Binary Boson Stars in Strong Field: Post-Minkowskian, Numerical Relativity and Effective-one-body formalism

Tamanna Jain
LabEx Junior Research Chair, LPENS, Paris
Junior Research Fellow, King's College, University of Cambridge

arxiv:2512.00945 with T Damour and U Sperhake

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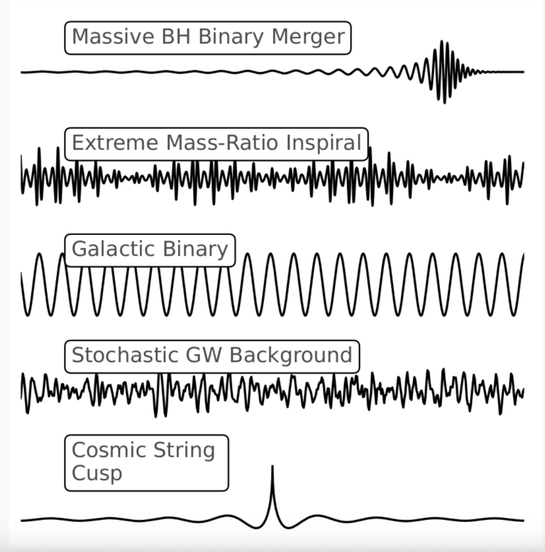
Goals

Gravitational Wave (GW) observations $\rightarrow$ Probe fundamental physics

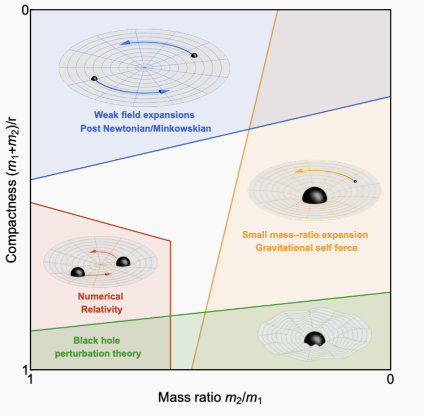
Key ingredient for GW physics $\rightarrow$ waveform templates (numerical+analytical) and parameter estimation (PE) to infer source properties & test beyond GR+SM

Gravitational Waveforms



LISA definition study report 2023

Different techniques depending on the region of the parameter space.



L Bernard's Talk

Two broad categories - Analytical Relativity (AR) and Numerical Relativity (NR)

"Ingredients"

- Theoretical Model: Einstein's Equations in 4-dimensions
- Spacetime decomposition: 3+1 decomposition of field equations
- Initial conditions: Bowen–York, two-punctures, etc
- Gauge choices: 1+log slicing condition, etc
- Extract physical observables: Newman-Penrose scalar Ψ_4 (GW strain), radiated energy, angular momentum, etc

Hence, solve the full Einstein's equations.

Drawbacks: computationally expensive and cover a patch of parameter space. Longevity of signals.

Importance: Help us probe extreme gravity regime

- Exact analytical solutions only for symmetry assumptions such as Schwarzschild, Kerr, etc
- Different approximation techniques:
  - post-Newtonian (PN): point-particle approximation, expansion in $\epsilon = \frac{v^2}{c^2} \sim \frac{GM}{rc^2} << 1$
  - post-Minkowskian (PM): point-particle approximation, expansion in G (weak-gravity)
  - self-force (SF): extreme mass ratio inspiral (EMRI), expansion in $q = \frac{m_2}{m_1} << 1$
- Analytically describes the dynamics of the system and physical observables such as waveform, radiated energy, etc

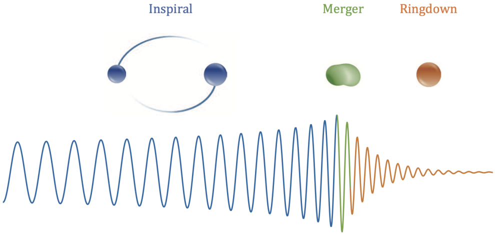
Hence, analytical solutions are only known perturbatively.

