Are Smart Beta indexes valid for hedge fund portfolio allocation?

Asmerilda Hitaj* Giovanni Zambruno*

*University of Milano Bicocca

Second Young researchers' meeting on BSDEs, Numerics and Finance July 2014

Outline

Investor Problem and Markowitz model

- Markowitz (1952) formulated the portfolio problem as a trade-off between mean and variance of portfolios.

Maximize μ_p for a given σ_p

$$\begin{cases} \max \mu_p = \sum_{i=1}^{N} w_i \; R_i \\ s.t. \\ w' \; \Sigma \; w = c \\ \sum_{i=1}^{N} w_i = 1. \end{cases}$$

Minimize σ_p for a given μ_p

$$\begin{cases} \min \sigma_p = w' \; \Sigma \; w \\ s.t. \\ \mu_p = c \\ \sum_{i=1}^{N} w_i = 1. \end{cases}$$

Investor problem cont...

- Markowitz (1959) also proposed the semivariance as a measure of risk.
- Athyde and Flores (2002) constructed the efficient frontier based on the first four moments of the portfolio return distribution.
- Jondeau et al. (2007), Martellini and Ziemann (2010), Hitaj et al.(2012) etc, used the EU to introduce higher moments in portfolio allocation.
- Davies et al. (2009) adopted a multi-objective optimization problem to introduce higher moments in portfolio allocation.
- Kahneman and Tversky (1979, 1992) developed the Prospect theory which incorporates real human decision patterns and psychology, in explaining how individuals make economic choices.
- In this work we consider two approaches for introducing higher moments into portfolio allocation.

Investor problem cont...

- Markowitz (1959) also proposed the semivariance as a measure of risk.
- Athyde and Flores (2002) constructed the efficient frontier based on the first four moments of the portfolio return distribution.
- Jondeau et al. (2007), Martellini and Ziemann (2010), Hitaj et al.(2012) etc, used the EU to introduce higher moments in portfolio allocation.
- Davies et al. (2009) adopted a multi-objective optimization problem to introduce higher moments in portfolio allocation.
- Kahneman and Tversky (1979, 1992) developed the Prospect theory which incorporates real human decision patterns and psychology, in explaining how individuals make economic choices.
- In this work we consider two approaches for introducing higher moments into portfolio allocation.

Investor problem cont...

- Markowitz (1959) also proposed the semivariance as a measure of risk.
- Athyde and Flores (2002) constructed the efficient frontier based on the first four moments of the portfolio return distribution.
- Jondeau et al. (2007), Martellini and Ziemann (2010), Hitaj et al.(2012) etc, used the EU to introduce higher moments in portfolio allocation.
- Davies et al. (2009) adopted a multi-objective optimization problem to introduce higher moments in portfolio allocation.
- Kahneman and Tversky (1979, 1992) developed the Prospect theory which incorporates real human decision patterns and psychology, in explaining how individuals make economic choices.
- In this work we consider two approaches for introducing higher moments into portfolio allocation.

Investor problem cont...

- Markowitz (1959) also proposed the semivariance as a measure of risk.
- Athyde and Flores (2002) constructed the efficient frontier based on the first four moments of the portfolio return distribution.
- Jondeau et al. (2007), Martellini and Ziemann (2010), Hitaj et al.(2012) etc, used the EU to introduce higher moments in portfolio allocation.
- Davies et al. (2009) adopted a multi-objective optimization problem to introduce higher moments in portfolio allocation.
- Kahneman and Tversky (1979, 1992) developed the Prospect theory which incorporates real human decision patterns and psychology, in explaining how individuals make economic choices.
- In this work we consider two approaches for introducing higher moments into portfolio allocation.

Investor problem cont...

- Markowitz (1959) also proposed the semivariance as a measure of risk.
- Athyde and Flores (2002) constructed the efficient frontier based on the first four moments of the portfolio return distribution.
- Jondeau et al. (2007), Martellini and Ziemann (2010), Hitaj et al.(2012) etc, used the EU to introduce higher moments in portfolio allocation.
- Davies et al. (2009) adopted a multi-objective optimization problem to introduce higher moments in portfolio allocation.
- Kahneman and Tversky (1979, 1992) developed the Prospect theory which incorporates real human decision patterns and psychology, in explaining how individuals make economic choices.
- In this work we consider two approaches for introducing higher moments into portfolio allocation.

Investor problem cont...

- Markowitz (1959) also proposed the semivariance as a measure of risk.
- Athyde and Flores (2002) constructed the efficient frontier based on the first four moments of the portfolio return distribution.
- Jondeau et al. (2007), Martellini and Ziemann (2010), Hitaj et al.(2012) etc, used the EU to introduce higher moments in portfolio allocation.
- Davies et al. (2009) adopted a multi-objective optimization problem to introduce higher moments in portfolio allocation.
- Kahneman and Tversky (1979, 1992) developed the Prospect theory which incorporates real human decision patterns and psychology, in explaining how individuals make economic choices.
- In this work we consider two approaches for introducing higher moments into portfolio allocation.

Expected Utility Theory (EUT)

- According to EUT the objective of the investor is to maximize his expected utility, under certain constraints:

$$\begin{cases} \max_{w} \mathbb{E}U(\tilde{X}) \\ s.t. \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \leq 1. \end{cases}$$

- In the empirical part we consider 'negative exponential' utility function:

$$U(\tilde{X}) = -e^{-\lambda \tilde{X}}$$

EUT cont..

