Numerical approximation of doubly reflected BSDEs with jumps and RCLL obstacles

Céline Labart (Université de Savoie)
Joint work with Roxana Dumitrescu

Second Young researchers meeting on BSDEs, Numerics and Finance
Bordeaux, July 9th 2014

Plan

Reflected Backward Stochastic Differential Equations with jumps

We consider the following equation

$$(\mathcal{E})\begin{cases} \text{(i) } Y_t = \xi_T + \int_t^T g(s, Y_s, Z_s, U_s)ds + (A_T - A_t) - (K_T - K_t) - \int_t^T Z_s dW_s - \int_t^T U_s d\tilde{N}_s, \\ \text{(ii) } \forall t \in [0,T], \xi_t \leq Y_t \leq \zeta_t \text{ a.s.}, \\ \text{(iii) } \int_0^T (Y_{t^-} - \xi_{t^-}) dA_t = 0 \text{ a.s. and } \int_0^T (\zeta_{t^-} - Y_{t^-}) dK_t = 0 \text{ a.s.} \end{cases}$$

where

- W is a Brownian motion and $\tilde{N}_t := N_t - \lambda t$ is a compensated Poisson process. W and $\tilde{N}$ are independent,
- ξ and ζ are RCLL processes, with predictable and inaccessible jumps, such that $\xi_t \leq \zeta_t$, $\xi_T = \zeta_T$

We assume

- g is Lipschitz in space and uniformly continuous in time
- g is such that $g(t,y_t,z_t,u_t^1) - g(t,y_t,z_t,u_t^2) \geq \gamma_t(u_t^1 - u_t^2)$ where $-1 \leq \gamma_t \leq C$
- Mokobodski's condition : ξ and ζ are in $\mathscr{S}^2$ and there exist two nonnegative RCLL supermartingales H and H' in $\mathscr{S}^2$ such that $\forall t \in [0,T],\ \xi_t \mathbf{1}_{t<T} \leq H_t - H'_t \leq \zeta_t \mathbf{1}_{t<T}$ a.s.

Existence and Uniqueness (Dumitrescu, Quenez, Sulem 2014)

$(\mathscr{E})$ admits a unique solution (Y,Z,U,α) in $\mathscr{S}^2 \times \mathbb{H}^2 \times \mathbb{H}^2 \times \mathscr{S}^2$, where $\alpha := A - K$ with A and K in $\mathscr{A}^2$.

$\mathscr{S}^2$: real-valued RCLL adapted processes ϕ s.t. $\mathbb{E}(\sup_{0\leq t\leq T}|\phi_t|^2) < \infty$
$\mathbb{H}^2$: predictable processes ϕ such that $\mathbb{E}\left[\int_0^T \phi_t^2 dt\right] < \infty$.
$\mathscr{A}^2$: real-valued non decreasing RCLL predictable processes A with $A_0 = 0$ and $\mathbb{E}(A_T^2) < \infty$.

Other numerical methods

In case of BSDE with jumps, we refer to

- Dynamic programming equation (Bouchard and Elie, 2007)
- random tree method (Lejay, Mordecki and Torres 2013)

In case of reflected BSDEs without jumps, we refer to

- discrete reflected version of the RBSDE and DPE with projection (Chassagneux, 2009)
- random tree method (Xu, 2011)

Idea of the algorithm

- We approximate the Brownian motion and the Poisson process by two independent random walks
- We introduce a sequence of penalized BSDEs to approximate the reflected BSDE.

Random walk approximation of $(W, \tilde{N})$

- Discretization of $[0, T]$ on a regular grid with step size $\delta := \frac{T}{n}$
- Approximation of W by random walk

$$\begin{cases} W_0^n = 0 \\ W_t^n = \sqrt{\delta} \sum_{i=1}^{[t/\delta]} e_i^n \end{cases}$$

where $(e_i^n)_{1 \le i \le n}$ are i.i.d. variables s.t. $P(e_1^n = 1) = P(e_1^n = -1) = \frac{1}{2}$.