The different methods: AR

Drawbacks: limited regime of validity, slow and oscillating convergence

Importance: physical understanding of binary dynamics, need many thousands of templates for detection+data analysis

How to construct bank of waveform templates?

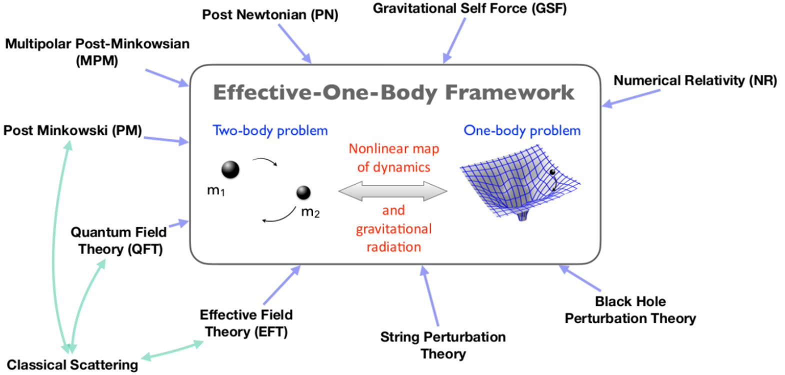


Antelis & Moreno 2017, arXiv:1610.03567

Solution: Synergy between AR and NR such as

- Effective-one-body (EOB) models
- Phenomenological Models (IMRPhenom) - PN for inspiral+NR for merger & ringdown
- Surrogate Models

EOB use a “cocktail” of various schemes



LISA waveform white paper 2023

Motivation:

Motion is not exactly spherical → plunge → PN can not model it effectively.

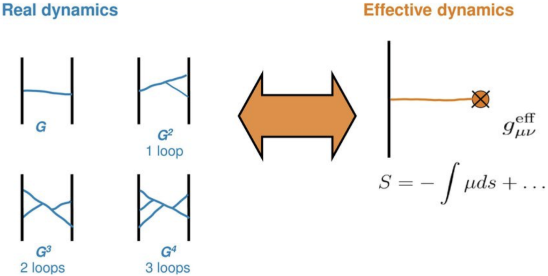
PN an asymptotic series → slow convergence (e.g: consider the test mass around a Schwarzschild BH) → needs resummation (e.g: Padé resummation)

Damour, Iyer, Sathyaprakash (1998, 2000, 2001)

Analytical approach to understand the global structure of the waveform? Or should we rely on NR?

Buonanno-Damour (1999, 2001) proposed EOB → resumms the PN information → extend the regime of validity of results

Dynamics of two bodies $(m_1, m_2) \rightarrow$ dynamics of an effective particle of mass $\mu = \frac{m_1 m_2}{m_1+m_2}$ moving in some effective metric $g^{\text{eff}}_{\mu\nu}$



where ds is the line element of effective metric (spherically symmetric),

$$ds^2 = ds^2_{\text{eff}} = -A(r)dt^2 + B(r)dr^2 + r^2\left(d\theta^2 + d\phi^2\right) \ .$$

All the information contained in the Hamiltonian of the real 2-body system is encoded in the metric potentials $A(r),\ B(r)$.

Why scattering?

- Constructing waveform templates for bound orbits require analytical+numerical information for inspiral, merger and ringdown.
- State-of-the-art of PN in GR is 4PN full with partial 5PN and 6PN results ($\approx$40years)$\rightarrow$ Is there an alternate analyitcal method? PM expansion (suitable for scattering)
- Scattering configurations are cleaner and offer direct comparison between Analytics-NR
- Direct map between unbound and bound orbits using EOB by computing effective gravitational potentials $\rightarrow$ generate robust analytical waveform templates
- The NR vs analytical comparison has been well tested for black holes, i.e. the point particle approximation Damour and Rettegno (2022)
- Can we also test the effect due to *tidal interactions? Can we use compact objects other than neutron stars to measure the effect?*

- Increased interest in beyond GR+SM models $\rightarrow$ exotic compact objects $\rightarrow$ Boson stars
- Well explored numerically, simpler to evolve than neutron stars and tidally deformable $\rightarrow$ ideal system to measure tides
- The PM approximation suitable for describing scattering systems $\rightarrow$ help model GW models from binary signals (un-bound or bound)
- Various techniques to do PM computations such as scattering amplitudes, eikonalization, PM-effective field theory and worldline field theory.
- For point-particle dynamics full results known up to 4PM with partial results at 5PM order and for tidal effects at 6PM and 7PM order.

