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Workshop on derived foliations

Aussois, 16–20 November 2026

Program

The aim of this workshop is to learn the language of derived foliations and applications. The main reference is Toën–Vezzosi's book [TV], complemented by the papers of Alfieri [Alf] and Tomić [Tom1, Tom2].

The program assumes basic algebraic geometry and homological algebra. The elements of derived geometry involved will all be introduced during the workshop.

Time	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
9h00-10h15	Graded-mixed/filtered complexes	Derived stacks	Derived $\mathbb{C}$-analytic geometry	GAGA	Direct images of derived foliations
10h45–12h00	de Rham complex	Derived foliations on stacks	$\mathbb{C}$-analytic derived foliations; integrability	Existence of analytic leaf space	AKSZ for Poisson structures
12h15–16h00	Lunch and discussion				
16h00–17h15	Derived foliations: affine case	Derived enhancements of singular foliations	Free afternoon	Shifted symplectic / Lagrangian structures	X
17h45–19h00	Shifted Poisson structures: affine case	Formal integrability		Poisson structures on stacks	X

List of talks

(1) *Graded mixed and filtered complexes.* Introduce graded complexes, graded mixed complexes, with $\otimes$ and model structure. Introduce filtered complexes, their associated graded objects and completion. The main objective is the equivalence between the categories of graded mixed complexes and complete filtered complexes via Tate realization (Cor. 1.3.4.4).

Refs: [TV, Sections 1.1–1.3].

(2) *de Rham complex.* Recall (connective) commutative dg-algebras, derived tensor products and the cotangent complex, with its relation to (formally) smooth/étale morphisms [PV, §2]. Define the derived de Rham algebra by its universal property and identify its underlying graded algebra with $\mathrm{Sym}_A(\mathbb{L}_{A/k}[1])$.

Refs: [PV, §2], [TV, Section 1.4] or [PTVV, §1].

(3) *Derived foliations: affine case.* Introduce the definition of a derived foliation over $\mathrm{Spec}(A)$, and give examples. Discuss the relation with Lie- or L_∞-algebroids.

Refs: [TV, §2.1.1-2.1.2 and 2.2]

(4) *Shifted Poisson structures: affine case.* Introduce the definition of a shifted Poisson structure on $\mathrm{Spec}(A)$ in terms of polyvectors/operadically [Mel]. Describe the Poisson center [MS, §3] and explain how it can be viewed as a derived foliation. If time permits: coisotropic structures, in particular non-degenerate coisotropic structure on $Z(A) \to A$.

Refs: [Mel], [CPTVV, §1.4.2, 1.4.3] for shifted Poisson structures, [MS, §3], [Tom1, §4.4-4.6] for center and coisotropic structures.

(5) *Derived stacks.* Recollection on DAG (over $\mathbb{Q}$, with affines modelled by connective cdgas), in particular: introduce derived stacks and Artin stacks (examples), quasi-coherent sheaves and cotangent complex (especially [TV, Theorem A.5.0.1]). If time permits: mapping stacks and representability theorem.

Refs: e.g. [PV, §3, 4 (5)] for a short summary, [Toe, §3] for more on derived stacks, [TV, §A.4, A.5] for QCoh and cotangent complex.

(6) *Derived foliations on stacks.* Introduce the global definition of a derived foliation + basic examples and properties. Structure of derived foliations on Artin/DM stacks. If there is time left: one could mention foliated de Rham cohomology/crystals, but we will not need this.

Refs: [PV, §2.1.3-2.1.5, evt. §3.1-3.2].

(7) *Derived enhancements.* Discuss (derived) foliations on smooth schemes that are transversely smooth and rigid, and show that they are formally integrable (Cor. 2.3.4.6). Then discuss when a classical foliation can be enhanced to a transversely smooth and rigid derived foliation (Prop. 2.4.2.2, Cor. 2.4.2.3).

Refs: [PV, §2.3.4, 2.4].

(8) *Formal integrability.* The goal would be to explain how formally around a geometric point of an Artin stack, every almost perfect derived foliation admits a formal leaf space [TV, Cor. 2.3.3.3 and surrounding discussion]. This requires a discussion of (derived foliations on) formal moduli problems.

