Autour de quelques idées de Jean-Claude

Journées scientifiques en l'honneur de Jean-Claude Latché

Marseille, 10-11 Juin 2026

Première rencontre, CMI, cerca 2000

Context: "Finite volume methods", HNA, by EGH finished in 1997, published in 2000.
Extension to incompressible Stokes and Navier-Stokes

$$\partial_t u_i - \nu \Delta u_i + \sum_{j=1}^{d} u_j \partial_j u_i = f_i, \quad i = 1, \ldots, d,$$

$$\mathrm{div}\, \boldsymbol{u} = 0.$$

for a co-located scheme with stabilisation.

EH	Stokes	CRAS, Ser. I 337 125–128, 2003
EHL	Stokes	*ESAIM: M2AN* 40:501 - 527, 2006
EHL	NS	*SIAM J. Numer. Anal.* 46:1006–1051, 2007
EHLP	NS	*ESAIM: M2AN* 43 (2009) 889-927

Ph.D advisor of many students (HDR JCL 2010)

- **Laura Gastaldo** ASNR 29/11/2007
  Méthodes de correction de pression pour les écoulements compressibles : application aux équations de Navier–Stokes barotropes et au modèle de dérive
- **Guillaume Ansanay-Alex** Fortuneo 17/06/2009
  Un schéma éléments finis non conformes / volumes finis pour l'approximation sur maillages non structurés des écoulements à faible nombre de Mach
- **Walid Kheriji** VEDECOM 28/11/2011
  Méthodes de correction de pression pour les équations de Navier–Stokes compressibles
- **Tan Trung Nguyen** Vietnam 12/02/2013
  Schémas numériques explicites à mailles décalées pour le calcul d'écoulements compressibles
- **Chady Zaza** ANSYS (Synopsis) (avec Ph. Angot et M. Belliard) 02/02/2015
  Contribution à la résolution numérique d'écoulements à tout nombre de Mach et au couplage fluide–poreux en vue de la simulation d'écoulements diphasiques homogénéisés dans les composants nucléaires
- **Nicolas Therme** (avec L. Gastaldo) CEA 10/12/2015
  Schémas numériques pour la simulation de l'explosion
- **Khadidja Mallem** ass.prof. Algérie 15/12/2015
  Convergence du schéma Marker-and-Cell pour les équations de Navier-Stokes incompressible
- **Dionysios Grapsas** Software Engineed, ESI group Lyon. 11/12/2017
  Schémas à mailles décalées pour des modèles d'écoulements compressibles réactifs
- **Youssouf Nasseri** postdoc Strasbourg 29/03/2021
  Analyse numérique de schémas volumes finis à mailles décalées pour certains systèmes hyperboliques issus de la mécanique des fluides
- **Aubin Brunel** ASNR 12/12/2022
  Schémas à mailles décalées sur maillages généraux pour les écoulements incompressibles et compressibles
- **Thomas Harbreteau** 3ème année de thèse 2026
  Schémas pour le calcul haute-performance des écoulements compressibles

Chateau-Gombert ou St Charles

Soutenance de thèse d'Aubin, décembre 2022

L'escapade

L'écomotive

Fundamental equations

Compressible Euler / Navier-Stokes Equations

$\rho, e, p : \Omega \times (0, T) \to \mathbb{R}$, $\boldsymbol{u} : \Omega \times (0, T) \to \mathbb{R}^d$

$$
\begin{aligned}
&\partial_t \rho + \mathrm{div}(\rho \boldsymbol{u}) = 0 \\
&\partial_t (\rho \boldsymbol{u}) + \mathrm{div}(\rho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p = \mathrm{div}(\tau(\boldsymbol{u})) \\
&\partial_t (\rho E) + \mathrm{div}\big((\rho E + p)\boldsymbol{u}\big) = \mathrm{div}(\tau(\boldsymbol{u}) \cdot \boldsymbol{u}) - \mathrm{div}\, \boldsymbol{q} \\
&E = \tfrac{1}{2}|\boldsymbol{u}|^2 + e, \qquad p = p(\rho, e)
\end{aligned}
$$

+ BCs + ICs

Key Properties to be preserved by the schemes

- $\rho > 0$, $e > 0$
- Total energy conservation: $\displaystyle \frac{d}{dt} \int_\Omega \Big(\tfrac{1}{2}\rho |\boldsymbol{u}|^2 + \rho e \Big)\, dx = 0$
- Entropy inequality (Euler, perfect gas): $\partial_t \eta(\rho, e) + \mathrm{div}[\eta(\rho, e)\boldsymbol{u}] \le 0$

Goal

Discrete analogues + Lax–Wendroff consistency

The Chef's Choices and the Resulting Menus

The Choices

- staggered meshes
- internal energy formulation
- no Riemann solver
- structure preserving schemes
- explicit or (semi-)implicit schemes

Scheme Properties

- Lax-Wendroff consistency for hyperbolic problems
- Positivity preservation $\rho \geq 0$, $e \geq 0$ by upwinding w.r.t. material velocity
- Simple fluxes, no Riemann solver needed $\Rightarrow$ Easy extension to general hyperbolic systems
- Unconditional stability and Low-Mach consistency for implicit or semi-implicit schemes
  - Pressure correction $\rightarrow$ standard stable schemes as $\mathrm{Ma} \rightarrow 0$
  - Proven rigorously (barotropic case) [Saleh *et al* 2018]
  - Numerical diffusion $\sim$ material velocity, not wave celerity

About the goal $\rho > 0$ and $e > 0$

Mass balance

- Mass balance: $\partial_t \rho + \mathrm{div}(\rho \boldsymbol{u}) = 0$
  If $\rho_0 > 0$, then $\rho > 0$
- Discrete setting ?
  OK for finite volume scheme with upwind choice.