- The action

$$S_{\text{bulk}} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\bar{\varphi} + V\left(|\varphi|^2\right)\right] ,$$

where $V\left(|\varphi|^2\right)$ is the potential

$$V\left(|\varphi|^2\right) = \frac{\mu^2}{2}|\varphi|^2\left[1 - \frac{2|\varphi|^2}{\sigma_0^2}\right]^2 .$$

with μ bosonic field mass and σ_0 determining the strength of self interactions.

- The ansatz to obtain spherically symmetrical solution of boson star is $\varphi(t,r) = A(r)e^{i(\epsilon\omega t+\Phi)}$, where $A(r)$ is the (real) scalar field amplitude profile, ω is the frequency, and Φ is an initial phase.

Binary Boson Stars: Numerics

- Solve 3+1 equations for the action of boson stars S_{bulk}

$$\partial_t \chi = \beta^m \partial_m \chi + \frac{2}{3}\chi(\alpha K - \partial_m \beta^m)\,, \tag{8}$$

$$\partial_t \tilde{\gamma}_{ij} = \beta^m \partial_m \tilde{\gamma}_{ij} + 2\tilde{\gamma}_{m(i}\partial_{j)}\beta^m - \frac{2}{3}\tilde{\gamma}_{ij}\partial_m \beta^m - 2\alpha \tilde{A}_{ij}\,, \tag{9}$$

$$\partial_t K = \beta^m \partial_m K - \chi \tilde{\gamma}^{mn} D_m D_n \alpha + \alpha \tilde{A}^{mn}\tilde{A}_{mn} + \frac{1}{3}\alpha K^2 + 4\pi G \alpha (S + \rho)\,, \tag{10}$$

$$\begin{aligned} \partial_t \tilde{A}_{ij} = {} & \beta^m \partial_m \tilde{A}_{ij} + 2\tilde{A}_{m(i}\partial_{j)}\beta^m - \frac{2}{3}\tilde{A}_{ij}\partial_m \beta^m + \alpha K \tilde{A}_{ij} - 2\alpha \tilde{A}_{im}\tilde{A}^m{}_j \\ & + \chi(\alpha \mathcal{R}_{ij} - D_i D_j \alpha - 8\pi G \alpha S_{ij})^{\mathrm{TF}}\,, \end{aligned} \tag{11}$$

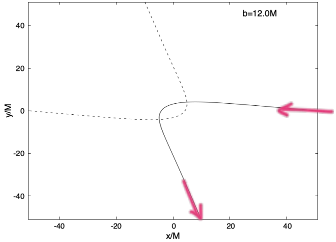
$$\begin{aligned} \partial_t \tilde{\Gamma}^i = {} & \beta^m \partial_m \tilde{\Gamma}^i + \frac{2}{3}\tilde{\Gamma}^i \partial_m \beta^m - \tilde{\Gamma}^m \partial_m \beta^i + \tilde{\gamma}^{mn}\partial_m \partial_n \beta^i + \frac{1}{3}\tilde{\gamma}^{im}\partial_m \partial_n \beta^n \\ & - \tilde{A}^{im}\left(3\alpha \frac{\partial_m \chi}{\chi} + 2\partial_m \alpha\right) + 2\alpha \tilde{\Gamma}^i_{mn}\tilde{A}^{mn} - \frac{4}{3}\alpha \tilde{\gamma}^{im}\partial_m K - 16\pi G \frac{\alpha}{\chi} j^i\,, \end{aligned} \tag{12}$$

