

Averaging techniques and uniformly accurate numerical approximations of Vlasov equations with strong magnetic field.

lecture by Mohammed LEMOU (CNRS IRMAR, Rennes)

Abstract:

Part I: Averaging techniques for highly oscillatory transport equations:

We will first present some methods in the literature which are designed to derive asymptotic models from Vlasov equations with strong magnetic field. Then we come to the first goal of this first part, which is to present a new strategy that allows a systematic derivation of high order averaging models with a complete recovery of the distribution function. The method will be presented in the general context of highly oscillatory transport equations: more precisely, we will show that a combination of standard averaging techniques with an equation on a suitably chosen oscillation phase, allows one to achieve this goal. We finally show how the methodology can be applied, not only to the case of Vlasov equation with constant magnetic field, but also to the case where this field has a non trivial space dependence. A particular attention will be paid to gyrokinetic asymptotics of Vlasov equation.

Part II: Uniformly accurate numerical approximations of Vlasov equation with strong magnetic field:

We first present the usual strategy to get numerical schemes having the so called asymptotic preserving property. In the strong magnetic field limit, these approximations degenerate into numerical schemes which are consistent with the averaged models. We then show how to go beyond this property and get numerical schemes with uniform accuracy. In particular a suitable two-scale reformulation will be presented in detail, which is at the heart of the construction of uniformly accurate numerical schemes. In this way, one may capture oscillations in the system without resolving the high frequencies, and in particular the accuracy of these numerical schemes is uniform with respect to the oscillation frequency. Some numerical experiments will finally be presented to illustrate the efficiency of the above strategy, mainly in the framework of a gyrokinetic scaling of kinetic equations.