- In order to account for higher moments in portfolio allocation the Taylor expansion of the utility function up to the fourth order is used.
- The investor problem can be written as:

$$\begin{cases} \max_{w} \mathbb{E}U(\tilde{X}) = -e^{-\lambda(\mu w')} \left[1 + \frac{\lambda^2}{2} w M_2 w' \; - \frac{\lambda^3}{6} w M_3 (w' \otimes w') \right] \\ s.t. \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \leq 1. \end{cases}$$

Multi Objective Portfolio Optimization

- Davies et al. (2009) used the multi objective approach to introduce higher moments to portfolio allocation.
- Their approach is based on a two step procedure:
- First step: solves separately each single optimization problem, that is maximize the mean and skewness and minimize the variance:

P_1

$$\begin{cases} \max_{w} w' \bar{R} = \mu_P^* \\ s.t. \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \quad \forall i. \end{cases}$$

P_2

$$\begin{cases} \min_{w} w' M_2 w = \left(\sigma_P^2\right)^* \\ s.t. \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \quad \forall i. \end{cases}$$

Multi objective approach cont...

P_3

$$\begin{cases} \max_{w} w' \; M_3 \; (w \otimes w) = \; S_P^* \\ s.t. \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \quad \forall i. \end{cases}$$

- Solving these three problems separately we find the aspiration levels of the investor for the mean, variance and skewness $(\mu_P^*, \left(\sigma_P^2\right)^*, S_P^*)$

Multi objective approach cont...

P_3

$$\begin{cases} \max\limits_{w} w' \; M_3 \; (w \otimes w) = \; S_P^* \\ s.t. \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \quad \forall i. \end{cases}$$

- Solving these three problems separately we find the aspiration levels of the investor for the mean, variance and skewness $(\mu_P^*, \left(\sigma_P^2\right)^*, S_P^*)$

Multi objective approach cont...

- Second step: construct a 'multi objective' problem (MO), where the portfolio allocation decision is given by the solution of the MO that minimizes the Minkowski-like distance from the aspiration levels, namely:

$$\begin{cases} \min\limits_{w_i} Z = \left| \frac{\mu_P^* - w'\bar{R}}{\mu_P^*} \right|^{\gamma_1} + \left| \frac{w' M_2 w - \left(\sigma_P^2\right)^*}{\left(\sigma_P^2\right)^*} \right|^{\gamma_2} \\ + \left| \frac{S_P^* - w' \ M_3 \ (w \otimes w)}{S_P^*} \right|^{\gamma_3} \\ \text{s.t.} \\ \quad \sum_{i=1}^{N} w_i = 1 \\ \quad 0 \leq w_i \ \forall i \end{cases}$$

where $\gamma_1, \ \gamma_2$ and γ_3 represent the investor's subjective parameters.

Equally weighted and Global Minimum Variance

Equally weighted

- Equally weighted strategy consists in holding a portfolio with weight $1/N$ in each component, where N is the number of assets.
- This strategy does not involve any optimization or estimation procedure.

Global Minimum Variance

- GMV portfolio is the one that has the minimum variance in absolute, without taking into account the expected return of the portfolio.

$$\begin{cases} \min_{w} \sigma_P^2 = w' \; M_2 \; w \\ \text{s.t.} \quad \sum_{i=1}^{N} w_i = 1, \; 0 \leq w_i \; \forall i, \end{cases}$$

Equally weighted and Global Minimum Variance

Equally weighted

- Equally weighted strategy consists in holding a portfolio with weight $1/N$ in each component, where N is the number of assets.
- This strategy does not involve any optimization or estimation procedure.

Global Minimum Variance

- GMV portfolio is the one that has the minimum variance in absolute, without taking into account the expected return of the portfolio.

$$\begin{cases} \min_{w} \sigma_P^2 = w' \; M_2 \; w \\ \text{s.t.} \quad \sum_{i=1}^{N} w_i = 1, \; 0 \leq w_i \; \forall i, \end{cases}$$

Equal Risk Contribution (ERC)

- Qian (2006) proposed the ERC, where weights are such that each asset has the same contribution to portfolio risk.
- Maillard et al. (2010) analyzed the properties of an unconstrained analytic solution of the ERC.

The marginal risk contribution of asset i is defined as:

$$\partial_{w_i}\sigma_P = \frac{\partial \sigma_P}{\partial w_i} = \frac{w_i\sigma_i^2 + \sum_{i\neq j} w_j\sigma_{ij}}{\sigma_P}.$$

- The total risk contribution of the i^{th} asset is:

$$\sigma_i(w) = w_i\partial_{w_i}\sigma_P.$$

- The portfolio risk can be seen as the sum of total risk contributions:

$$\sigma_P = \sum_i^N \sigma_i(w).$$

Equal Risk Contribution (ERC)

- Qian (2006) proposed the ERC, where weights are such that each asset has the same contribution to portfolio risk.
- Maillard et al. (2010) analyzed the properties of an unconstrained analytic solution of the ERC.

The marginal risk contribution of asset i is defined as:

$$\partial_{w_i}\sigma_P = \frac{\partial \sigma_P}{\partial w_i} = \frac{w_i\sigma_i^2 + \sum_{i\neq j} w_j\sigma_{ij}}{\sigma_P}.$$

- The total risk contribution of the i^{th} asset is:

$$\sigma_i(w) = w_i \partial_{w_i}\sigma_P.$$

- The portfolio risk can be seen as the sum of total risk contributions:

$$\sigma_P = \sum_i^N \sigma_i(w).$$

Equal Risk Contribution (ERC)

- Qian (2006) proposed the ERC, where weights are such that each asset has the same contribution to portfolio risk.
- Maillard et al. (2010) analyzed the properties of an unconstrained analytic solution of the ERC.

The marginal risk contribution of asset i is defined as:

$$\partial_{w_i}\sigma_P = \frac{\partial \sigma_P}{\partial w_i} = \frac{w_i\sigma_i^2 + \sum_{i\neq j} w_j\sigma_{ij}}{\sigma_P}.$$

- The total risk contribution of the i^{th} asset is:

$$\sigma_i(w) = w_i \partial_{w_i}\sigma_P.$$

- The portfolio risk can be seen as the sum of total risk contributions:

$$\sigma_P = \sum_{i}^{N} \sigma_i(w).$$

ERC cont...