$$\partial_t \Pi = \beta^m \partial_m \Pi + \alpha \left[\Pi K + \frac{1}{2}V'\varphi + \frac{1}{4}\tilde{\gamma}^{mn}\left(\partial_m \varphi \partial_n \chi - 2\chi \tilde{D}_m \tilde{D}_n \varphi\right)\right] - \frac{1}{2}\chi \tilde{\gamma}^{mn}\partial_m \varphi \partial_n \alpha\,, \tag{17}$$

$$\partial_t \varphi = \beta^m \partial_m \varphi - 2\alpha \Pi\,,$$

- Initial data construction: Plain superposition of individual solutions of two boson stars to construct BS binary system metric ($\gamma_{ij} = \gamma^A_{ij} + \gamma^B_{ij} - \delta ij$) leads to larger constraint violations
- Need to account for influence of one star onto other $\rightarrow$ improved superposition ($\gamma_{ij} = \gamma^A_{ij} + \gamma^B_{ij} - \gamma^B_{ij}(x^i_A)$) Helfer et al. 2009

- Considered a binary boson star configuration with velocity $= 0.23c$, separation $X = 50.49GM$, initial phase $\Phi = 0$ and $\epsilon = 1$.
- Extract initial energy ($E_{\rm ADM}$) and angular momentum ($J_{\rm ADM}$) from the NR simulations $\rightarrow$ the data is not constraint satisfied there are numerical errors $\leq 1\%$.
- Extrapolate the trajectories to infinity and extract the numerical scattering angle



- The analytic and numerical results differed by ~ 40 degrees when we included *only* tidal effects $\rightarrow$ suggesting *missing* physics

- Let us go back to the action,

$$S_{\text{bulk}} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\bar{\varphi} + V\left(|\varphi|^2\right)\right] .$$

- From faraway, boson stars act as point-like particle with scalar source of strength $c_A e^{i\omega_A\tau_A}$ $\rightarrow$ similar to scalar-tensor theories where the scalar source strength is determined by $m_A(\phi)$ and its derivatives
- However, for boson stars the scalar field is complex, massive and harmonic (oscillatory in time)
- EFT point-particle action

$$S_{pp} = -\sum_A \int \left\{m_A - 2\pi\left[\varphi(\mathbf{z}_A)\bar{s}_A(\tau_A) + \bar{\varphi}(\mathbf{z}_A)s_A(\tau_A)\right]\right\}d\tau_A$$

where τ_A is the proper time. The relation between proper and coordinate time is $d\tau_A = \sqrt{-g_{\mu\nu}\dot{z}^\mu\dot{z}_\nu}d\sigma_A$

- To simplify our computations we want to work in quadratic point particle action.
- Reduce the non-locality of the point-particle action to local action $\rightarrow$ introduce Lagrange-multiplier (S_L) $\rightarrow$ Lagrange-EFT
- New point particle action

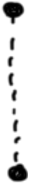
$$S_{pp}^{\rm tot} = S_{pp}^{\rm new}(\theta_A) + S_L \ .$$

with

$$S_{pp}^{\rm new}(\theta_A) = -\sum_A \int \left\{ m_A - 2\pi \left[\varphi(\mathbf{z}_A)\bar{s}_A(\theta_A) + \bar{\varphi}(\mathbf{z}_A) s_A(\theta_A) \right] \right\} e_a \left(\frac{g_{\mu\nu}\dot{z}^\mu \dot{z}_\nu}{e_a^2} + 1 \right) d\sigma_A \ .$$

Here, $e_a = \sqrt{-g_{\mu\nu}\dot{z}^\mu \dot{z}_\nu}$ is the einbein and S_L is the Lagrange action which fixes θ_A

Feynman Rules → world-line vertices, bulk vertices and propagators



- For one scalar-exchange interaction, we obtain

$$\Delta p_{1\,\mathrm{LO}}^{\nu\,\mathrm{scalar}} = -\frac{8\pi\tilde{\mu}c_1c_2}{\sqrt{\gamma^2-1}}K_1\left(\tilde{\mu}|b|\right)\frac{b^\nu}{|b|}\cos(\Phi_{21}) - \frac{8\pi c_1c_2}{\sqrt{\gamma^2-1}}(\omega_1\gamma-\omega_2)\check{u}_2^\mu\sin(\Phi_{21})K_0(\tilde{\mu}|b|)\ .$$

