**Title:**

"Germs of singular holomorphic two dimensional foliations"

**Abstract:** A holomorphic singular foliation $\mathcal{F}$ of codimension $q$ on a polydisc $P$ of $\mathbb{C}^n$ (where $n \geq 2$ and $0 < q < n$), with singular set $sing(\mathcal{F})$ of codimension $\geq 2$, can always be defined by a holomorphic integrable $q$-form on $P$, say $\eta$, with the property that for any $z \in P \setminus sing(\mathcal{F})$ we have $\eta(z) \neq 0$ and

$$T_z\mathcal{F} = \{v \in T_zP \,|\, i_v\eta(z) = 0\} \ , \tag{1}$$

where $T_z$ denotes the tangent space at $z$ and $i$ the interior product. In particular, a two dimensional foliation $\mathcal{F}$ on $P$, can be defined by a $(n-2)$-form $\eta$ satisfying (1). When $d\eta \not\equiv 0$ we can define a 1-dimensional singular foliation on $P$ by the vector field $X$ given by

$$d\eta = i_X\, dz_1 \wedge ... \wedge dz_n \ . \tag{2}$$

The integrability condition implies that $i_X\eta = 0$. When $cod_{\mathbb{C}}(sing(X)) \geq 3$ then the division theorem implies that there exists another holomorphic vector field $Y$ on $P$ such that

$$\eta = i_Y\, i_X\, dz_1 \wedge ... \wedge dz_n \tag{3}$$

and $\mathcal{F}$ is defined by the involutive system $\langle X, Y\rangle$. The situation that we consider in our main results is when $X$ has an isolated singularity at $0 \in P \subset \mathbb{C}^n$. In this case, necessarily $Y(0) = 0$. We have essentially two results:

**Theorem 1.** *Suppose that $DX(0)$ is semi-simple with eigenvalues $\lambda_1, ..., \lambda_n$ such that $\lambda_j \neq 0$, $\forall\, 1 \leq j \leq n$. Assume also that there exists $\tau \in \mathbb{C}$ such that the linear part of $Z := Y + \tau.X$, $DZ(0)$, has eigenvalues $\mu_1, ..., \mu_n$ satisfying Brjuno's condition of small denominators and also $\lambda_i.\,\mu_j - \lambda_j.\,\mu_i \neq 0$ for all $1 \leq i < j \leq n$. Then $\mathcal{F}$ can be defined in some neighborhood of $0 \in \mathbb{C}^n$ by a local action of $\mathbb{C}^2$ generated by two vector fields $S$ and $T$, which in some local holomorphic coordinate system around $0$, say $w = (w_1, ..., w_n)$, are $S = \sum_j \lambda_j\, w_j\, \partial_{w_j}$ and $T = \sum_j \mu_j\, w_j\, \partial_{w_j}$.*

**Theorem 2.** *Assume that $0 \in \mathbb{C}^n$ is an isolated singularity of $X$ and that $DX(0)$ is nilpotent. Then there exists a coordinate system around $0$, say $w = (w_1, ..., w_n)$, where $\eta$ is polynomial. More precisely, in the coordinate system $w$ we can write $\eta = i_L\, i_{\tilde{X}}\, dw_1 \wedge ... \wedge dw_n$, where $L$ is linear with eigenvalues $k_1, ..., k_n \in \mathbb{N}$ and $\tilde{X}$ satisfies*

$$[L, \tilde{X}] = \ell.\,\tilde{X} \ ,$$

*where $\ell \in \mathbb{N}$. In particular, $\mathcal{F}$ can be defined by a local action of the affine group in a soma neighborhood of $0 \in \mathbb{C}^n$.*

**Remark 1.** Theorem 2 in the case $n = 3$ was proved originally in [LN]. We would like to observe that in [LN] we prove that the linear vector field $L$ is necessarily semi-simple. However, for $n \geq 4$ we could not prove this fact, although we think it is true in general. In fact, we have proven that under a non-resonant condition that depends only on $X$ and is generic then $L$ is semi-simple.

If I have time, I will give an application of theorem 2 to the theory of irreducible components of two dimensional foliations on $\mathbb{P}^n$, $n \geq 4$.