- Approximation of $\tilde{N}$ by a second random walk

$$\begin{cases} \tilde{N}_0^n = 0 \\ \tilde{N}_t^n = \sum_{i=1}^{[t/\delta]} \eta_i^n \end{cases}$$

where $(\eta_i^n)_{1 \le i \le n}$ are i.i.d. variables s.t. $P(\eta_1^n = \kappa_n - 1) = 1 - P(\eta_1^n = k_n) = \kappa_n$ where $\kappa_n = e^{-\frac{\lambda}{n}}$.

$(e_i^n)_{1\leq i\leq n}$ and $(\eta_i^n)_{1\leq i\leq n}$ are defined on the original probability space $(\Omega, \mathbb{F}, \mathbb{P})$.

Lejay, Mordecki, Torres 2013

$(W^n, \widetilde{N}^n)$ converges in probability to $(W, \widetilde{N})$ for the J_1-Skorokhod topology.

We say that ϕ^n converges to ϕ in probability for the J_1-Skorokhod topology if there exists a family $(\psi^n)_{n\in\mathbb{N}}$ of one-to-one random time changes from $[0,T]$ to $[0,T]$ such that $\sup_{t\in[0,T]}|\psi^n(t)-t| \xrightarrow[n\to\infty]{} 0$ a.s. and $\sup_{t\in[0,T]}|\phi^n_{\psi^n(t)}-\phi_t| \xrightarrow[n\to\infty]{} 0$ in probability.

Martingale representation

- Let $\mathscr{F}_j^n = \sigma\{e_1^n, \dots, e_j^n, \eta_1^n, \dots, \eta_j^n\}$ and y_{j+1} $\mathscr{F}_{j+1}^n$-measurable.
- We need a set of three strongly orthogonal martingales to represent the martingale difference $m_{j+1} := y_{j+1} - \mathbb{E}(y_{j+1}|\mathscr{F}_j^n)$

There exists a unique triplet (z_j, u_j, v_j) of $\mathscr{F}_j^n$-random variables such that

$$m_{j+1} := y_{j+1} - \mathbb{E}(y_{j+1}|\mathscr{F}_j^n) = \sqrt{\delta} z_j e_{j+1}^n + u_j \eta_{j+1}^n + v_j \mu_{j+1}^n,$$

where $\mu_{j+1}^n = e_{j+1}^n \eta_{j+1}^n$ and

$$\begin{cases} z_j = \frac{1}{\sqrt{\delta}} \mathbb{E}(y_{j+1} e_{j+1}^n | \mathscr{F}_j^n), \\ u_j = \frac{1}{\kappa_n(1-\kappa_n)} \mathbb{E}(y_{j+1} \eta_{j+1}^n | \mathscr{F}_j^n), \\ v_j = \frac{1}{\kappa_n(1-\kappa_n)} \mathbb{E}(y_{j+1} \mu_{j+1}^n | \mathscr{F}_j^n) \end{cases}$$

Computation of conditional expectations

Let Φ denote a function from $\mathbb{R}^{2j+2}$ to $\mathbb{R}$. We use the following formula to compute the conditional expectations

$$\begin{aligned}\mathbb{E}(\Phi(e_1^n,\cdots,e_{j+1}^n,\eta_1^n,\cdots,\eta_{j+1}^n)|\mathscr{F}_j^n) = &\frac{\kappa_n}{2}\Phi(e_1^n,\cdots,e_j^n,1,\eta_1^n,\cdots,\eta_j^n,\kappa_n-1)\\ &+\frac{\kappa_n}{2}\Phi(e_1^n,\cdots,e_j^n,-1,\eta_1^n,\cdots,\eta_j^n,\kappa_n-1)\\ &+\frac{1-\kappa_n}{2}\Phi(e_1^n,\cdots,e_j^n,1,\eta_1^n,\cdots,\eta_j^n,\kappa_n)\\ &+\frac{1-\kappa_n}{2}\Phi(e_1^n,\cdots,e_j^n,-1,\eta_1^n,\cdots,\eta_j^n,\kappa_n).\end{aligned}$$

