

Number theory days



Report of Contributions

Contribution ID: 0

Type: **not specified**

Generalised perfectoid rings and perfectoid spaces

Thursday, June 26, 2014 3:30 PM (1 hour)

Scholze's theory of perfectoid rings and perfectoid spaces is rather recent, but it has already had some spectacular applications to étale cohomology, p-adic Hodge theory and p-adic representations. I will present a generalization of this theory that I am developing in collaboration with Ofer Gabber. I will also explain the questions that have led us to this generalization.

Presenter: RAMERO, Lorenzo (Université Lille 1)

Session Classification: Arithmetic geometry and Galois theory

Track Classification: Arithmetic geometry and Galois theory

Contribution ID: 1

Type: **not specified**

The arithmetic of tori over various fields

Thursday, June 26, 2014 11:45 AM (1 hour)

Let T be an algebraic torus defined over a number field K . In the case of a number field, obstructions to local-global principles for T are well understood thanks to work by Voskresenskii and Sansuc. We consider the case $K=k(t)$ for different fields k (quasi-finite, p -adic) and extend the classical results in this context.

Presenter: HARARI, David (Université Paris-Sud)

Session Classification: Arithmetic geometry and Galois theory

Contribution ID: 2

Type: **not specified**

Invariant d'Arason et complexe de Peyre

Friday, June 27, 2014 11:45 AM (1 hour)

La dimension, le discriminant, et l'invariant de Clifford sont des invariants classiques des formes quadratiques, qui s'étendent au contexte plus général des algèbres centrales simples à involution orthogonale. Sous certaines conditions, on peut aussi définir un invariant d'Arason; mais contrairement à ce qui se passe pour les formes quadratiques, celui-ci n'est pas toujours représenté par une classe de cohomologie tuée par 2. Dans un article commun avec Anne Quéguiner-Mathieu, on étudie cet invariant pour les algèbres de degré 12 et d'indice 2. Dans ce contexte, il apparaît étroitement lié à l'homologie d'un complexe de cohomologie galoisienne introduit et étudié par Emmanuel Peyre. Ce point de vue permet, notamment, de donner de nouveaux exemples dans lesquels l'homologie de ce complexe est non triviale, et qui sont d'une nature un peu différente des exemples décrits dans l'article de Peyre.

Presenter: TIGNOL, Jean-Pierre (Université catholique de Louvain)

Session Classification: Quadratic forms

Track Classification: Quadratic forms

Contribution ID: 3

Type: **not specified**

Faisceaux à réciprocity

Friday, June 27, 2014 10:30 AM (1 hour)

On définit une notion de réciprocity sur les préfaisceaux avec transferts (PST) de Voevodsky. Pour cela, on enrichit les groupes de 0-cycles avec module de Kerz-Saito en leur conférant une structure de PST. Les PST invariants par homotopie sont à réciprocity, ainsi que ceux représentables par un groupe algébrique commutatif : ce dernier point généralise un théorème classique de Rosenlicht qui est à l'origine de ce travail. On généralise aux PST à réciprocity un certain nombre des propriétés démontrées par Voevodsky pour les PST invariants par homotopie : préservation de cette propriété par faisceautisation Zariski ou Nisnevich, injectivité semi-locale.

Il s'agit d'un travail en collaboration avec Shuji Saito et Takao Yamazaki.

Presenter: KAHN, Bruno (Institut de Mathématiques de Jussieu)

Session Classification: Quadratic forms

Track Classification: Quadratic forms

Contribution ID: 4

Type: **not specified**

Classification of Torsors and Subtle Stiefel-Whitney classes

Friday, June 27, 2014 9:00 AM (1 hour)

This is a joint work with Alexander Smirnov. I will describe a new homotopic approach to the classification of torsors of algebraic Groups. It extends the approach of Morel-Voevodsky, where torsors are interpreted as Hom's to the classifying space of the group in the A^1 -homotopy category of Morel-Voevodsky. In the case of the orthogonal group $O(n)$, we introduce new invariants: "Subtle Stiefel-Whitney classes" which are much more informative than the classical ones (defined by J.Milnor). These invariants distinguish the triviality of the torsor (quadratic form), see powers I^n of the fundamental ideal, contain Arason and higher invariants, and are related to the J-invariant of quadrics (thus, connecting previously isolated areas). These classes are also essential for the motivic description of some natural varieties related to a quadratic form.

