30 *Coffee break*  
10:45 **Kevin COSTELLO**  
A twisted form of the ADS/CFT correspondence  
12:00 **Graeme SEGAL**  
Wick rotation and the positivity of energy in quantum field theory  
13:00 *Buffet lunch*  
14:30 **Mohammed ABOUZAIID**  
Family Floer cohomology and mirror symmetry  
15:45 **Anton KAPUSTIN**  
Higher Symmetry, TQFT, and Gapped Phases of Matter  
16:45 *Coffee break*  
17:15 **Thomas WILLWACHER**  
Recent progress and open problems in graph cohomology

### Mercredi 25 juin 2014

- 09:00 *Welcome coffee*  
09:30 **Kenji FUKAYA**  
Can one use virtual fundamental chain in (topological) quantum field theory ?  
10:30 *Coffee break*  
10:45 **Paul SEIDEL**  
Lefschetz pencils and noncommutative geometry  
12:00 **Simon DONALDSON**  
The Ding functional, Berndtsson convexity and moment maps

- 13:00 *Buffet lunch*
- 14:30 **Claire VOISIN**  
Universal Chow group of 0-cycles and stable rationality
- 15:45 **Tom BRIDGELAND**  
Stability structures on Calabi-Yau algebras
- 16:45 *Coffee break*
- 17:15 **Alexander EFIMOV**  
Relative derived categories and matrix factorizations

## Jeudi 26 juin 2014

- 09:00 Welcome coffee
- 09:30 **Shing-Tung YAU**  
Non-Kähler Calabi-Yau Mirror Symmetry and Symplectic Structures
- 10:30 Coffee break
- 10:45 **Richard THOMAS**  
The Katz-Klemm-Vafa formula
- 12:00 **Carlos SIMPSON**  
An overview of the structure at infinity of representation spaces
- 13:00 *Buffet lunch*
- 14:30 **Alexander GONCHAROV**  
Quantum Hodge field theory
- 15:45 **Anton ZORICH**  
Lyapunov exponents of the Hodge bundle, volumes of moduli spaces,  
and diffusion in periodic billiards
- 16:45 **Coffee break**
- 17:15 **Denis AUROUX**  
Towards homological mirror symmetry for affine varieties
- 18:30 Remarks by **Eric Chaney** Chief economist at AXA
- 19:00 *Cocktail*

## Vendredi 27 juin 2014

- 09:00 *Welcome coffee*
- 09:30 Alain CONNES  
The Arithmetic Site
- 10:30 *Coffee break*
- 10:45 **Don ZAGIER**  
Partitions, quasimodular forms, and Siegel-Veech constants
- 12:00 **François LOESER**  
Motivic curve counting
- 13:00 *Buffet lunch*
- 14:30 **Bertrand TOËN**  
Deformation quantization and derived algebraic geometry
- 15:45 Yan SOIBELMAN  
Cohomological Hall algebra and its representations
- 16:45 *Coffee break*
- 17:15 **Tony PANTEV**  
Derived foliations and shifted potentials

# Algèbre, Géométrie et Physique : une conférence en l'honneur de Maxim Kontsevitch

## Résumés

**Mohammed ABOUZAIID** (Columbia Univ., New York USA)

### **Family Floer cohomology and mirror symmetry**

One can associate to a Lagrangian torus fibration on a symplectic manifold  $X$  a rigid analytic space  $Y$  whose points are the unitary local systems on the fibres. Assuming that there are no singular fibres, I will explain how family Floer cohomology gives rise to a functor which assigns to an (unobstructed) Lagrangian in  $X$  an object in a (twisted) derived category of  $Y$ , and that this functor is faithful.

**Denis AUROUX** (U.C. Berkeley, USA)

### **Towards homological mirror symmetry for affine varieties**

I will report on a program to construct Landau-Ginzburg mirrors to affine varieties from the perspective of the SYZ conjecture and prove homological mirror symmetry for these mirror pairs. The wrapped Fukaya categories that appear in the statement of homological mirror symmetry in this setting will be discussed, focusing especially on the simplest non-trivial example: the pair of pants.