Internal energy balance

- Convection operator $\mathcal{C}(e) = \partial_t(\rho e) + \mathrm{div}(\rho e \boldsymbol{u}) = 0$
  If $\rho_0 e_0 > 0$, then $\rho e > 0 \rightsquigarrow e > 0$.
- Internal energy balance $\mathcal{C}(e) + p\,\mathrm{div}\,\boldsymbol{u} = 0$
  Assuming $p = \mathfrak{p}(\rho, e)$, with $\mathfrak{p}(\rho, e) = 0$ if $\rho \leq 0$ or $e \leq 0$,
  if $\rho_0 e_0 > 0$ $p = 0$ when $e = 0$, then $e > 0$.
- Discrete level : OK for finite volume, upwind choice

Internal energy

Internal energy for Euler equations

$$\partial_t(\rho e) + \mathrm{div}\big((\rho e)\boldsymbol{u}\big) + \underbrace{p\ \mathrm{div}\ \boldsymbol{u}}_{\text{non-conservative term}} = 0$$

- Toro: schemes for non-conservative equations yield wrong shock speeds and wrong weak solutions.
- Possible cure: add a suitable non-conservative diffusion (Gallouët et al., FVCA 2011).

JC's idea

- Use the decomposition Internal energy = Total energy − Kinetic energy.
- Kinetic energy equation obtained from mass and momentum:

$$\partial_t\left(\tfrac{1}{2}\rho|\boldsymbol{u}|^2\right) + \mathrm{div}\left(\tfrac{1}{2}\rho|\boldsymbol{u}|^2\boldsymbol{u}\right) + \boldsymbol{u}\cdot\nabla p = 0$$

Recover discrete kinetic energy from discrete mass and momentum:

$$(\text{kinetic energy})_d + R_d = 0, \quad R_d \geq 0$$

- $(\text{total energy})_d = (\text{internal energy})_d - R_d + (\text{kinetic energy})_d + R_d$
- solve $(\text{internal energy})_d = R_d \geq 0 \Rightarrow e \geq 0$

Time discretisation, implicit scheme

Euler equations

$$
\begin{aligned}
&\partial_t \rho + \operatorname{div}(\rho \boldsymbol{u}) = 0\\
&\partial_t(\rho \boldsymbol{u}) + \operatorname{div}(\rho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p = 0\\
&\partial_t(\rho e) + \operatorname{div}\big((\rho e)\boldsymbol{u}\big) + p \operatorname{div} \boldsymbol{u} = 0\\
&p = \mathfrak{p}(\rho, e)
\end{aligned}
$$

Implicit scheme $\eth_t^{n+1} z = \frac{1}{\delta t}(z^{n+1} - z^n)$

δt: time step

$$
\begin{aligned}
&\eth_t^{n+1} \rho + \operatorname{div}(\rho^{n+1} \boldsymbol{u}^{n+1}) = 0\\
&\partial_t(\rho^{n+1} \boldsymbol{u}^{n+1}) + \operatorname{div}(\rho \boldsymbol{u}^{n+1} \otimes \boldsymbol{u}^{n+1}) + \nabla p^{n+1} = 0\\
&\partial_t(\rho^{n+1} e^{n+1}) + \operatorname{div}\big((\rho^{n+1} e^{n+1})\boldsymbol{u}^{n+1}\big) + p^{n+1} \operatorname{div} \boldsymbol{u} = S^{n+1}\\
&p = \mathfrak{p}(\rho, e)
\end{aligned}
$$

$+$ staggered discretisation

Discrete kinetic enery inequality, Unconditional stability

[HKL On some implicit and semi-implicit staggered schemes for the shallow water and Euler equations.

