Algebraic Topology and Representation Theory – Lille, 26-30 June 2017

ABSTRACTS

Mini-courses

Joseph Chuang (London)
Derived localisation

Localisation of commutative algebras is straightforward and well understood. Noncommutative localisation is more subtle, in part because it is not an exact functor. Andrey Lazarev, Chris Braun and I have been studying the derived localisation of a (not necessarily commutative) algebra A at a subset S; it is a differential graded algebra obtained from A by inverting the elements of S in a universal, homotopy invariant, way.

In my mini-course I will describe our main theorem, that the derived category of the derived localisation is a Bousfield localisation of the derived category of A, and provide several examples and applications. As one particular example, I will explain, following Dave Benson, how the differential graded algebra of chains on the loop space of the Bousfield-Kan p-completion of the classifying space of a finite group G with coefficients in a field k of characteristic p is a derived localisation of the group algebra kG. The main source for the mini-course is arXiv:1505.01146.

Jesper Grodal (Copenhagen)
Endotrivial modules via homotopy theory

For G a finite group and k a field of characteristic p, an endo-trivial module is a kG-module M such that $\mathrm{End}(M)$ is isomorphic to a trivial module plus a projective module. Equivalence classes of such modules form a group under tensor product which identifies with the Picard group of the stable module category.

Endo-trivial modules occur in many parts of representation theory as "almost 1-dimensional modules", and there has been a quest to classify them spanning the last 40 years, starting from the work of Dade in the 70s.

In my mini-course I'll explain a way to calculate this group of endotrivial modules via homotopy theory, based on my recent preprint arXiv:1608.00499. Time permitting, I'll also discuss ongoing further developments.

Talks

Tobias Barthel (Copenhagen)
Ascent for stratifications

Starting with Quillen's work on stratifications for group cohomology, descent techniques have played a fundamental role in the study of the global and local structure of important

categories appearing in representation theory and stable homotopy theory. In this talk, we explain a novel approach to such structural phenomena which is based on ascent instead. Our methods provide a different perspective on known results and also apply to many new examples, for instance those coming from homotopical group theory. This is joint work in progress with Castellana, Heard, and Valenzuela.

Emilie Dufresne (Nottingham)
Mapping toric varieties into small dimensional spaces

A smooth d-dimensional projective variety X can always be embedded into $(2d+1)$-dimensional space. In contrast, a singular variety may require an arbitrary large ambient space. If we relax our requirement and ask only that the map is injective, then any d-dimensional projective (resp. affine) variety can be mapped injectively to $(2d+1)$-dimensional projective space (resp. affine). A natural question then arises: what is the minimal m such that a projective variety can be mapped injectively to m-dimensional projective space? In this talk I discuss this question for the affine cones over normal toric varieties, with the most complete results being for the affine cones over Segre-Veronese varieties. (Joint work with Jack Jeffries.)

Jonathan Elmer (London)
Symmetric powers and modular invariants of elementary abelian p-groups

In the 1970's, Almkvist and Fossum discovered formulae which describe the indecomposable summands of arbitrary symmetric powers of modular representations of cyclic groups of order p. We will show how (in spite of the wildness of the representation type) these formulae can be generalised to elementary abelian p-groups. We will also give some applications to invariant theory.

Eric M. Friedlander (Los Angeles)
Cohomology of unipotent algebraic groups

Following my recent construction of support varieties for linear algebraic groups which avoids using cohomology, I have been investigating the cohomology of unipotent algebraic groups. Even in the case of U_3, the Heisenberg group, the computation of cohomology has been very challenging. Much of this talk will focus on methods, results, and remaining challenges for such computations.

John Greenlees (Sheffield)
Morita theory and singularity categories

(Joint with Greg Stevenson.) The main purpose of the talk is to describe how to define the singularity category of $C^*(BG)$ for compact Lie groups G. The definition works for ring spectra with a suitable normalization in a suitable Gorenstein context. Currently few calculational tools are available, but there is the analogue of the BGG correspondence. The talk will give a number of examples: for example if G is a p-group the stable module category of kG is equivalent to $D^b(Proj(C^*(BG))$.