- A characteristic of this strategy is that:

$$w_i \, \partial_{w_i} \sigma_P = w_j \, \partial_{w_j} \sigma_P \qquad \forall \; i, \; j$$

- The investor problem in this case is:

$$\begin{cases} \min\limits_{w} \; \sum_{i=1}^{N} \; \sum_{j=1}^{N} \; \left(w_i (M_2 \; w)_i - w_j (M_2 \; w)_j\right)^2 \\ \text{s.t.} \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \; \forall i, \end{cases}$$

ERC cont...

- A characteristic of this strategy is that:

$$w_i \, \partial_{w_i} \sigma_P = w_j \, \partial_{w_j} \sigma_P \qquad \forall \; i, \; j$$

- The investor problem in this case is:

$$\begin{cases} \min\limits_{w} \; \sum_{i=1}^{N} \; \sum_{j=1}^{N} \; \left(w_i (M_2 \; w)_i - w_j (M_2 \; w)_j \right)^2 \\ \text{s.t.} \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \; \forall i, \end{cases}$$

Maximum Diversified Portfolio

- The objective of this strategy is to construct a portfolio that maximizes the benefits from diversification (see Choueifaty and Coignard (2008)). Where the *diversification ratio is defined as:*

$$DR = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sqrt{w' M_2 w}}$$

- The investor problem in the MDP context is used is:

$$\begin{cases} \max_{w_i} DR = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sqrt{w' M_2 w}} \\ s.t. \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \ \forall i. \end{cases}$$

- De Miguel et. al (2009), Optimal Versus Naive Diversification: How inefficient is the $\frac{1}{N}$ Portfolio Strategy?

Maximum Diversified Portfolio

- The objective of this strategy is to construct a portfolio that maximizes the benefits from diversification (see Choueifaty and Coignard (2008)). Where the *diversification ratio is defined as:*

$$DR = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sqrt{w' M_2 w}}$$

- The investor problem in the MDP context is used is:

$$\begin{cases} \max_{w_i} DR = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sqrt{w' M_2 w}} \\ s.t. \\ \sum_{i=1}^{N} w_i = 1 \\ 0 \leq w_i \ \forall i. \end{cases}$$

- De Miguel et. al (2009), Optimal Versus Naive Diversification: How inefficient is the $\frac{1}{N}$ Portfolio Strategy?

Shrinkage estimator

- Estimation of moments and comoments is needed for portfolio allocation.
- In the empirical part we use the shrinkage estimators (see for e.g. Ledoit and Wolf (2003) and Martellini & Ziemann(2010):
  - For the mean we use the shrinkage toward the Grand Mean.

$$\mu^{shrink} = (1-\phi) * \bar{\mu} + \phi * \mu^{target} \qquad (0 \leq \phi \leq 1)$$

where (Jorion 86):

$$\phi = \min\left(1, \max\left(0, \frac{1}{T}\frac{(N-2)}{(\bar{\mu}-\mu^{target})'\Sigma^{-1}(\bar{\mu}-\mu^{target})}\right)\right),$$

and:

$$\mu^{target} = \frac{1'\ \Sigma^{-1}}{1'\ \Sigma^{-1}1}\bar{\mu}$$

Shrinkage estimator cont...

- For mean, variance and skewness we use the shrinkage towards the constant correlation approach, Elton and Gruber (1973).

$$M^{shrink} = (1-\phi) * \bar{M} + \phi * M^{CC}$$

Empirical analysis and Statistics

- We consider 4 hedge funds portfolio and 1 equity portfolio.
- The statistics of each index in portfolio are:

Dow Jones Credit Suisse Hedge Funds indexes

Period under consideration $Jan/1994\ to\ Dec/2011$

General statistics for each component in HFP_1 portfolio

	Annual Mean	Annual STD	Skewness	Kurtosis	JB-test
Hedge Fund Index	0.084	0.076	-0.327	5.437	57.295
Convertible Arbitrage H. F.	0.072	0.072	-2.997	21.048	3254.979
Dedicated Short Bias H. F.	-0.035	0.168	0.449	3.745	12.252
Emerging Markets H. F.	0.071	0.152	-1.201	9.714	457.581
Event Driven H. F.	0.088	0.065	-2.451	15.221	1560.520
Event Driven Distressed H. F.	0.097	0.068	-2.388	15.579	1629.257
Event Driven Multi-Strategy H. F.	0.083	0.070	-1.944	11.603	802.213
Event Driven Risk Arbitrage H. F.	0.065	0.042	-1.097	8.028	270.850
Fixed Income Arbitrage H. F.	0.051	0.060	-4.700	36.613	10963.463
Global Macro H. F.	0.114	0.097	-0.246	6.888	138.245
Long/Short Equity H. F.	0.088	0.099	-0.218	6.205	94.174
Managed Futures H. F.	0.058	0.117	-0.079	2.979	0.227

Table: General statistics for HFP_1

Statistics for portfolio HFP_2

Period under consideration $Jan/1997\ to\ Jan/2011$
General statistics for each component in HFP_2 portfolio

	Annual Mean	Annual STD	Skewness	Kurtosis	JB-test
'Convertible Arbitrage'	0.079	0.065	-2.626	20.019	2550.845
'CTA Global'	0.066	0.085	0.151	2.795	1.071
'Distressed Securities'	0.098	0.063	-1.504	8.296	298.244
'Emerging Markets'	0.092	0.126	-1.218	8.202	265.339
'Equity Market Neutral'	0.064	0.030	-2.396	18.066	2010.055
'Event Driven'	0.089	0.062	-1.561	8.228	298.111
'Fixed Income Arbitrage'	0.059	0.045	-3.770	25.089	4380.870
'Global Macro'	0.082	0.056	0.851	4.984	54.942
'Long/Short Equity'	0.085	0.076	-0.408	4.042	14.080
'Merger Arbitrage'	0.074	0.036	-1.493	8.669	330.163
'Relative Value'	0.079	0.044	-1.955	11.523	707.179
'Short Selling'	0.014	0.181	0.644	5.384	59.057
'Funds Of Funds'	0.061	0.059	-0.410	6.418	99.369