Sequence of penalized BSDEs

$$(\mathcal{E})\begin{cases} \text{(i) } Y_t = \xi_T + \int_t^T g(s, Y_s, Z_s, U_s)ds + (A_T - A_t) - (K_T - K_t) - \int_t^T Z_s dW_s - \int_t^T U_s d\tilde{N}_s, \\ \text{(ii) } \forall t \in [0,T], \xi_t \leq Y_t \leq \zeta_t \text{ a.s.}, \\ \text{(iii) } \int_0^T (Y_{t^-} - \xi_{t^-}) dA_t = 0 \text{ a.s. and } \int_0^T (\zeta_{t^-} - Y_{t^-}) dK_t = 0 \text{ a.s.} \end{cases}$$

We approximate $(\mathcal{E})$ by a sequence of penalized BSDEs

$$Y_t^p = \xi + \int_t^T g(s, Y_s^p, Z_s^p, U_s^p) ds + A_T^p - A_t^p - (K_T^p - K_t^p) - \int_t^T Z_s^p dW_s - \int_t^T U_s^p d\tilde{N}_s,$$

with $A_t^p := p\int_0^t (Y_s^p - \xi_s)^- ds$ and $K_t^p := p\int_0^t (\zeta_s - Y_s^p)^- ds$, and $\alpha_t^p := A_t^p - K_t^p$ for all $t \in [0,T]$.

Numerical scheme

$$Y_t^p = \xi + \int_t^T g(s, Y_s^p, Z_s^p, U_s^p) ds + A_T^p - A_t^p - (K_T^p - K_t^p) - \int_t^T Z_s^p dW_s - \int_t^T U_s^p d\widetilde{N}_s,$$

with $A_t^p := p\int_0^t (Y_s^p - \xi_s)^- ds$ and $K_t^p := p\int_0^t (\zeta_s - Y_s^p)^- ds$
We integrate the previous equation between t_j and t_{j+1}

$$Y_{t_j}^p = Y_{t_{j+1}}^p + \int_{t_j}^{t_{j+1}} g(s, Y_s^p, Z_s^p, U_s^p) ds + A_{t_{j+1}}^p - A_{t_j}^p - (K_{t_{j+1}}^p - K_{t_j}^p)$$
$$- \int_{t_j}^{t_{j+1}} Z_s^p dW_s - \int_{]t_j, t_{j+1}]} U_s^p d\widetilde{N}_s,$$

$$A_{t_{j+1}}^p - A_{t_j}^p := p\int_{t_j}^{t_{j+1}} (Y_s^p - \xi_s)^- ds,\ K_{t_{j+1}}^p - K_{t_j}^p := p\int_{t_j}^{t_{j+1}} (\zeta_s - Y_s^p)^- ds$$

Numerical scheme

$$Y_t^p = \xi + \int_t^T g(s, Y_s^p, Z_s^p, U_s^p) ds + A_T^p - A_t^p - (K_T^p - K_t^p) - \int_t^T Z_s^p dW_s - \int_t^T U_s^p d\tilde{N}_s,$$

with $A_t^p := p \int_0^t (Y_s^p - \xi_s)^- ds$ and $K_t^p := p \int_0^t (\zeta_s - Y_s^p)^- ds$

We integrate the previous equation between t_j and t_{j+1}

$$Y_{t_j}^p \sim Y_{t_{j+1}}^p + \delta g(t_j, \mathbb{E}(Y_{t_{j+1}}^p | \mathcal{F}_j), Z_{t_j}^p, U_{t_j}^p) + A_{t_{j+1}}^p - A_{t_j}^p - (K_{t_{j+1}}^p - K_{t_j}^p) \underbrace{- \int_{t_j}^{t_{j+1}} Z_s^p dW_s - \int_{]t_j, t_{j+1}]} U_s^p d\tilde{N}_s,}_{\text{martingale representation}}$$

$$A_{t_{j+1}}^p - A_{t_j}^p \sim p\delta (Y_{t_j}^p - \xi_{t_j})^-, \; K_{t_{j+1}}^p - K_{t_j}^p \sim p\delta(\zeta_{t_j} - Y_{t_j}^p)^-$$