Ellen Henke (Aberdeen)
Normal inclusions of fusion systems and linking systems

The study of saturated fusion systems relates to questions in homotopy theory, the modular representation theory of finite groups, and the group theory surrounding the proof of the classification of finite simple groups. Broto, Levi and Oliver introduced centric linking systems associated to saturated fusion systems to define and study classifying spaces of fusion systems. The longstanding conjecture that there is a unique centric linking system associated to each saturated fusion system was finally proved by Chermak in 2011. Already before that, a rich theory of fusion systems was built up mostly in analogy to the theory of finite groups. In particular, there is a notion of normal subsystems of saturated fusion systems.

After an introduction to the theory, I will report about a theorem proved jointly with Chermak. It implies that, given a normal subsystem E of a saturated fusion system F, there is a canonical way to define a functor from a linking system of E to a linking system of F. It seems that this functor should be regarded as an "inclusion map".

Radha Kessar (London)
On Morita equivalences with endopermutation source

Morita equivalences with endopermutation source are a special and useful type of categorical equivalence in the modular representation theory of finite groups, connecting algebra theoretic and p-local group theoretic information. I will give an introduction to the topic and will report on ongoing joint work with Robert Boltje and Markus Linckelmann.

Nick Kuhn (Charlottesville)
Split Hopf algebras and the cohomology of the James construction

Everyone knows the algebra structure of the homology of JX, but how about the cohomology algebra structure? For example, if X is infinite complex projective space, is the cohomology of JX polynomial? Other questions: When do JX and JY have isomorphic mod p cohomology as algebras? as Hopf algebras?

Pondering these questions, or rather the dual versions about homology, leads to questions about graded connected cocommutative Hopf algebras in characteristic p, that are free as algebras. The Hopf algebras I am interested in turn out to have an additional property, that I term "split", and I can completely classify these, giving definitive answers to questions like those above. I make use of the "non-commutative Witt vectors" of Goerss-Lannes-Morel. There is a parallel classification in the bi-commutative case, that also appears to be new.

Caroline Lassueur (Kaiserslautern)
Endo-trivial modules via character theory: a short survey

Endo-trivial kG-modules were first defined independently by Alperin and Dade in 1978 (where kG is the group algebra of a finite group G). By definition they are modules whose k-endomorphism ring is trivial modulo projectives. Dade classified them for abelian p-groups in his original article. Twenty-five years later, endo-trivial modules were classified

in the case of p-groups, using heavy cohomological machinery. A classification for arbitrary groups is still unknown. However, over the past ten years several new techniques have emerged, which shed some new light on the subject. Contrasting with the methods presented in Jesper's series of talks, the aim of this talk is to give a short survey of the kind of results that can be obtained using character theory.

Nguyen Dang Ho Hai (Hue)
On the eigenvectors of Lannes' T-functor

Let U denote the category of unstable modules over the mod p Steenrod algebra. J. Lannes defined an endofunctor T of U to be the left adjoint to the functor given by taking the tensor product with the mod p cohomology of the classifying space of a cyclic group of order p. The functor T preserves injective objects of U, and thus induces an endomorphism on the Grothendieck group of reduced injective unstable modules.

L. Schwartz conjectures that, over the field of rational numbers, this endomorphism is diagonalizable with eigenvalues the powers of p. In this talk we first show that the Deligne-Lusztig characters of the finite general linear groups can be used to compute the eigenvalues and eigenvectors of T. We then give some remarks on the eigenvectors of T, including the fact that the Poincaré series of an eigenvector associated to the eigenvalue p^k has a pole at 1 of order at most k.

Bob Oliver (Paris)
Fusion systems, classifying spaces, and partial groups

Fix a prime p. The fusion system of a finite group G with respect to a Sylow subgroup S of G is the category $F = F_S(G)$ whose objects are the subgroups of S, and whose morphisms are the homomorphisms induced by conjugation in G. Its orbit category $O(F)$ is the category with the same objects, and where morphisms from P to Q are Q-conjugacy classes of homomorphisms in F.