Table: General statistics for HFP_2

Statistics for portfolio HFP_3

Period under consideration $Jan/1990$ to $Sep/2013$

General statistics for each component in HFP_3 portfolio

	Annual Mean	Annual STD	Skewness	Kurtosis	JB-test
HFRI ED: Distressed/Restructuring	0.116	0.065	-1.022	7.795	322.669
HFRI ED: Merger Arbitrage	0.082	0.040	-2.065	11.777	1117.400
HFRI EH: Equity Market Neutral'	0.066	0.032	-0.251	4.570	32.254
HFRI EH: Quantitative Directional	0.121	0.128	-0.432	3.818	16.805
HFRI EH: Short Bias	0.005	0.185	0.248	5.261	63.629
HFRI Emerging Markets (Total)	0.125	0.141	-0.832	6.604	187.096
HFRI Emerging Markets: Asia ex-Japan	0.098	0.135	-0.092	3.813	8.240
HFRI Equity Hedge (Total)	0.124	0.091	-0.253	4.789	41.021
HFRI Event-Driven (Total)	0.112	0.068	-1.294	6.973	266.972
HFRI FOF: Conservative	0.062	0.039	-1.691	10.497	803.204
HFRI FOF: Diversified	0.068	0.059	-0.446	7.020	201.339
HFRI FOF: Market Defensive	0.073	0.058	0.235	3.891	12.042

Table: General statistics for HFP_3

Statistics for portfolio HFP_4

Period under consideration $Jan/1990\ to\ Sep/2013$

General statistics for each component in HFP_4 portfolio

	Annual Mean	Annual STD	Skewness	Kurtosis	JB-test
'HFRI FOF: Strategic Index'	0.095	0.086	-0.463	6.393	146.916
'HFRI FOF Composite Index'	0.072	0.058	-0.666	6.869	198.828
'HFRI Fund Weighted Composite Index'	0.106	0.069	-0.681	5.425	91.828
'HFRI Fund Weighted Composite Index CHF'	0.093	0.070	-0.711	5.342	89.181
'HFRI Fund Weighted Composite Index GBP'	0.122	0.070	-0.698	5.364	89.498
'HFRI Fund Weighted Composite Index JPY'	0.082	0.069	-0.671	5.108	74.185
'HFRI Macro (Total) Index'	0.113	0.075	0.559	3.972	26.069
'HFRI Macro: Systematic Diversified Index'	0.102	0.075	0.150	2.659	2.460
'HFRI Relative Value (Total) Index'	0.098	0.044	-2.112	16.381	2338.090
'HFRI RV: Fixed Income-Convertible Arbitrage Index'	0.085	0.066	-3.049	31.509	10093.109
'HFRI RV: Fixed Income-Corporate Index'	0.078	0.065	-1.336	10.937	832.768
'HFRI RV: Multi-Strategy Index'	0.082	0.044	-2.063	16.151	2255.866

Table: General statistics for HFP_4

Statistics for Equity portfolio taken from S&P 500

Period under consideration $Feb/1985\ to\ May/2013$
General statistics for each component in $Equity$ portfolio

	Annual Mean	Annual STD	Skewness	Kurtosis	JB-test
'AMD UN Equity'	-0.052	0.645	-0.200	3.566	6.820
'APA UN Equity'	0.102	0.347	-0.240	3.658	9.383
'CMCSA UW Equity'	0.129	0.310	-0.186	3.383	4.037
'ED UN Equity'	0.048	0.174	-0.244	3.703	10.387
'FDX UN Equity'	0.086	0.298	0.077	3.357	2.141
'GIS UN Equity'	0.099	0.189	-0.072	3.304	1.605
'JNJ UN Equity'	0.125	0.203	-0.181	3.598	6.911
'L UN Equity'	0.091	0.255	-0.270	3.600	9.240
'NUE UN Equity'	0.119	0.340	-0.225	3.608	8.098
'PAYX UW Equity'	0.183	0.299	-0.056	3.357	1.981
'PFE UN Equity'	0.099	0.244	-0.259	3.656	9.886
'SO UN Equity'	0.073	0.171	-0.076	3.794	9.256
'LUV UN Equity'	0.090	0.331	-0.109	3.140	0.952
'WAG UN Equity'	0.121	0.266	-0.059	3.611	5.487

Table: General statistics for $Equity$

Portfolio allocation procedure

- A static portfolio allocation procedure is used:
- Different rolling-window lengths are considered:
  - 24 months in-sample and 3 months out of sample.
  - 24 months in-sample and 6 months out of sample.
  - 48 months in-sample and 3 months out of sample.
  - 48 months in-sample and 6 months out of sample.
- For each portfolio we have calculated the in-sample optimal weights using the different approaches discussed:
  - Smart Beta strategies: EW, MDP, ERC and GMV.
  - Multi Objective approach using 2 and 3 moments, varying $\gamma_i = 1, \ldots, 5$. Therefore in case of MO approach with two moments we have 25 portfolios and when three moments are considered we have 125 portfolios.
  - EUT with 2 and 3 moments with $\lambda = 1, \ldots, 30$.

Portfolio allocation procedure

- A static portfolio allocation procedure is used:
- Different rolling-window lengths are considered:
  - 24 months in-sample and 3 months out of sample.
  - 24 months in-sample and 6 months out of sample.
  - 48 months in-sample and 3 months out of sample.
  - 48 months in-sample and 6 months out of sample.
- For each portfolio we have calculated the in-sample optimal weights using the different approaches discussed:
  - Smart Beta strategies: EW, MDP, ERC and GMV.
  - Multi Objective approach using 2 and 3 moments, varying $\gamma_i = 1, \ldots, 5$. Therefore in case of MO approach with two moments we have 25 portfolios and when three moments are considered we have 125 portfolios.
  - EUT with 2 and 3 moments with $\lambda = 1, \ldots, 30$.