Presenter: VISHIK, Alexander (University of Nottingham)

Session Classification: Quadratic forms

Track Classification: Quadratic forms

Contribution ID: 5

Type: **not specified**

On the modularity of reducible mod l Galois representations

Tuesday, June 24, 2014 11:45 AM (1 hour)

In this talk, I'll give a modularity result for reducible mod l Galois representations. By analogy with the irreducible case, I'll state some questions regarding characterization and optimization of the different types of modular forms attached to such a given representation. Finally, I'll give an application of these results to the determination of an explicit lower bound for the highest degree of the coefficient fields of newforms of prime level and trivial Nebentypus. This is a joint work with Ricardo Menares.

Presenter: BILLEREY, Nicolas (Université Clermont-Ferrand 2)

Session Classification: Galois representations and modular forms

Track Classification: Galois representations and modular forms

Contribution ID: 6

Type: **not specified**

Rings and images occurring from universal deformations of profinite groups

Tuesday, June 24, 2014 9:00 AM (1 hour)

Recently Dorobisz, Eardley-Manoharmayum and Manoharmayum have proved abstract results (a) on the shape of possible deformation rings and (b) on the image of universal deformations of profinite groups, for representations into GL_n . The result regarding (a) were motivated by questions of Bleher, Chinburg and de Smit. We place these results in an axiomatic framework that in principle applies to all reductive groups that are simple modulo their center. In this framework, we give alternative proofs of the above results. For the case GL_n we recover the results by Dorobisz, Eardley and Manoharmayum. For other cases, it remains to verify the stated axioms for these groups, for instance results on the first and second cohomology for the adjoint action of these groups. This is work in progress jointly with Sara Arias de Reyna.

Presenter: BOECKLE, Gebhard (Heidelberg University)

Session Classification: Galois representations and modular forms

Track Classification: Galois representations and modular forms

Contribution ID: 7

Type: **not specified**

The future of modularity

Tuesday, June 24, 2014 3:30 PM (1 hour)

This is joint work with Ariel Pacetti. We present generalizations to totally real number fields of the construction done by the speaker some years ago over \mathbb{Q} that allows to connect to each other any given pair of newforms through chains of modular compatible systems of Galois representations. We also discuss applications of this, and we consider the case of abstract Galois representations and we explain what we can do there, and what we conjecture, and finally (time permitting) how this conjecture should be attacked. This provides a new path to attack Langlands functoriality and modularity conjectures.

Presenter: DIEULEFAIT, Luis (Universitat de Barcelona)

Session Classification: Galois representations and modular forms

Track Classification: Galois representations and modular forms

Contribution ID: 8

Type: **not specified**

Automorphic Galois representations in the inverse Galois problem

Tuesday, June 24, 2014 5:00 PM (1 hour)

In the talk I will report on recent results on the inverse Galois problem based on compatible systems of Galois representations coming from modular and automorphic forms. The focus will be on ideas and strategies as well as the obstacles that are preventing us from proving much stronger theorems. In this context, the role of coefficient fields will be particularly highlighted. Most parts are joint work with Sara Arias-de-Reyna, Luis Dieulefait and Sug-Woo Shin.

Presenter: WIESE, Gabor (Université du Luxembourg)

Session Classification: Galois representations and modular forms

Track Classification: Galois representations and modular forms

Contribution ID: 9

Type: **not specified**

Some particular direct-sum decompositions and direct-product decompositions

Wednesday, June 25, 2014 10:30 AM (1 hour)

We will describe direct-sum decompositions and direct-product decompositions for some classes of modules. We will be mainly interested in direct sums and direct products of modules whose endomorphism rings have at most two maximal ideals.

Presenter: FACCHINI, Alberto (University of Padova)

Session Classification: Noncommutative algebra

Track Classification: Noncommutative algebra

Contribution ID: 10

Type: **not specified**

Almost involutive Hopf algebras: are there additional symmetries in Hopf algebras besides the antipode?

Wednesday, June 25, 2014 9:00 AM (1 hour)

An involutory Hopf algebra is a Hopf algebra whose antipode squared equals the identity, $S^2 = \text{id}$.

The identity map is an automorphism of Hopf algebras, hence it is tempting to substitute $\text{id} \mapsto \sigma$ where σ is an arbitrary Hopf morphism and consider Hopf algebras whose antipode (that is an antimorphism of Hopf algebras) squared is the square of a Hopf automorphism,

$\text{ant}^2 = \sigma^2$. A map such as σ if it exists, is called a companion morphism.

