Recent trends in harmonic and complex analysis

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Université d'Orléans, Mathématiques

Book of Abstracts

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A limiting case for the divergence equation and related problems

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Let $d \geq 2$, $\Omega \subset R^d$ be a smooth bounded domain and $f \in L^d(\Omega)$
with $\int_R f(x)dx = 0$. Bourgain
and Brezis proved that there exists a vector field $X \in W^{1;d}(\Omega) \cap L^\infty(\Omega)$
such that $divX = f$ and
$||f||_{W^{1;d}} + ||f||_{L^\infty} \leq C||f||_{L^d}$.
We will discuss various extensions of this result to more general functions
spaces, and present some related inequalities. This talk is based on results obtained in collaboration with P. Bousquet, P. Mironescu, Y. Wang and P. L. Yung.

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A useful black box: the theory of weighted Sobolev spaces

Author: Petru Mironescu[1]

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The smoothness of a function f is described by the rate of convergence of the smoothings of f to f. Another way to measure smoothness is given by the decay properties of the derivatives of these smoothings: this is encoded by the theory of weighted Sobolev spaces, developed in the 60s.
In the first part of the lecture, we will recall some striking results of this theory, with focus on Sobolev and Besov spaces.
We will next explain how this can be used as a starting point for deriving standard properties of function spaces, such as the Gagliardo-Nirenberg inequalities or the functional calculus.
The final part will be devoted to the role of the theory of weighted Sobolev spaces in the study of geometric quantities like the degree or the Jacobian. In this part, we will show how a combination of analytic and geometric arguments leads to estimates for topological invariants. We will also explain the factorization of unimodular maps, whose proof is based in part on such arguments.
The global flavor is that weighted spaces and elementary additional ingredients, possibly as simple as integration by parts, can be very effective.
The lecture will be elementary and hopefully at a master level. It is based on a series of results of Uspenski'i, Maz'ya, Bourgain, Brezis, Nguyen, Russ, and the lecturer.

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Affine measures in harmonic analysis: geometric interpretations

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Affine measures have been introduced in the past to facilitate the study of Fourier Restriction and the related question of the L^p smoothing properties of averages along submanifolds (convolution Radon transforms). They capture in a geometric way the role of curvature. In this talk we present the Affine Measures and then discuss the geometric interpretation of these objects - a line of research that started with a result of D. Oberlin relating such measures to a Hausdorff-like ambient measure. We discuss some new results in the same spirit (this is joint work w/ J. Hickman): in particular,

we give a geometric interpretation for the case of hypersurfaces with vanishing curvature. If time allows, we discuss how one could move on to study certain non-translation-invariant cases.

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Asymptotic behaviour of powers of composition operators

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We study the asymptotic behaviour of the powers T^n of a continuous composition operator T on an arbitrary Banach space X of holomorphic functions on the open unit disc of the complex plane. We show that for composition operators, one has the following dichotomy: either the powers converge uniformly or they do not converge even strongly. We also show that uniform convergence of the powers of an operator $T \in L(X)$ is very much related to the behaviour of the poles of the resolvent of T on the unit circle and that all poles of the resolvent of the composition operator T on X are algebraically simple. Our results are applied to study the asymptotic behaviour of semigroups of composition operators associated with holomorphic semiflows.

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BMO and Calderon-Zygmund operators: a sparse approach

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Commutators and paraproducts beyond Calderón-Zygmund theory

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Commutators, Calderon-Zygmund Operators, and BMO

Lecture 3

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Commutators, Calderon-Zygmund Operators, and BMO

Lecture 2

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Commutators, Calderon-Zygmund Operators, and BMO

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Lecture 1: An important theorem in harmonic analysis connects the commutator of multiplication by a function and Calderon-Zygmund operators and the functions of bounded mean oscillation. And as a dual statement, it connects the Hardy space with a certain "factorization" of Lebesgue spaces. During these lectures we will give proofs of these theorems using tools from dyadic harmonic analysis. Connections with analytic function theory will be provided and extensions to the case of weighted estimates and to the case of multi-parameter harmonic analysis will also be discussed.

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Density of translates in weighted L^p spaces on locally compact groups

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Let G be a locally compact group, and let $1 \leq p < \infty$. Consider the weighted L^p-space $L^p(G,\omega) = \{f : \int |f\omega|^p < \infty\}$, where $\omega : G \to$ R is a positive measurable function. Under appropriate conditions on ω, G acts on $L^p(G,\omega)$ by translations. When is this action hypercyclic, that is, there is a function in this space such that the set of all its translations is dense in $L^p(G,\omega)$? H.Salas (1995) gave a criterion of hypercyclicity in the case $G =$ Z . Under mild assumptions, we present a corresponding characterization for a general locally compact group G. Our results are obtained in a more general setting when the translations only by a subset $S \subset G$ are considered.
Joint work with E. Abakumov (Paris-Est).

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Interpolation et échantillonage multiple dans l'espace de Fock

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Nous étudions les ensembles d'interpolation, d'unicité et d'échantillonnage multiple pour les espaces de Fock classiques dans le cas où la multiplicité est non bornée. Nous montrons, dans le cas hilbertien ainsi que celui de la norme uniforme, qu'il n'y a pas de suites simultanément d'échantillonnage et d'interpolation lorsque la multiplicité tend vers l'infini. Ceci répond partiellement à une question posée par Brekke et Seip. Travail conjoint avec A. Borichev, A.Hartmann et X.Massaneda

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Lower bounds for the Hilbert transform

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In a recent paper Alaifari, Pierce & S. Steinerberger conjectured a lower bound for the Hilbert transform H of the form
$||\mathrm{Hf}||_{L^2(J)} \geq \exp(-c_{I,J}||f'||_1/||f||_2)||f||_{L^2(I)}$

when I, J are disjoint intervals and $f \in L^2$, $f' \in L^1$.

The aim of this talk is to present the motivation of this conjecture as an invitation to study lower bounds for Calderon Zygmund operators.

We will then switch to the dyadic Hilbert transform as a toy model for the above conjecture.
In this case, the problem has been entirely solved in my paper with E. Pozzi and B. Wick [2].

Refs

[1] Alaifari, Pierce & Steinerberger
Lower bounds for the truncated Hilbert transform. Rev. Mat. Iberoamericana 32 (2016), 23–56.)

[2] Ph. Jaming, E. Pozzi & B.D. Wick
Lower bounds for the dyadic Hilbert transform
Annales de la Faculté des Sciences de Toulouse (to appear)
ARXIV : 1605.05511

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Sharp off-diagonal weighted estimates for the Bergman projection.

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In 1978, D. Békollé and A. Bonami characterized the exact range of weights for which the Bergman projection is bounded on weighted Lebesgue spaces. In 2013, S. Pott and M. C. Reguera found the exact dependence of the norm of the Bergman projection on the Békollé-Bonami characteristic of the weight. In this talk, we discuss extension of these results to the upper-triangle case.