Portfolio allocation procedure

- A static portfolio allocation procedure is used:
- Different rolling-window lengths are considered:
  - 24 months in-sample and 3 months out of sample.
  - 24 months in-sample and 6 months out of sample.
  - 48 months in-sample and 3 months out of sample.
  - 48 months in-sample and 6 months out of sample.
- For each portfolio we have calculated the in-sample optimal weights using the different approaches discussed:
  - Smart Beta strategies: EW, MDP, ERC and GMV.
  - Multi Objective approach using 2 and 3 moments, varying $\gamma_i = 1, \ldots, 5$. Therefore in case of MO approach with two moments we have 25 portfolios and when three moments are considered we have 125 portfolios.
  - EUT with 2 and 3 moments with $\lambda = 1, \ldots, 30$.

Portfolio allocation procedure (cont...)

- Once the in-sample weights are calculated we keep them constant for the next out of-sample period and calculate the out-of sample returns obtained with the different strategies.
- From the out-of-sample returns obtained with each strategy we calculate:
  - Sharpe Ratio, with $R_f = 2\%$ annual

$$Sh_R = \frac{\overline{R}_P - R_f}{\sigma_P}$$

  - Excess Return on VaR Ratio, with $R_f = 2\%$ annual and $\alpha = 5\%$.

$$EVaR_R = \frac{\overline{R}_P - R_f}{VaR_P(\alpha)}$$

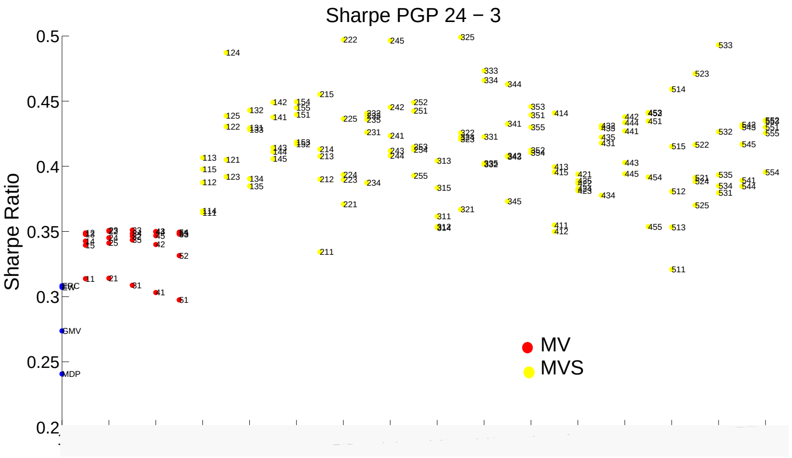
  - Information Ratio, where the Smart Beta obtained indexes are used as benchmark .

$$I_R = \frac{\overline{R}_P - \overline{R}_B}{\sigma(R_P - R_B)}.$$

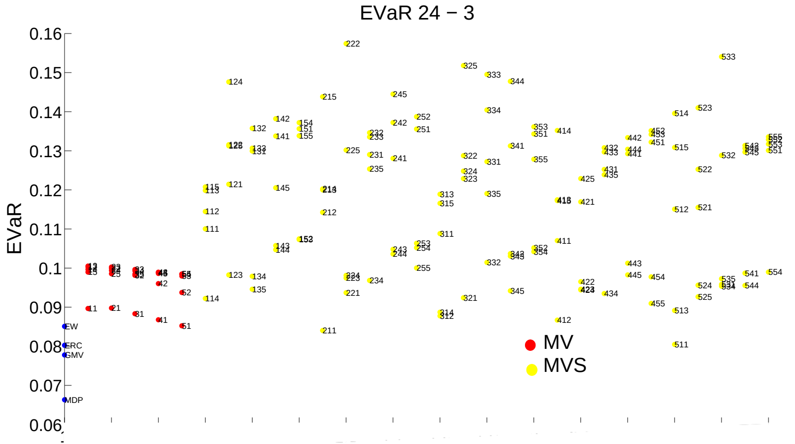
- To represent the concentration (lack of diversification) of each portfolio we calculate the modified Herfindahl index.

$$H_I = \frac{\sum_{i=1}^{N} w_i^2 - \frac{1}{N}}{1 - \frac{1}{N}}.$$

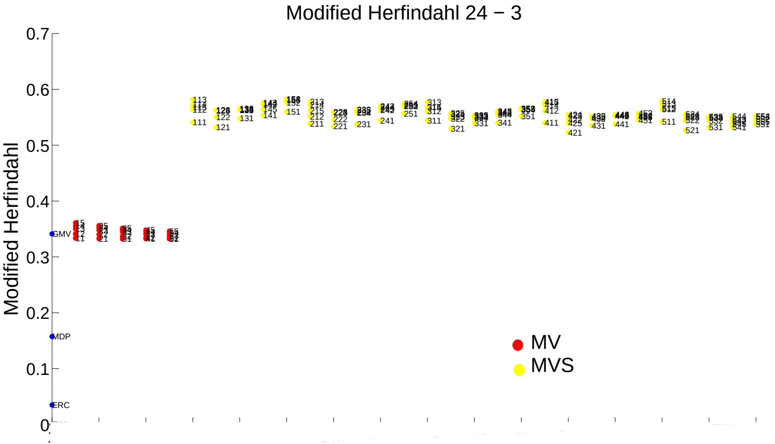
Results for HFP_1, using MO approach (24 - 3)