If

ant has finite order, so does σ . A morphism of a given mathematical structure that is of finite order may be interpreted as a symmetry of the structure.

Hence, the companion morphism can be interpreted as an additional symmetry of the structure of H . If the Hopf algebra H admits a companion morphism, we say that it is almost involutory (AI).

The purpose of this talk, is to define and consider the initial properties of almost involutory Hopf algebras. We prove that up to dimension 15 all Hopf algebras except a few types in dimensions eight and twelve are AI.

Presenter: FERRER-SANTOS, Walter (Universidad de la Republica, Montevideo)

Session Classification: Noncommutative algebra

Track Classification: Noncommutative algebra

Contribution ID: 11

Type: **not specified**

Phénomènes de seuil pour les suites de pseudo puissances

Monday, June 23, 2014 5:15 PM (1 hour)

Nous étudions des phénomènes de seuil en théorie additive des nombres. L'objet central est les pseudo puissances s -ièmes introduites par Erdos et Renyi en 1960. In 1975, Goguel a montré que, presque sûrement, une telle suite n'était pas une base asymptotique d'ordre s . On verra qu'elle est presque sûrement base d'ordre $s+\epsilon$. On étudie aussi la taille du plus petit complément additif de sA , c'est à dire du plus petit B tel que $sA+B$ contienne tout entier assez grand. Nous caractérisons précisément la taille seuil d'un tel complément.

Presenter: PLAGNE, Alain (École polytechnique)

Session Classification: Analytic-Additive Number Theory

Track Classification: Analytic-Additive Number Theory

Contribution ID: 13

Type: **not specified**

Generalised Fourier coefficients of multiplicative functions

Monday, June 23, 2014 11:30 AM (1 hour)

The aim of this talk is to explain a strategy that allows us to bound the Fourier coefficients of a large class of not necessarily bounded multiplicative functions. The interest in this result lies in the fact that the strategy can be adapted to show that these multiplicative functions give rise to functions that are orthogonal to linear nilsequences when applying a ‘W-trick’. This, in turn, provides one of two necessary steps for an application of the Green–Tao methods, which can be employed to asymptotically evaluate linear correlations of these multiplicative functions. Such correlations appear naturally in many arithmetic problems.

Presenter: MATTHIESEN, Lilian (Institut de Mathématiques de Jussieu)

Session Classification: Analytic-Additive Number Theory

Track Classification: Analytic-Additive Number Theory

Contribution ID: 14

Type: **not specified**

Skew Generalized Quasi-Cyclic Codes

Wednesday, June 25, 2014 11:45 AM (1 hour)

In this article we introduce skew generalized quasi-cyclic codes over finite field F with Galois automorphism θ . This is a generalization of quasi-cyclic codes and skew polynomial codes. These codes have an added advantage over quasi-cyclic codes, since the length of the code C need not be a multiple of the index of C . After a brief description of the skew polynomial ring $F[x; \theta]$, it is shown that a skew generalized quasi-cyclic code C is a left submodule of $R_1 \times R_2 \times \cdots \times R_l$, where $R_i = F[x; \theta]/(x^{m_i} - 1)$, $|\langle \theta \rangle| = m$ and $m | m_i$ for all $i = 1, \dots, l$. This method provides a direct construction of many codes with best known parameters over $GF(4)$.

Joint work with T. Abualrub, P. Seneviratne

Presenter: SOLÉ, Patrick (Telecom ParisTech)

Session Classification: Noncommutative algebra

Track Classification: Noncommutative algebra

Contribution ID: 15

Type: **not specified**

Some Problems in Analytic Number Theory for Polynomials over Finite Fields

Monday, June 23, 2014 3:45 PM (1 hour)

In this talk I will explore some traditional problems of analytic number theory in the context of function fields over a finite field. Several such problems which are currently viewed as intractable can, in the function field scenario, be attacked with vastly different tools than those of traditional analytic number theory. The resulting theorems in the function field setting can be used to check existing conjectures in the classical case, and to generate new ones. The problems I will discuss include: the twin prime conjecture, the additive divisor problem, moments of L-functions and connections with random matrix theory.