$$Y^p_{t_j} \sim Y^p_{t_{j+1}} + \delta g(t_j, \mathbb{E}(Y^p_{t_{j+1}}|\mathcal{F}_j), Z^p_{t_j}, U^p_{t_j}) + A^p_{t_{j+1}} - A^p_{t_j} - (K^p_{t_{j+1}} - K^p_{t_j})$$

$$\underbrace{- \int_{t_j}^{t_{j+1}} Z^p_s dW_s - \int_{]t_j, t_{j+1}]} U^p_s d\tilde{N}_s,}_{\text{martingale representation}}$$

$$A^p_{t_{j+1}} - A^p_{t_j} \sim p\delta(Y^p_{t_j} - \xi_{t_j})^-, \; K^p_{t_{j+1}} - K^p_{t_j} \sim p\delta(\zeta_{t_j} - Y^p_{t_j})^-$$

$$\begin{cases} \overline{y}^{p,n}_n := \xi^n, \\ \overline{y}^{p,n}_j = \overline{y}^{p,n}_{j+1} + \delta g(t_j, \mathbb{E}(\overline{y}^{p,n}_{j+1}|\mathcal{F}^n_j), \overline{z}^{p,n}_j, \overline{u}^{p,n}_j) + \overline{a}^{p,n}_j - \overline{k}^{p,n}_j \\ \qquad\qquad - (\overline{z}^{p,n}_j \sqrt{\delta} e^n_{j+1} + \overline{u}^{p,n}_j \eta^n_{j+1} + \overline{v}^{p,n}_j \mu^n_{j+1}) \\ \overline{a}^{p,n}_j = p\delta(\overline{y}^{p,n}_j - \xi^n_j)^- \,;\, \overline{k}^{p,n}_j = p\delta(\zeta^n_j - \overline{y}^{p,n}_j)^-, \end{cases}$$

$$\begin{cases} \overline{y}_n^{p,n} := \xi^n, \\ \overline{y}_j^{p,n} = \overline{y}_{j+1}^{p,n} + \delta g(t_j, \mathbb{E}(\overline{y}_{j+1}^{p,n}|\mathcal{F}_j^n), \overline{z}_j^{p,n}, \overline{u}_j^{p,n}) + \overline{a}_j^{p,n} - \overline{k}_j^{p,n} \\ \qquad -(\overline{z}_j^{p,n}\sqrt{\delta}e_{j+1}^n + \overline{u}_j^{p,n}\eta_{j+1}^n + \overline{v}_j^{p,n}\mu_{j+1}^n) \\ \overline{a}_j^{p,n} = p\delta(\overline{y}_j^{p,n} - \xi_j^n)^- ; \overline{k}_j^{p,n} = p\delta(\zeta_j^n - \overline{y}_j^{p,n})^-, \end{cases} \qquad (\star)$$

where ξ^n (resp. ζ^n) approximates ξ (resp. ζ). Taking conditional expectation in $(\star)$

$$\begin{cases} \overline{y}_n^{p,n} := \xi_n^n \\ \overline{y}_j^{p,n} = \mathbb{E}[\overline{y}_{j+1}^{p,n}|\mathcal{F}_j^n] + g(t_j, \mathbb{E}(\overline{y}_{j+1}^{p,n}|\mathcal{F}_j^n), \overline{z}_j^{p,n}, \overline{u}_j^{p,n})\delta + \overline{a}_j^{p,n} - \overline{k}_j^{p,n}, \\ \overline{a}_j^{p,n} = \frac{p\delta}{1+p\delta}\left(\mathbb{E}[\overline{y}_{j+1}^{p,n}|\mathcal{F}_j^n] + g(t_j, \mathbb{E}[\overline{y}_{j+1}^{p,n}|\mathcal{F}_j^n], \overline{z}_j^{p,n}, \overline{u}_j^{p,n}) - \xi_j^n\right)^-, \\ \overline{k}_j^{p,n} = \frac{p\delta}{1+p\delta}\left(\zeta_j^n - \mathbb{E}[\overline{y}_{j+1}^{p,n}|\mathcal{F}_j^n] - g(t_j, \mathbb{E}[\overline{y}_{j+1}^{p,n}|\mathcal{F}_j^n], \overline{z}_j^{p,n}, \overline{u}_j^{p,n})\right)^- \\ \overline{z}_j^{p,n} = \frac{1}{\sqrt{\delta}}\mathbb{E}(\overline{y}_{j+1}^{p,n}e_{j+1}^n|\mathcal{F}_j^n), \\ \overline{u}_j^{p,n} = \frac{1}{\kappa_n(1-\kappa_n)}\mathbb{E}(\overline{y}_{j+1}^{p,n}\eta_{j+1}^n|\mathcal{F}_j^n). \end{cases}$$