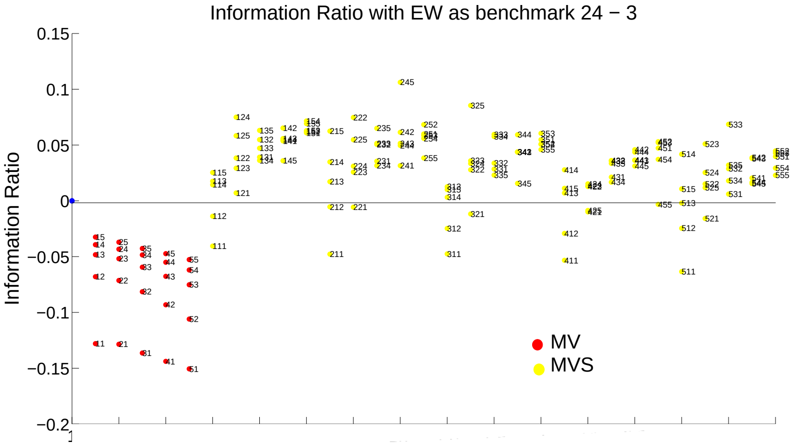
MO for HFP_1, EVaR with $R_f = 2\%$ annual and $\alpha = 5\%$



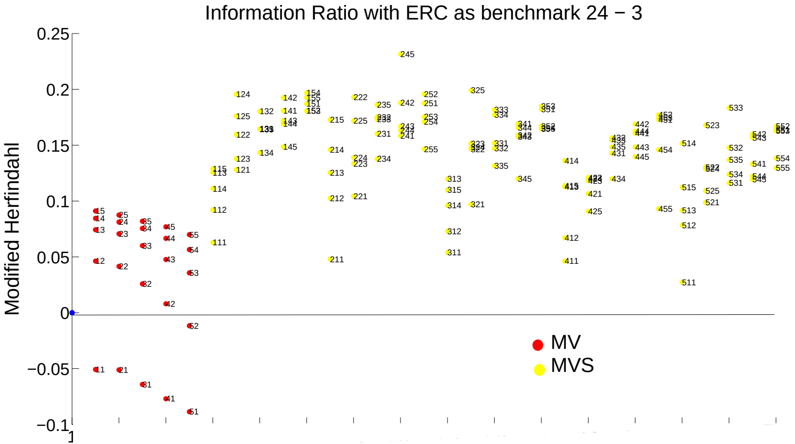
MO for HFP_1, Modified Herfindhal index



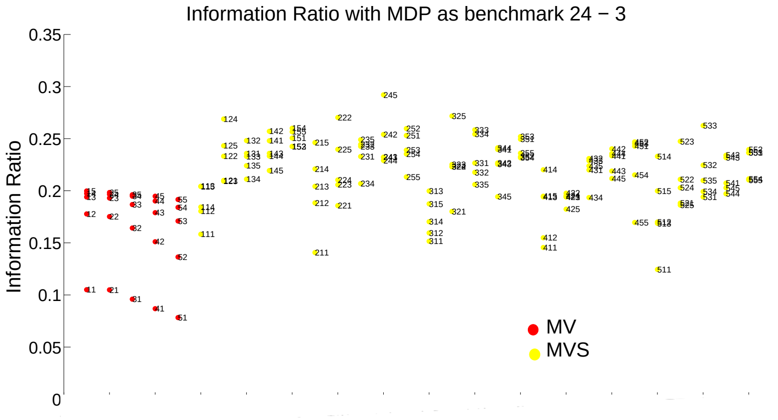
MO for HFP_1, IR using as benchmark EW



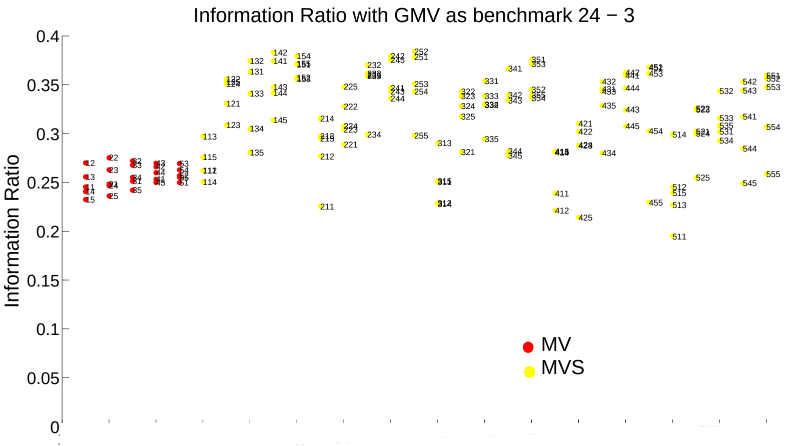
MO for HFP_1, IR using as benchmark ERC



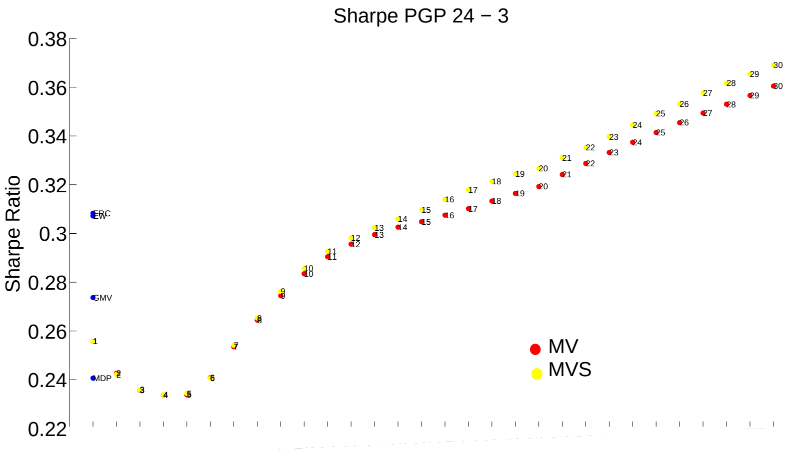
MO for HFP_1, IR using as benchmark MDP



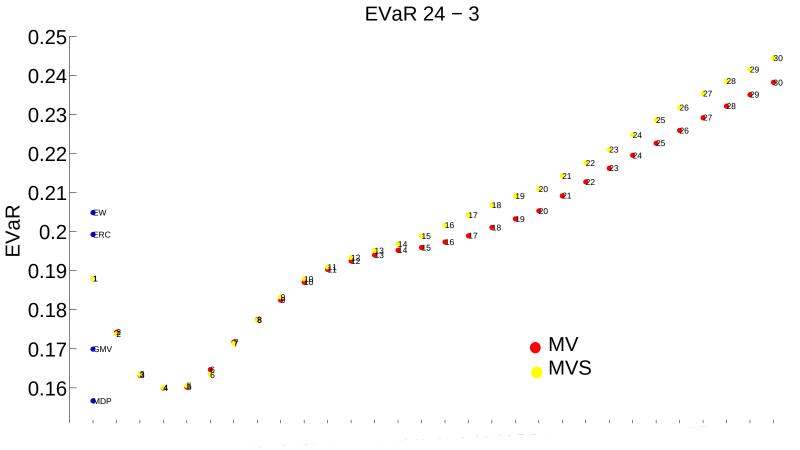
MO for HFP_1, IR using as benchmark GMV



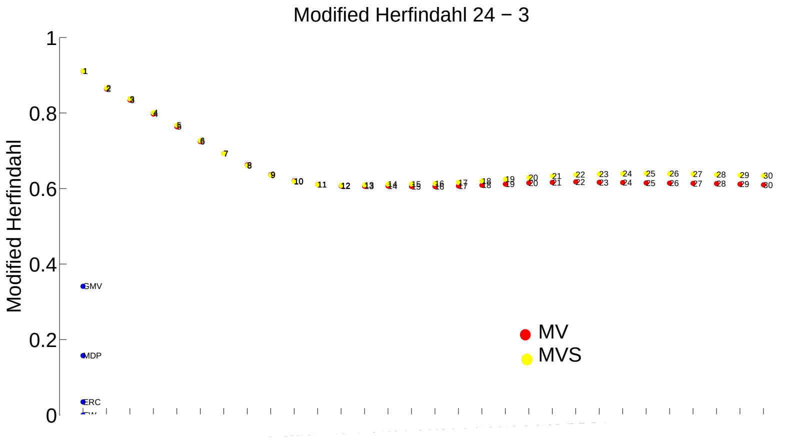