Parts of this work are joint with M. Abouzaid, A. Efimov, L. Katzarkov and D. Orlov.

**Tom BRIDGELAND** (Univ. of Sheffield, UK)

### **Stability structures on Calabi-Yau algebras**

I will begin by briefly recalling the definition of the space of stability structures on a triangulated category before moving on to describe this space in the case of the  $n$ -Calabi-Yau derived category of the  $A_2$  quiver. In the second part of the talk I will recall the Kontsevich-Soibelman wall-crossing formula and speculate about the existence of Frobenius structures on stability spaces.

**Alain CONNES** (IHÉS-Collège de France, Paris, France)

### ***The Arithmetic Site***

We show that the non-commutative geometric approach to the Riemann zeta function has an algebraic geometric incarnation: the « Arithmetic Site ». This site involves the tropical semiring viewed as a sheaf on the topos dual to the multiplicative semigroup of positive integers. We realize the Frobenius correspondences in the square of the « Arithmetic Site ».

**Kevin COSTELLO** (Northwestern Univ., Evanston, USA)

### **A twisted form of the ADS/CFT correspondence**

**Emanuel DIACONESCU** (Univ. of Alberta, Canada)

***Donaldson-Thomas invariants and character varieties***

A relation between the motivic Donaldson-Thomas invariants of Kontsevich and Soibelman and the cohomology of character varieties will be shown to arise naturally in string theory. This yields in particular a string theoretic derivation of a conjecture of Hausel, Letellier and Rodriguez-Villegas and generalizations.

This is based on work with Wu-yen Chuang, Ron Donagi and Tony Pantev.

**Simon DONALDSON** (Simons Center for Geometry and Physics, Stony Brook, USA)

**The Ding functional, Berndtsson convexity and moment maps**

This will be mainly an expository talk. The Ding functional is an important notion in the study of Kahler-Einstein metrics and the Kahler-Ricci flow. Berndtsson showed that the Ding functional is convex in a certain sense, a result which has many important consequences. The main point of the talk will be to explain how the Ding functional and Berndtsson's result can be fitted into a general framework involving a moment map for the action of the group of Hamiltonian diffeomorphisms. As far as time allows, we will explain some general background on existence problems for Kahler-Einstein and constant scalar curvature metrics and "stability" in algebraic geometry.

**Alexander EFIMOV** (Steklov Mathematical Institute, RAS, Moscow, Russia)

**Relative derived categories and matrix factorizations**

For a morphism of schemes  $X \rightarrow Y$  of finite Tor-dimension we will define certain "relative derived categories", which are intermediate triangulated categories between the perfect derived category and derived category of coherent sheaves on  $X$ . They are closely related with categories of matrix factorizations on a singular scheme.

We will discuss the Thomason localization theorem for these categories. In particular we will see that localization fails for the naive version of the category of "perfect" matrix factorizations.

**Yakov ELIASHBERG** (Stanford Univ., USA)

**Limits of Symplectic Topology**

I will discuss several examples illustrating the following general principle: in symplectic topology everything is possible if it is not prohibited by the theory of holomorphic curves.

**Kenji FUKAYA** (Simons Center for Geometry and Physics, Stony Brook, USA)

**Can one use virtual fundamental chain in (topological) quantum field theory**

**Alexander GONCHAROV** (Yale Univ., New Haven, USA)

***Quantum Hodge field theory***

We show that periods of the rational homotopy type of a complex variety  $X$  can be described as correlators of a functional integral related to  $X$ , called Hodge correlators.

Moreover, the Hodge correlators describe a real mixed Hodge structure on the rational homotopy type.

We upgraded them to motivic correlators, lying in the motivic Lie coalgebra.

The simplest motivic correlators on the universal modular curve deliver the Beilinson-Kato Euler system.