Main result - Assumptions

1. For some $r>2$, $\sup_{n\in\mathbb{N}}\max_{j\leq n}\mathbb{E}(|\xi^n_j|^r)+\sup_{n\in\mathbb{N}}\max_{j\leq n}\mathbb{E}(|\zeta^n_j|^r)+\sup_{t\leq T}\mathbb{E}|\xi_t|^r+\sup_{t\leq T}\mathbb{E}|\zeta_t|^r<\infty$
2. $\overline{\xi}^n$ (resp $\overline{\zeta}^n$) converges in probability to ξ (resp. ζ) for the J1-Skorokhod topology,

where $\overline{\xi}^n_t := \xi^n_{[t/\delta]}$, $\overline{\zeta}^n_t := \zeta^n_{[t/\delta]}$.
These assumptions are satisfied as soon as ξ and ζ are of the following form

- they satisfy an SDE $X_t = X_0 + \int_0^t b_X(X_{s^-})ds + \int_0^t \sigma_X(X_{s^-})dW_s + \int_0^t c_X(X_{s^-})d\tilde{N}_s$ with Lipschitz coefficients
- $X_t = \Phi(t, W_t, \widetilde{N}_t)$

Main result

$$\overline{Y}_t^{p,n} = \overline{y}_{[t/\delta]}^{p,n}, \overline{Z}_t^{p,n} = \overline{z}_{[t/\delta]}^{p,n}, \overline{U}_t^{p,n} = \overline{u}_{[t/\delta]}^{p,n}, \overline{A}_t^{p,n} = \sum_{j=0}^{[t/\delta]-1} \overline{a}_j^{p,n}, \overline{K}_t^{p,n} = \sum_{j=0}^{[t/\delta]-1} \overline{k}_j^{p,n}.$$

We also introduce $\overline{\alpha}_t^{p,n} := \overline{A}_t^{p,n} - \overline{K}_t^{p,n}$.

Theorem

The sequence $(\overline{Y}^{p,n}, \overline{Z}^{p,n}, \overline{U}^{p,n})$ converges to (Y, Z, U), in the following sense : $\forall r \in [1,2[$

$$\lim_{p\to\infty}\lim_{n\to\infty}\left(\mathbb{E}\left[\int_0^T |\overline{Y}_s^{p,n} - Y_s|^2 ds\right] + \mathbb{E}\left[\int_0^T |\overline{Z}_s^{p,n} - Z_s|^r ds\right] + \mathbb{E}\left[\int_0^T |\overline{U}_s^{p,n} - U_s|^r ds\right]\right) = 0.$$

Moreover, $\overline{Z}^{p,n}$ (resp. $\overline{U}^{p,n}$) weakly converges in $\mathbb{H}^2$ to Z (resp. to U) and for $0 \le t \le T$, $\overline{\alpha}_{\psi^n(t)}^{p,n}$ converges weakly to α_t in L^2 as $n \to \infty$ and $p \to \infty$.