Here, $\check{u}_2^\mu = (u_2^\mu - u_1^\mu\gamma)/(\gamma^2-1)$, $b^\nu = b_1^\nu - b_2^\nu$ is the impact parameter, K_1, K_0 are modified Bessel's functions, $\Phi_{21} = \Phi_2 - \Phi_1$ is the phase difference, and $\widetilde{\mu}^2 \equiv \mu^2 - (2\omega_1\omega_2\gamma - \omega_1^2 - \omega_2^2)/(\gamma^2-1)$.

- Therefore, scalar-exchange at LO is Yukawa-type $\rightarrow$ the important contributions at next order are due gravitationally dressing of scalar exchange. The Feynman diagrams up to $\mathcal{O}(Gc_Ac_B)$ are

- At $\mathcal{O}(Gc_Ac_B)$ no closed analytical solution $\rightarrow$ expansion in small-v and large -b to compute the impulse and hence scattering angle.
- The PM scattering angles showed poor agreement with NR results for small angular momentum (impact parameter) $\rightarrow$ PM-expansion is an expansion of $1/j$ which loses accuracy is strong field systems.
- Ressumation of the PM scattering angles making use of an EOB gravitational potential w. T. Damour and P Rettegno 2023 for binary BHs

$$\pi + \chi(\gamma, j) = 2j \int_0^{\bar{u}_{\max}(\gamma,j)} \frac{d\bar{u}}{\sqrt{p_\infty^2 + w(\bar{u},\gamma) - j^2\bar{u}^2}} \ ,$$

- For our case including scalar and tidal interactions, we decompose

$$w(\gamma,\bar{r}) = w^{\text{BH}}(\gamma,\bar{r}) + w^{\text{tid}}(\gamma,\bar{r}) + w^{\text{scalar}}(\gamma,\bar{r}) \ .$$

- Determine w using the EOB-map relating expansion of w and χ.

- At LO for scalar contributions, the relation is

$$\chi^{\text{scalar}}(j) = -\frac{\partial}{\partial j}\int_{\bar{r}_{\min}}^{\infty} d\bar{r}\,\frac{w^{\text{scalar}}(\bar{r})}{\sqrt{p_\infty^2 - j^2/\bar{r}^2}}\ ,$$

  obtaining

$$w_{\text{LO}}^{\text{scalar}} = \frac{8\pi c_1 c_2}{G m_1 m_2}\frac{e^{-\bar{m}\bar{r}}}{\bar{r}}\cos(\Phi_{21})\ ,$$

  where $\bar{m} \equiv GMh\widetilde{\mu}$.

- For high-orders, we shall assume that the w can be approximated by the factorised form

$$w_{\text{HO}}^{\text{scalar}} = w_{\text{LO}}^{\text{scalar}} w_{\text{g-dressing}}^{\text{scalar}}\ ,$$

- The exponentiated gravitational dressing factor identified from the leading order expansion of $\mathcal{O}(Gc_A c_B)$ is,

$$w_{\text{g-dressing}}^{\text{scalar}} = e^{GM((2\omega^2-\mu^2)/\widetilde{\mu})\log(2e^{\gamma_{\text{E}}}GM\widetilde{\mu}\bar{r})}\ .$$

- For tidal interactions

$$\chi_6^{\text{tid}} = \frac{15\pi}{32} p_\infty^4 w_6^{\text{tid}}\,, \quad \chi_7^{\text{tid}} = 4 p_\infty^3 w_1 w_6^{\text{tid}} + \frac{8}{5} p_\infty^5 w_7^{\text{tid}}\,.$$

- Solve for scattering angle using the EOB potentials to obtain the resummed angles → validate with NR simulations

- Considered a binary boson star configuration with velocity $= 0.23c$, separation $X = 50.49GM$.