EUT for HFP_1, SR with $R_f = 2\%$ on annual.



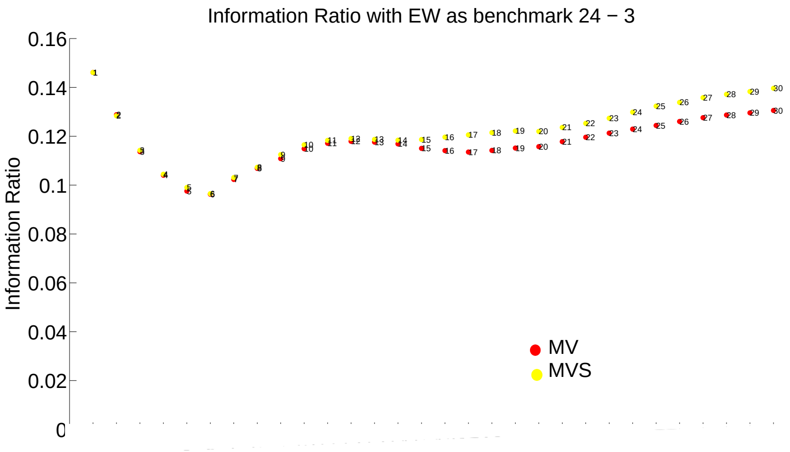
EUT for HFP_1, EVaR with $R_f = 2\%$ and$\alpha = 5\%$ on annual.



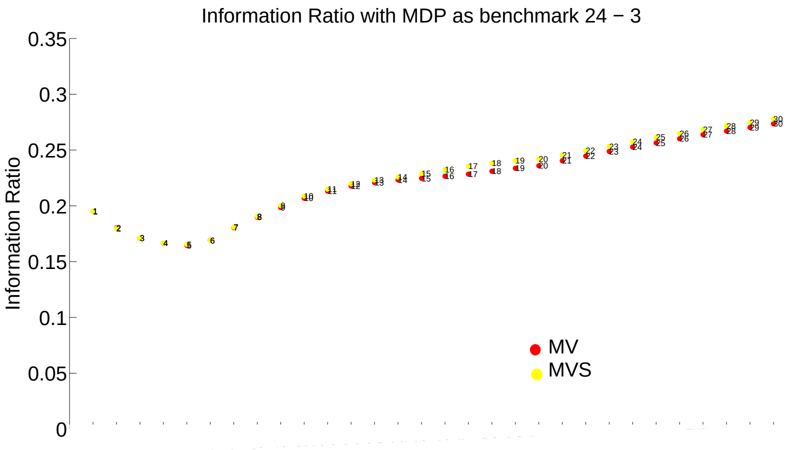
EUT for HFP_1, Modified Herfindahl.



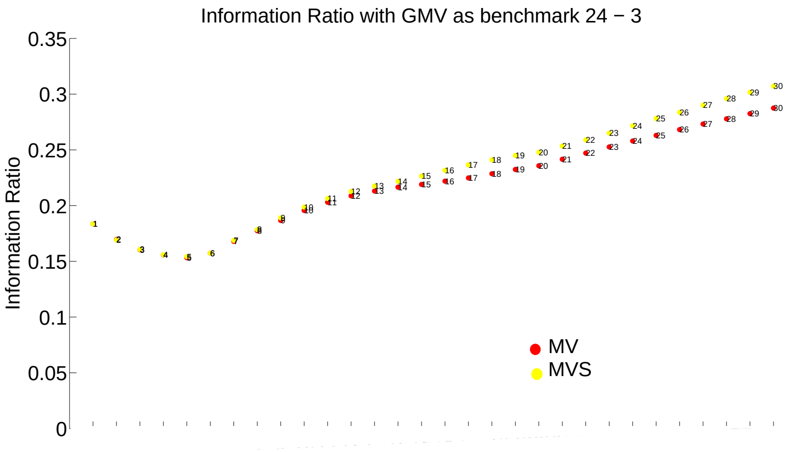
EUT for HFP_1, IR using as benchmark EW



EUT for HFP_1, IR using as benchmark MDP



EUT for HFP_1, IR using as benchmark GMV



Conclusions

Multi Objective approach

- For all the considered portfolios and independently from the rolling window strategy including higher moments decrease the portfolio diversification.