A subgroup P of S is centric in F if for each $Q \leq S$ that is G-conjugate to P, $C_S(Q) = Z(Q)$. Let $O(F^c)$ be the full subcategory of $O(F)$ with objects the subgroups centric in F. The p-completion of the classifying space BG can be recovered from F by choosing a functor $O(F^c) \to \mathit{Top}$ that lifts the homotopy functor $(P \mapsto BP)$, and then taking the p-completion of its homotopy colimit. By a recent theorem of Chermak, Glauberman, and Lynd, this is independent of the choice of lifting.

All of these structures and constructions can be carried out in a more abstract setting. Abstract fusion systems were originally defined by Puig, motivated by problems in representation theory, but they also have topological interest because of the properties of their classifying spaces, and are of interest in finite group theory because of connections with parts of the proof of the classification of finite simple groups. This last connection comes via "partial groups" as defined by Chermak: sets with multiplication defined only on certain n-tuples of elements, satisfying axioms analogous to those for a group, and which can be associated to fusion systems in a way closely related to the association with classifying spaces described above.

I want to explain these connections during the talk, and hopefully will have time to

illustrate them by 1-2 examples.

Nicolas Ricka (Detroit)
Study of the stable category of modules over graded connected Hopf algebras via descent

Let A be a connected graded Hopf algebra over a finite field of characteristic p. In this talk, I will show how to use descent techniques to answer questions motivated by algebraic topology in the stable category of A-modules. In particular, I will set up a spectral sequence to compute the stable Picard group of such a category. Motivated by the study of local Picard group of the stable category of spectra, P. Bhattacharya and I use these techniques to compute the stable Picard group of the subalgebra $A(2)$ of the modulo 2 Steenrod algebra. Finally, I will discuss the more difficult problem of constructing a stable A-module with prescribed restriction to subalgebras of A.

Beren Sanders (Copenhagen)
The Wirthmüller isomorphism and the compactness locus of a geometric functor

In many equivariant settings, restricting from a finite group G to a subgroup H is often "ambidextrous" in the sense that the restriction functor's left adjoint (induction) is isomorphic to its right adjoint (coinduction). When one considers non-finite groups things become a bit more interesting: Often one still has an isomorphism between induction and coinduction, but only up to a twist by a special invertible object. A prominent example is the so-called "Wirthmüller isomorphism" in equivariant stable homotopy theory. In this talk, I will begin by discussing some old results (joint with Paul Balmer and Ivo Dell'Ambrogio) which clarify the nature of these isomorphisms between adjoints, before moving on to some more recent work related to these ideas. We will work at the level of tensor triangulated categories and may have a few words to say about the spectrum (in the sense of Balmer) of such categories.

Radu Stancu (Amiens)
Projective dimensions of Mackey functors

In this talk I present a joint work with Serge Bouc and Peter Webb in which we examine the projective dimensions of Mackey functors and cohomological Mackey functors for a finite group G, defined over a field of characteristic p. The results for the two families of functors are of quite different nature. The cohomological Mackey functors are Gorenstein if and only if a Sylow p-subgroup of G is cyclic or dihedral, and they have finite global dimension if and only if the order of G is prime to p or a Sylow subgroup of G is cyclic of order 2. By contrast, the Mackey functors of finite projective dimension are projective.

Jan Šťovíček (Prague)
On the telescope conjecture for compactly generated triangulated categories

The telescope conjecture originated in the study of the stable homotopy category of spectra, but it is in fact a general question about localisations of compactly generated trian-

gulated categories. As such, it has interpretations in various related areas of mathematics where these are used. I will discuss cases where the conjecture is known to hold or fail: commutative noetherian rings, hereditary rings and rings of weak global dimension one. I will also explain some conceptual ideas to deal with the problem, involving support theory.

Peter Symonds (Manchester)
Stable categories of modules for infinite groups

Joint work with Nadia Mazza. We construct a stable module category for a large class of infinite groups using complete resolutions. This is similar to the construction of Tate cohomology. We then restrict to the group of endotrivial (or invertible) modules, in other words the Picard group. This has been intensely studied for finite groups. We develop enough machinery to allow us to make calculations for certain infinite groups.