Refs: [PV, §2.3.1-2.3.3].

(9) *Derived $\mathbb{C}$-analytic geometry.* Review of talks 2 and 5 in the complex analytic setting, starting from holomorphic cdgas as the affines; explain (in the non-derived case) the link with Stein manifolds [Pri, §1.2]. Then discuss the analogues of the cotangent complex/de Rham algebra/derived stacks in this setting.

Refs: [TV, §5.1-5.2], [Pri] (§3.3 describes the link with the approach from [Por]).

(10) *Analytic integrability.* Introduce analytic derived foliations and describe the analytic refinements of the results on transversely smooth and rigid derived foliations from talk 7. Depending on time, it may be useful to review some of the classical results from [Mal] that are used.

Refs: [TV, §5.3-5.4], [Mal].

(11) *GAGA.* Review the GAGA theorem for almost perfect complexes [Por] and discuss the GAGA theorem for derived foliations [TV, Theorem 5.5.0.4].

Refs: [TV, §5.5], [Por, §7].

(12) *Analytic leaf spaces.* Discuss the existence theorem for analytic leaf spaces (Thm. 6.2.2.1).
(If time permits: one may also wish to discuss the definition of the monodromic leaf space.)

Refs: [TV, §6.1-6.2 (evt. 6.3)].

(13) *Shifted symplectic / Lagrangian structures.* Review of shifted symplectic and Lagrangian structures from [PTVV]. In particular, discuss Lagrangian intersections (Thm. 2.9) and, if time permits, the AKSZ theorem (Thm. 2.5)

Refs: [PTVV, §1.2, 2.1, 2.2].

(14) *Shifted Poisson structures.* Explain the formal localisation procedure from [CPTVV], and discuss how this can be used to define shifted Poisson structures on Artin stacks.

Refs: [CPTVV, §2, 3.1].

(15) *Direct images of derived foliations.* Discuss the model for derived foliations in terms of equivariant linear stacks over the graded loop stack, and (using this) the mapping theorem for derived foliations on mapping stacks.

Refs: [Alf], [TV, §2.5] for a short summary.

(16) *AKSZ for Poisson structures.* Discuss how every shifted Poisson structure admits a (formal) shifted Lagrangian thickening, and if time permits, how this can be used to prove an AKSZ theorem for shifted Poisson structures.

Refs: [Tom1] for shifted Lagrangian thickenings (scheme case), [Tom2] for the application to the AKSZ construction.

References

Main

[TV] B. Toën and G. Vezzosi, *Derived Foliations*, arXiv:2305.08212v2.
[Alf] V. Alfieri, *A Mapping Theorem for Derived Foliations*, arXiv:2510.21497v2.
[Tom1] N. Tomić, *Shifted Lagrangian thickenings of shifted Poisson derived schemes*, arXiv:2506.23348v2.
[Tom2] N. Tomić, *AKSZ construction for shifted Poisson structures*, arXiv:2601.04064v3.

More on DAG

[PV] T. Pantev, G. Vezzosi, *Introductory topics in derived algebraic geometry.* link.
[Toe] B. Toën, *Derived algebraic geometry.* arXiv:1401.1044.

Complex analytic setting

[Mal] B. Malgrange, *Frobenius avec singularités II.* Inventiones 39 (1977).
[Por] M. Porta, GAGA theorems in derived complex geometry. arXiv:1506.09042.
[Pri] J.P. Pridham, *A differential graded model for derived analytic geometry.* arXiv:1805.08538.

Shifted Poisson/symplectic structures

[CPTVV] D. Calaque, T. Pantev, B. Toën, M. Vaquié and G. Vezzosi, *Shifted Poisson structures and deformation quantization.* arXiv:1506.03699.
[Mel] V. Melani, *Poisson bivectors and Poisson brackets on affine derived stacks.* arXiv:1409.1863.
[MS] V. Melani, P. Safronov, *Derived coisotropic structures I: affine case.* arXiv:1608.01482.
[PTVV] T. Pantev, B. Toën, M. Vaquié and G. Vezzosi, *Shifted symplectic structures.* arXiv:1111.3209.