Scheme of the proof

We split the proof in the following steps

- Error between the exact solution of the BSDE (Y, Z, U, α) and the penalized BSDE $(Y^p, Z^p, U^p, \alpha^p)$
- Error between the penalized BSDE $(Y^p, Z^p, U^p, \alpha^p)$ and its time discretization $(\overline{Y}^{p,n}, \overline{Z}^{p,n}, \overline{U}^{p,n}, \overline{\alpha}^{p,n})$. To do it, we introduce an intermediate implicit penalized discrete scheme $(Y^{p,n}, Z^{p,n}, U^{p,n}, \alpha^{p,n})$ and we study
  - the error between $(Y^p, Z^p, U^p, \alpha^p)$ and $(Y^{p,n}, Z^{p,n}, U^{p,n}, \alpha^{p,n})$
  - the error between $(Y^{p,n}, Z^{p,n}, U^{p,n}, \alpha^{p,n})$ and $(\overline{Y}^{p,n}, \overline{Z}^{p,n}, \overline{U}^{p,n}, \overline{\alpha}^{p,n})$

Convergence of the penalized BSDE to the reflected BSDE

We prove this result in the general case of jumps driven by a Poisson random measure.

$$\begin{cases} Y_t = \xi_T + \int_t^T g(s, Y_s, Z_s, U_s)ds + (A_T - A_t) - (K_T - K_t) - \int_t^T Z_s dW_s - \int_t^T \int_{\mathbb{R}^*} U_s(e)\tilde{N}(ds, de) \\ \forall t \in [0,T], \xi_t \le Y_t \le \zeta_t \\ \int_0^T (Y_{t^-} - \xi_{t^-})dA_t = 0 \text{ a.s. and } \int_0^T (\zeta_{t^-} - Y_{t^-})dK_t = 0 \end{cases}$$

Proposition

For all $r \in [1,2[$, the following strong convergence holds

$$\lim_{p\to\infty} \mathbb{E}\left[\int_0^T |Y_s^p - Y_s|^2 ds\right] + \mathbb{E}\left[\int_0^T |Z_s^p - Z_s|^r ds + \int_0^T \left(\int_{\mathbb{R}^*} |U_s^p - U_s|^2 \nu(de)\right)^{\frac{r}{2}} ds\right] = 0.$$

Moreover, Z^p weakly converges in $\mathbb{H}^2$ to Z, U^p weakly converges in $\mathbb{H}^2_\nu$ to U, and $\alpha_t^p := A_t^p - K_t^p$ weakly converges to α_t in $L^2(\mathcal{F}_t)$.

Convergence of the penalized BSDE to the reflected BSDE

In fact we give an other proof of existence of solutions to doubly reflected BSDEs with jumps and RCLL barriers, based on penalization (without using a fixed point argument) and

- comparison theorem for BSDEs with jumps (Quenez, Sulem 2013)
- generalized monotonic theorem
- characterisation of the solution of a doubly reflected BSDE as a value function of a stochastic game and Snell enveloppe theory

Convergence of the discrete time setting to the continuous time setting

This result ensues from Lejay, Mordecki, Torres (2013)

Proposition

For any $p \in \mathbb{N}^*$, the sequence $(Y_t^{p,n}, Z_t^{p,n}, U_t^{p,n})$ converges to (Y_t^p, Z_t^p, U_t^p) in the following sense :

$$\lim_{n\to\infty} \mathbb{E}[\sup_{0\le t\le T} |Y_{\psi^n(t)}^{p,n} - Y_t^p|^2 + \int_0^T |Z_s^{p,n} - Z_s^p|^2 ds + \int_0^T |U_s^{p,n} - U_s^p|^2 ds] = 0,$$

where ψ^n is a random one-to-one continuous mapping from $[0, T]$ to $[0, T]$ such that $\sup_{t\in[0,T]} |\psi^n(t) - t| \xrightarrow[n\to\infty]{} 0$ a.s..

