SMOOTH FOLIATIONS ON COMPACT HOMOGENEOUS KÄHLER VARIETIES

FEDERICO LO BIANCO

Codimension 1 (possibly singular) foliations on complex tori have been classified in a work by Brunella, whereas Ghys studied codimension 1 smooth foliations on homogeneous varieties, and managed to give a complete classification in the Kähler case.
Pereira and I tried to exploit the techniques used by Brunella to generalise Ghys's results to smooth foliation of arbitrary codimension on homogeneous Kähler varieties. In particular the key lemma for our proof is that, for any such foliated variety $(X, \mathcal{F})$, there exist a projection $\pi\colon X \to Y$ onto a homogeneous Kähler variety and an ample line bundle $\mathcal{L}$ on Y such that $\pi^*\mathcal{L} = \det(N\mathcal{F})$ (where $N\mathcal{F}$ denotes the normal bundle of $\mathcal{F}$).
Here are our main results.

Theorem A. *Let $\mathcal{F}$ be a smooth foliation on a homogeneous compact Kähler manifold X. Then there exists*

(1) *a locally trivial fibration $\psi : X \to X'$ onto a homogeneous compact Kähler manifold X';*
(2) *a locally trivial fibration $\pi : X' \to Y$ onto a homogeneous compact Kähler manifold with tori fibers; and*
(3) *a foliation $\mathcal{F}'$ on X'*

such that

(1) $\mathcal{F} = \psi^*\mathcal{F}'$;
(2) $\pi_* T\mathcal{F}'$ *is a locally free sheaf of rank* $\dim \mathcal{F}'$; *and*
(3) $\det \pi_* N\mathcal{F}'$ *is ample.*

Corollary B. *Smooth foliations on rational homogeneous varieties are locally trivial fibrations.*

Theorem C. *Let $\mathcal{F}$ be a smooth foliation on a homogeneous compact Kähler manifold X. If every leaf of $\mathcal{F}$ is analitically dense (i.e. not conatined in any proper subvariety of X) then there exist a locally trivial fibration $\pi : X \to Y$ with rational fibers onto a complex torus Y and a linear foliation $\mathcal{G}$ on Y such that $\mathcal{F} = \pi^*\mathcal{G}$.*

Date: September 23, 2014.