ϵ	1			−1
Φ	0	90	180	0

- Considering four different sequences: 3 sequences with phase difference ($\Phi =$) 0, 90 and 180 with $\epsilon = 1$ and one sequence with $\Phi = 0, \epsilon = -1$, i.e. an anti boson star configuration. In anti-boson star, second star is rotating in opposite direction with respect to the first star in complex plane.
- Extrapolate the trajectories to infinity and extract the numerical scattering angle
- Like previously extract energy and angular momentum from NR simulation for input into analytical results.
- The scalar charge c_A required for analytical results is $\sqrt{G}\mu c_A = 1.608$, which is obtained using asymptotic expression for the scalar profile

$$A(r) \approx \frac{c_A}{r} \exp\left[-\bar{\mu}_A r + G m_A \frac{2\omega_A^2 - \mu^2}{\bar{\mu}_A} \log(2e^{\gamma_E}\bar{\mu}_A r)\right] ,$$

with $\bar{\mu}_A = \sqrt{\mu^2 - \omega_A^2}$

TABLE II. Summary of the AR scattering angle for various EOB predictions with impact parameter b. Here, $\delta_\chi = (\chi_{\rm AR} - \chi_{\rm NR})/\Delta\chi_{\rm NR}$ measures the deviation of AR from NR values in units of the NR error estimate.

$\frac{b}{GM}$	$\chi_{\rm AR}^{\rm BBH}$ [°]	$\delta_\chi^{\rm BS\overline{BS}}$	$\delta_\chi^{\frac{\pi}{2}}$	$\chi_{\rm LO,AR}^{\rm scalar}$ [°]				$\chi_{\rm HO,AR}^{\rm scalar}$ [°]				$\chi_{{\rm HO},w_6,{\rm AR}}^{\rm scalar}$ [°]				$\chi_{{\rm HO},w_6+w_7,{\rm AR}}^{\rm scalar}$ [°]			
				BSBS	δ_χ	BSBS$^\pi$	δ_χ^π	BSBS	δ_χ	BSBS$^\pi$	δ_χ^π	BSBS	δ_χ	BSBS$^\pi$	δ_χ^π	BSBS	δ_χ	BSBS$^\pi$	δ_χ^π
9.9	212.44	-2.81	-1.24	-	-	110.41	-69.84	253.68	-5.38	190.44	-6.33	256.24	-2.69	191.12	-5.79	259.32	0.56	191.89	-5.20
10.5	165.62	0.25	0.42	-	-	115.45	-61.04	172.51	2.25	159.78	-1.94	172.83	2.61	160.01	-1.63	173.11	2.93	160.21	-1.36
11.0	143.10	1.30	1.36	371.98	367.86	115.60	-43.50	145.47	2.50	140.90	0.12	145.58	2.69	141.00	0.30	145.67	2.84	141.09	0.45
12.0	114.90	0.12	1.08	124.70	16.06	107.50	-9.93	115.27	1.32	114.53	0.90	115.31	1.37	114.56	0.93	115.32	1.40	114.58	0.96
13.0	97.21	0.38	0.76	99.16	2.70	95.39	-1.73	97.28	0.46	97.14	0.40	97.29	0.47	97.15	0.42	97.30	0.48	97.15	0.42
14.1	83.12	-0.23	-0.19	83.46	0.07	82.76	-0.50	83.12	-0.23	83.10	-0.20	83.12	-0.23	83.10	-0.20	83.13	-0.23	83.10	-0.20
15.0	75.39	-0.24	-0.18	75.50	-0.01	75.28	-0.27	75.39	-0.10	75.39	-0.18	75.40	-0.10	75.39	-0.18	75.40	-0.10	75.39	-0.18
16.0	68.03	-0.28	-0.05	68.06	-0.08	68.01	-0.34	68.03	-0.10	68.03	-0.32	68.03	-0.10	68.03	-0.32	68.04	-0.10	68.03	-0.32