Error between explicit and implicit penalization schemes

Proposition

We have

$$\lim_{n\to\infty} \sup_{0\leq t\leq T} \mathbb{E}[|\overline{Y}_t^{p,n} - Y_t^{p,n}|^2] + \mathbb{E}[\int_0^T |\overline{Z}_s^{p,n} - Z_s^{p,n}|^2 ds] + \mathbb{E}[\int_0^T |\overline{U}_s^{p,n} - U_s^{p,n}|^2 ds] = 0.$$

Moreover, $\lim_{n\to\infty}(\overline{\alpha}_t^{p,n} - \alpha_t^{p,n}) = 0$ in $L^2(\mathcal{F}_t)$, for $t \in [0, T]$.

Inaccessible jumps

$\xi_t := (W_t)^2 + \widetilde{N}_t + (T-t)$, $\zeta_t := (W_t)^2 + \widetilde{N}_t + 3(T-t)$,
$g(t,\omega,y,z,u) := -5|y+z| + 6u$, $\lambda = 5$ and $T = 1$.

TABLE: The solution $\bar{y}^{p,n}$ at time $t=0$

$Y_0^{p,n}$	n=100	n=200	n=400	n=500	n=600
p=20	1.2181	1.2245	1.2277	1.2283	1.2288
p=50	1.2648	1.2728	1.2767	1.2775	1.2780
p=100	1.2808	1.2894	1.2936	1.2945	1.2950
p=500	1.2939	1.3033	1.3079	1.3088	1.3094
p=1000	1.2957	1.3051	1.3098	1.3107	1.3113
p=5000	1.2971	1.3066	1.3113	1.3122	1.3129
p=20000	1.2974	1.3069	1.3116	1.3125	1.3132
CPU time for p=20000	0.0644	0.6622	6.3560	12.5970	20.0062

Inaccessible jumps

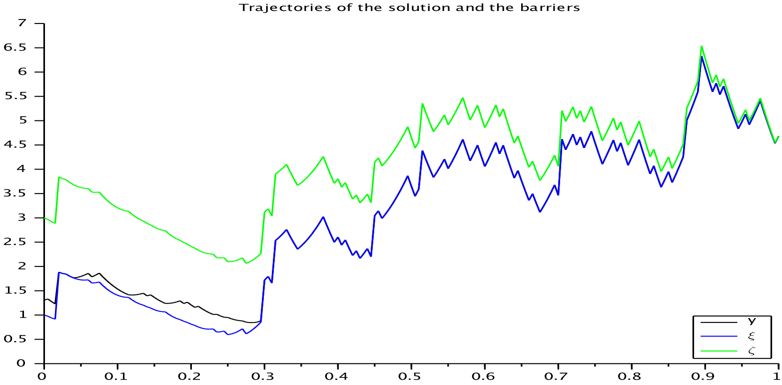

FIGURE: Trajectories of the solution $\overline{y}^{p,n}$ and the barriers $\overline{\xi}^n$ and $\overline{\zeta}^n$ for $\lambda = 5$, $N = 200$, $p = 20000$.

Predictable and totally inaccessible jumps

$\xi_t := (W_t)^2 + \widetilde{N}_t + (T-t)(1 - \mathbf{1}_{W_t \geq -1})$, $\zeta_t := (W_t)^2 + \widetilde{N}_t + (T-t)(2 + \mathbf{1}_{W_t \geq -1})$,
$g(t,\omega,y,z,u) := -5|y+z| + 6u$, $\lambda = 5$, $T = 1$.

$Y_0^{p,n}$	n=100	n=200	n=400	n=500	n=600
p=20	1.0745	1.0698	1.0782	1.0748	1.0759
p=50	1.1138	1.1103	1.1191	1.1159	1.1170
p=100	1.1266	1.1238	1.1328	1.1297	1.1308
p=500	1.1373	1.1353	1.1448	1.1419	1.1431
p=1000	1.1387	1.1369	1.1465	1.1437	1.1449
p=5000	1.1399	1.1382	1.1481	1.1453	1.1466
p=20000	1.1401	1.1385	1.1484	1.1456	1.1469

Conclusion

We have presented an algorithm to solve doubly reflected BSDEs with RCLL barriers based on random trees and penalization.

- ☺ The conditional expectations are easily computed
- ☹ It suffers from the curse of dimensionality
- How to extend it to random Poisson measure ?