**Sergei GUKOV** (Caltech, Pasadena, USA)  
*Fivebranes and 4-manifolds*

**Mikhail KAPRANOV** (Yale Univ., New Haven, USA)  
*Algebra of the infrared and secondary polytopes*

**Anton KAPUSTIN** (Caltech, Pasadena, USA)  
**Higher Symmetry, TQFT, and Gapped Phases of Matter**

I define and study Topological Quantum Field Theories (TQFTs) which describe the low-energy limit of gapped phases of gauge theories. These TQFTs generalize the Dijkgraaf-Witten TQFT and can be described on a lattice using discrete 1-form and 2-form gauge fields. The gauge group is replaced by a gauge 2-group (a 2-category with a single object and invertible 1-morphisms and 2-morphisms). It is proposed that 2-group TQFTs are associated with new types of symmetry-protected gapped phases of matter.

**François LOESER** (Univ. Pierre-et-Marie-Curie and IHÉS, France)  
*Motivic curve counting*

We shall explain how using a motivic Poisson formula due to Hrushovski and Kazhdan, it is possible to prove for equivariant compactifications of additive groups a geometric version of a conjecture of Manin on asymptotics of the number of points of bounded height. This is joint work with A. Chambert-Loir.

**Yuri MANIN** (MPIM Bonn, Germany)  
**Big Bang, Blow Up, and Modular Curves: Algebraic Geometry of Cyclic Cosmology**  
(joint with M. Marcolli)

We study some mathematical models of Penrose's idea to interpret the Big Bang as a sign of crossover from « the end of previous aeon" of the expanding and cooling Universe to the beginning of the next aeon ».

We suggest to model the kinematics of Big Bang as an algebraic geometric (or analytic) blow up of a point at the boundary of the next aeon space-time. Moreover, we argue that at the moment of crossover (infinite future of the previous aeon) time undergoes the Wick rotation and becomes purely imaginary on the crossover boundary.

The Mixmaster model of the early history of the next aeon is neatly included into this picture by postulating that the reverse Wick rotation follows a hyperbolic geodesic connecting imaginary time axis to the real one.

**Nikita NEKRASOV** (Simons Center for Geometry and Physics, Stony Brook, USA)  
**Non-perturbative Dyson-Schwinger equations and novel symmetries of quantum field theory**

**Tony PANTEV** (Univ. of Pennsylvania, Philadelphia, USA)  
*Derived foliations and shifted potentials*

I will discuss Lagrangian fibrations and foliations in shifted symplectic geometry and their relevance to the problem of global existence of potentials. I will describe constructions of isotropic foliations on moduli spaces and will discuss the associated potentials as well as some applications to quantum field theory and non-abelian Hodge theory. This is based on joint works with Calaque, Katzarkov, Toën, Vaquié, and Vezzosi.

**Graeme SEGAL** (Univ. of Oxford, UK)

**Wick rotation and the positivity of energy in quantum field theory**

**Paul SEIDEL** (MIT, Cambridge, USA)

**Lefschetz pencils and noncommutative geometry**

This talk concerns the symplectic geometry of Lefschetz pencils. Applying Floer cohomology results in a rich algebraic structure. We will explain a possible framework for understanding this structure, and its relation with mirror symmetry.

**Carlos SIMPSON** (CNRS-Univ. de Nice, France)

**An overview of the structure at infinity of representation spaces**

We give an overview of some features of the structure of moduli spaces of local systems, asymptotically near infinity.

This is motivated by the relationship with wallcrossing in the moduli spaces of stability conditions, investigated by Kontsevich and Soibelman.

In joint work with Katzarkov, Pandit and Noll, we look at harmonic maps to buildings, spectral networks, and the WKB approximation, structures which are attached to the asymptotics at infinity. We will look at how these things fit together and relate to the other rich structures of these spaces.