Boson Stars: Analytics vs Numerics

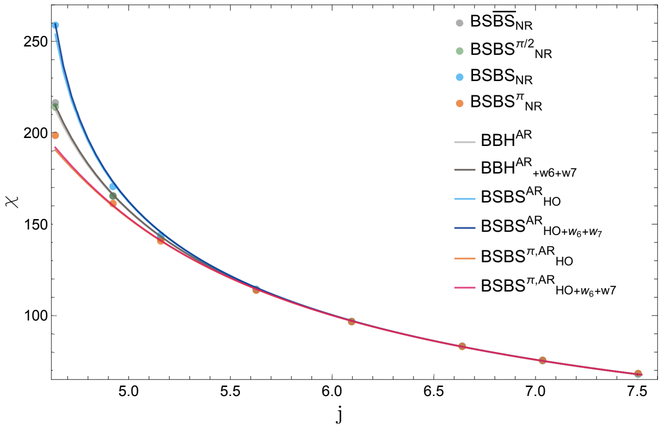


Figure: Scattering angle comparison between the NR data (filled circles) and the EOB-resummed, PM based analytical prediction (AR; solid lines) for equal-mass, non-spinning boson star configurations for various rescaled initial angular momentum ($j = J/(Gm_1m_2)$).

Boson Stars: Analytics vs Numerics

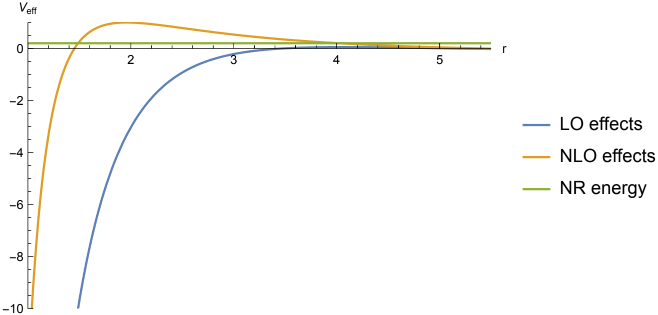


Figure: Effective Potential ($V_{\rm eff}$) vs r.

Subtle but real effect due to tidal interactions

TABLE II. Summary of the analytically predicted scattering angle along with the deviation from NR results including tidal interactions for $BS\overline{BS}$ and for $BSBS^{\frac{\pi}{2}}$ (denoted by $\frac{\pi}{2}$) configurations. For all configurations, the total energy is $E_{\rm in}/M = 1.02394$.

$b/(GM)$	$\chi_{\rm AR}^{\rm BBH+w6}$	$\delta_{\chi}^{\rm BS\overline{BS}}$	$\delta_{\chi}^{\frac{\pi}{2}}$	$\chi_{\rm AR}^{\rm BBH+w6+w7}$	$\delta_{\chi}^{\rm BS\overline{BS}}$	$\delta_{\chi}^{\frac{\pi}{2}}$
9.9	213.55	-2.03	-0.45	214.78	-1.16	0.43
10.5	165.89	0.58	0.75	166.13	0.87	1.04
11.0	143.21	1.49	1.55	143.30	1.64	1.70
12.0	114.93	1.16	1.14	114.95	1.20	1.18
13.0	97.22	0.39	0.78	97.22	0.40	0.79
14.1	83.12	-0.23	-0.19	83.11	-0.23	-0.19
15.0	75.39	-0.24	-0.18	75.39	-0.24	-0.17
16.0	68.03	-0.28	-0.06	68.03	-0.28	-0.06

Key results

- Treatment of binary boson stars as an EFT $\rightarrow$ exact LO scattering angle due to one scalar exchange
- Inclusion of gravitational dressing effects due to scalar-graviton interactions
- Determination of the scalar charge c_A
- Our results indicate that scalar effects for boson stars contribute more significantly to the dynamics than the tidal interactions.
- First EOB potential for constructing analytical waveform templates
- We also observed collapse of BSs into BHs after their close encounter and before separating to infinity $\rightarrow$ potentially signify role of time-dependent scalar perturbations

- On analytical side: Compute the full 2PM results, inclusion of the radiation effects due to recoil in the EOB potential, deriving the waveform, compute gravito-magnetic tidal interactions, map this EOB potential to bound orbits to generate waveform templates
- On numerical side: Improved constraint solver for better extraction of $E/J_{\rm adm}$, span the parameter space, i.e. fluffier models, unequal mass

Thank You!