Presenter: ANDRADE, Julio (IHES)

Session Classification: Analytic-Additive Number Theory

Track Classification: Analytic-Additive Number Theory

Contribution ID: 16

Type: **not specified**

On the history of analytic number theory

Monday, June 23, 2014 2:30 PM (1 hour)

Contrarily to other parts of number theory, the history of analytic number theory often appears as a collection of particular, even isolated, episodes, focussing on Euler or Riemann or Hadamard and de La Vallée-Poussin. The talk will discuss some of these gems, as well as less well-known ones, and comment on the discontinuous character of their history.

Presenter: GOLDSTEIN, Catherine (Institut de mathématiques de Jussieu)

Session Classification: Analytic-Additive Number Theory

Track Classification: Analytic-Additive Number Theory

Contribution ID: 17

Type: **not specified**

Points rationnels et zéro-cycles dans les fibrations

Thursday, June 26, 2014 9:00 AM (1 hour)

Si X est une variété projective et lisse définie sur un corps de nombres, la “méthode des fibrations” pour étudier l’ensemble des points rationnels de X ou le groupe de Chow des zéro-cycles de X vise à ramener les questions que l’on pose pour X (par exemple: existence d’un point ou d’un zéro-cycle de degré 1) aux mêmes questions pour les fibres d’un morphisme dominant $f: X \rightarrow \mathbb{P}^1$. Le but de l’exposé est un théorème général dans cette direction lorsque la fibre générique de f est rationnellement connexe, sans hypothèse sur les fibres singulières (travail en commun avec Y. Harpaz).

Presenter: WITTENBERG, Olivier (École normale supérieure)

Session Classification: Arithmetic geometry and Galois theory

Track Classification: Arithmetic geometry and Galois theory

Contribution ID: 18

Type: **not specified**

Oort groups and lifting problems

Thursday, June 26, 2014 10:30 AM (1 hour)

The Oort conjecture states that every cyclic branched cover of curves in characteristic p can be lifted to such a cover in characteristic zero. This raises the more general question of which finite groups G have the property that every G -Galois branched cover of curves in characteristic p can be lifted to characteristic zero. While this can be viewed as analogous to the inverse Galois problem, the situation here turns out to be very different, and the finite groups that have the lifting property are quite constrained. This talk will discuss the Oort conjecture and its generalizations, presenting in particular recent developments.

Presenter: HARBATER, David (University of Pennsylvania)

Session Classification: Arithmetic geometry and Galois theory

Track Classification: Arithmetic geometry and Galois theory

Contribution ID: 20

Type: **not specified**

Equivariant L-values of modular abelian varieties

Tuesday, June 24, 2014 10:30 AM (1 hour)

An abelian variety defined over a number field is called strongly modular when its L-function is the product of L-functions of modular forms of weight 2. In this talk, we will show a weak version of Beilinson's conjectures for non-critical L-values of strongly modular abelian varieties. We will explain the interest of formulating an equivariant version of these conjectures (after Burns and Flach), as well as the main ingredients of the proof: a Hecke-equivariant version of Beilinson's theorem on modular curves, and a modularity result for endomorphism algebras. As an application, we deduce a weak version of Zagier's conjecture on $L(E,2)$ when E is a \mathbb{Q} -curve without complex multiplication which is completely defined over a quadratic field.

Presenter: BRUNAUT, François (École normale supérieure de Lyon)

Session Classification: Galois representations and modular forms

Track Classification: Galois representations and modular forms

Contribution ID: 21

Type: **not specified**

Sign changes of Fourier coefficients of cusp forms

Monday, June 23, 2014 10:00 AM (1 hour)

We will give a survey on recent results about sign changes of Fourier coefficients of cusp forms in one and several variables.

Presenter: KOHNEN, Winfried (Universität Heidelberg)

Session Classification: Analytic-Additive Number Theory

Track Classification: Analytic-Additive Number Theory

Contribution ID: 22

Type: **not specified**

Recent Progress in Bogomolov's Program: A Survey

Thursday, June 26, 2014 5:00 PM (1 hour)

Given a field K , finitely generated and of transcendence degree 2 over the algebraic closure of a prime field, we may now reconstruct K from the maximal 2-step nilpotent pro- ℓ quotient of its absolute Galois group. This allows us to construct a complete (albeit countably infinite) set of geometric obstructions for an element of the Grothendieck-Teichmüller group to come from an element of the absolute Galois group of \mathbb{Q} .

Presenter: SILBERSTEIN, Aaron (University of Pennsylvania)

Session Classification: Arithmetic geometry and Galois theory

Track Classification: Arithmetic geometry and Galois theory