Equity portfolio

- Independently from the rolling window strategy, using the Smart Beta indexes is almost always better than the MV or MVS strategies.

HF portfolios

- There is no clear response to whether higher moments are better than Smart Beta indexes, this is probably due to the algorithm used in the optimization procedure. We are still working on this problem.

Conclusions

Multi Objective approach

- For all the considered portfolios and independently from the rolling window strategy including higher moments decrease the portfolio diversification.

Equity portfolio

- Independently from the rolling window strategy, using the Smart Beta indexes is almost always better than the MV or MVS strategies.

HF portfolios

- There is no clear response to whether higher moments are better than Smart Beta indexes, this is probably due to the algorithm used in the optimization procedure. We are still working on this problem.

Conclusions (cont...)

Expected Utility approach

- For Equity and Hedge Fund the portfolio is highly concentrated for low levels of risk aversion.
- For Equity portfolio, Higher moments are better than Smart Beta indexes when the in-sample-period is short. In this case almost always introducing 'higher moments' gives better results for low values of risk averse parameter λ.
- For Equity portfolio, Smart Beta indexes are better than Higher moments when the in-sample-period is long. In this case the higher the risk averse parameter λ the lower is the information ratio with respect to Smart Beta indexes.
- For HF portfolio,Higher moments are always better than Smart Beta indexes independently from the rolling window strategy, but the information ratio obtained when short in-sample period is used is higher than that obtained in case of long-in-sample period.

Conclusions (cont...)

Expected Utility approach

- For Equity and Hedge Fund the portfolio is highly concentrated for low levels of risk aversion.
- For Equity portfolio, Higher moments are better than Smart Beta indexes when the in-sample-period is short. In this case almost always introducing 'higher moments' gives better results for low values of risk averse parameter λ.
- For Equity portfolio, Smart Beta indexes are better than Higher moments when the in-sample-period is long. In this case the higher the risk averse parameter λ the lower is the information ratio with respect to Smart Beta indexes.
- For HF portfolio,Higher moments are always better than Smart Beta indexes independently from the rolling window strategy, but the information ratio obtained when short in-sample period is used is higher than that obtained in case of long-in-sample period.

Conclusions (cont...)

Expected Utility approach

- For Equity and Hedge Fund the portfolio is highly concentrated for low levels of risk aversion.
- For Equity portfolio, Higher moments are better than Smart Beta indexes when the in-sample-period is short. In this case almost always introducing 'higher moments' gives better results for low values of risk averse parameter λ.
- For Equity portfolio, Smart Beta indexes are better than Higher moments when the in-sample-period is long. In this case the higher the risk averse parameter λ the lower is the information ratio with respect to Smart Beta indexes.
- For HF portfolio,Higher moments are always better than Smart Beta indexes independently from the rolling window strategy, but the information ratio obtained when short in-sample period is used is higher than that obtained in case of long-in-sample period.

Thank You!

References

- Athayde G. and R. G. Flores (2002). The portfolio frontier with higher moments: The undiscovered country. Computing in Economics and Finance, Society for Computational Economics.
- Choueifaty, Y. and Coignard, Y. (2008). Towards maximum diversification. Journal of Portfolio Management, 35(1):pp. 40 / 51.
- Davies, R., M. K. Harry, and L.Sa, (2009). Fund of hedge funds portfolio selection: A multiple-objective approach. Journal of Derivatives & Hedge Funds.15:91 / 115, 2009.
- DeMiguel, V., L. Garlappi, and R. Uppal (2009). Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy?. In: Review of Financial Studies 22, pp. 1915/1953.
- Elton, E., and M. Gruber, (1973). Estimating the dependence structure of share prices - Implications for portfolio selection. The Journal of Finance, 28, 5, 1203 / 1232.

References

- Jorion, P., (1986). Bayes-Stein estimation for portfolio analysis. The Journal of Financial and Quantitative Analysis, 21, 3, 272 / 292.
- Kahneman, D. and A. Tversky, 1979. Prospect theory: An analysis of decisions under risk. Econometrica, pages 263 / 291.
- Ledoit, O. and M. Wolf, (2003). Improved estimation of the covariance matrix of stock returns with an application to portfolio. Journal of Empirical Finance, 10, 5, 603 / 621.
- Hitaj, A. and L. Mercuri, (2013). Portfolio allocation using multivariate variance gamma models. Financial Markets and Portfolio Management, 27(1):65 / 99.
- Jondeau,E., S. Poon., and M. Rockinger, (2007). Financial modeling under non-gaussian distributions. Springer Finance. Springer.

References

- Maillard, S., Roncalli, T., and Teiletche, J. (2010). On the properties of equally-weighted risk contributions portfolios. Journal of Portfolio Management,36(4):pp. 60 / 70.
- Markowitz, H. (1952). Portfolio selection. The Journal of Finance, 7(1):77 / 91.
- Markowitz, H. (1959). Portfolio Selection Efficient Diversification of Investments. Wiley. Wiley & Sons.
- Martellini, L., and V. Ziemann, (2010). Improved estimates of higher-order comoments and implications for portfolio selection. Review of Financial Studies, 23, 4, 1467 / 1502.