**Yan SOIBELMAN** (Kansas State Univ., Manhattan, USA)

**Cohomological Hall algebra and its representations**

**Richard THOMAS** (Imperial College London, UK)

***The Katz-Klemm-Vafa formula***

I will describe joint work with Rahul Pandharipande proving the Katz-Klemm-Vafa conjecture. This expresses the Gromov-Witten theory of K3 surfaces and K3-fibred 3-folds – in all genera and for all multiple covers – in terms of modular forms. We use Pandharipande-Pixton's proof of the MNOP conjecture for many 3-folds and a new result on stable pairs and Noether-Lefschetz loci to translate the problem into a computation of stable pair invariants. This is carried out using a degeneration formula and a vanishing result.

**Bertrand TOËN** (CNRS-Univ. de Montpellier 2, France)

***Deformation quantization and derived algebraic geometry***

In this talk I will present the recent developments concerning the interactions between deformation quantization and derived algebraic geometry. In a first part I'll briefly recall some elements of derived algebraic geometry, including the notions of shifted symplectic and Poisson structures. In the second part, I will focus on the existence of deformation quantization of shifted symplectic and Poisson structures. I will mainly present two approaches, a first one based on a generalization of Kontsevich's formality, and a second one inspired by Fedosov's quantization of symplectic manifolds.

**Yuri TSCHINKEL** (Courant Institute & Simons Foundation, New York, USA)

**On the arithmetic of K3 surfaces**

I will discuss recent developments in the study of K3 surfaces over non-closed fields (joint with B. Hassett)

**Claire VOISIN** (CNRS-École Polytechnique, Palaiseau, France)

***Universal Chow group of 0-cycles and stable rationality***

Summary: Much work has been done in the 70's to solve the Lüroth problem of distinguishing unirational from rational (or stably rational) varieties. For 3-dimensional unirational varieties, the only stable birational invariant used up to now has been the Artin-Mumford invariant. We show using the universal Chow group of zero-cycles that some unirational threefolds with trivial Artin-Mumford invariant are not stably rational.

Some of these examples have the following property: they do not admit a universal codimension 2 cycle, which prevents their stable rationality and exhibits a new phenomenon in the theory of algebraic cycles.

**Thomas WILLWACHER** (ETH Zürich, Switzerland)

**Recent progress and open problems in graph cohomology**

Graph complexes are some of the most mysterious objects in homological algebra.

Currently we do not even have conjectures describing the global structure of the graph cohomology.

I will report on some recent results constraining the cohomology of the "commutative" graph complexes.

**Shing-Tung YAU** (Chinese Univ. of Hong Kong, China and Harvard Univ., USA)

***Non-Kähler Calabi-Yau Mirror Symmetry and Symplectic Structures***

**Don ZAGIER** (Collège de France, Paris and MPIM Bonn, Germany)

**Partitions, quasimodular forms, and Siegel-Veech constants**

A beautiful theorem of Bloch and Okounkov, generalizing an earlier result of Kaneko and myself, says that the suitably defined generating functions of a large class of functions of partitions are quasimodular forms (i.e., polynomials in the classical Eisenstein series  $E_2$ ,  $E_4$  and  $E_6$ ). We will explain the statement and an extremely short new proof of this theorem, and also describe joint work with Martin Möller giving applications to the computations of Teichmüller volumes and Siegel-Veech constants in the theory of flat surfaces.

**Anton ZORICH** (Univ. Paris-Diderot, France)

**Lyapunov exponents of the Hodge bundle, volumes of moduli spaces, and diffusion in periodic billiards**

I will try to describe how dynamics of certain natural flows on surfaces is governed by dynamics of the Teichmüller flow on the moduli space and by geometry of the Hodge bundle. In particular, I will try to present recent results of J. Athreya, A. Eskin, M. Mirzakhani, M. Kontsevich, and myself making the relation between the two problems rather explicit.

As a model case I will consider billiards in the plane with periodic polygonal obstacles (developing ideas of V. Delecroix, P. Hubert, and S. Lelièvre).