M2AN,48(6):1807–1857,2014]

Time discretisation, prediction-correction scheme

Time discretisation, Euler only

$$\frac{1}{\delta t}(\rho^n \widetilde{\boldsymbol{u}}^{n+1} - \rho^{n-1}\boldsymbol{u}^n) + \mathrm{div}(\rho^{k-1}\widetilde{\boldsymbol{u}}^{n+1}\boldsymbol{u}^n) + \overline{\nabla p}^n = 0, \qquad \overline{\nabla p}^n = \Big(\frac{\rho^n}{\rho^{n-1}}\Big)^{1/2}\nabla p^n$$

solve for $\boldsymbol{u}^{n+1},\ \rho^{n+1},\ h^{n+1},\ p^{n+1}$:

$$\left|\begin{array}{l}
\dfrac{1}{\delta t}\rho^n(\boldsymbol{u}^{n+1} - \widetilde{\boldsymbol{u}}^{n+1}) + \nabla p^{n+1} - \overline{\nabla p}^n = 0, \qquad i = 1, \dots, d, \\
\dfrac{1}{\delta t}(\rho^{n+1} - \rho^n) + \mathrm{div}(\rho^k \boldsymbol{u}^{n+1}) = 0, \\
\dfrac{1}{\delta t}(\rho^{n+1}e^{n+1} - \rho^n e^n) + \mathrm{div}(\rho^k e^k \boldsymbol{u}^{n+1}) - u^{n+1}\cdot\nabla p^{n+1} = S^{n+1}. \\
p^{n+1} = \mathfrak{p}(\rho^{n+1}, e^{n+1})
\end{array}\right.$$

- $k = n+1$ Implicit correction step. Unconditional stability. First order space discretization, upwind choice.
  [HKL 2014], [GHKL An unconditionally stable staggered pressure correction scheme for the compressible Navier-Stokes equations. SMAI-JCM, 2:51–97, 2016
- $k = n$ Linearized correction step. Conditional stability. Quasi second order space discretization, MUSCL-like choice.
  [HHL A quasi second order in space pressure correction scheme for compressible flows, preprint]

Why $\overline{\nabla p}^n = \left(\frac{\rho^n}{\rho^{n-1}}\right)^{1/2} \nabla p^n$? – semi-discrete kinetic energy

Kinetic energy : $\partial_t\left(\frac{1}{2}\rho|\boldsymbol{u}|^2\right) + \mathrm{div}\left(\frac{1}{2}\rho|\boldsymbol{u}|^2\boldsymbol{u}\right) + \boldsymbol{u}\cdot\nabla p = 0$ (semi)-discrete case?

– Multiply prediction step (momentum) by $\widetilde{\boldsymbol{u}}^{n+1}$

– Multiply correction step by $(\frac{\delta t}{2\rho^n})^{\frac{1}{2}}$ and square

– Add both $\rightsquigarrow$

$$\frac{1}{2\delta t}(\rho^n|\boldsymbol{u}^{n+1}|^2 - \rho^{n-1}|\boldsymbol{u}^n|^2) + \frac{1}{2}\,\mathrm{div}(|\widetilde{\boldsymbol{u}}^{n+1}|^2\rho^n\boldsymbol{u}^n) + \nabla p^{n+1}\cdot\boldsymbol{u}^{n+1} = -r^{n+1} + \delta b^n$$

with $r^{n+1} \geq 0$ and

$$\delta b^n = \frac{\delta t}{2\rho^n}|\nabla p^{n+1}|^2 - \frac{\delta t}{2\rho^n}|\overline{\nabla p}^n|^2 \overset{?}{=} (\mathrm{bidule}^{n+1})^2 - (\mathrm{bidule}^n)^2$$

$$\text{yes if } \left(\frac{\delta t}{2\rho^n}\right)^{\frac{1}{2}}|\overline{\nabla p}^n| = \mathrm{bidule}^n = \left(\frac{\delta t}{2\rho^{n-1}}\right)^{\frac{1}{2}}|\nabla p^n| \text{ that is: } \overline{\nabla p}^n = \left(\frac{\rho^n}{\rho^{n-1}}\right)^{\frac{1}{2}}\nabla p^n$$

When summing in time, “bidule” terms cancel out.
$\rightsquigarrow$ total energy conservation

$\rightsquigarrow$ Lax Wendroff consistency

Prediction-correction scheme, explicit convection fluxes

semi-discrete scheme:

$$\overline{\nabla p}^n = \Big(\frac{\rho^n}{\rho^{n-1}}\Big)^{1/2} \nabla p^n$$

$$\frac{1}{\delta t}(\rho^n \tilde{\boldsymbol{u}}^{n+1} - \rho^{n-1} \boldsymbol{u}^n) + \operatorname{div}(\rho^{n-1} \tilde{\boldsymbol{u}}^{n+1} \boldsymbol{u}^n) + \overline{\nabla p}^n = 0,$$

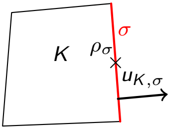
solve for $\boldsymbol{u}^{n+1},\ \rho^{n+1},\ h^{n+1},\ p^{n+1}$:

$$\left|\begin{array}{l} \dfrac{1}{\delta t}\rho^n(\boldsymbol{u}^{n+1} - \tilde{\boldsymbol{u}}^{n+1}) + \nabla p^{n+1} - \overline{\nabla p}^n = 0, \qquad i = 1, \ldots, d, \\ \dfrac{1}{\delta t}(\rho^{n+1} - \rho^n) + \operatorname{div}(\rho^n \boldsymbol{u}^{n+1}) = 0, \\ \dfrac{1}{\delta t}(\rho^{n+1} e^{n+1} - \rho^n e^n) + \operatorname{div}(\rho^n e^n \boldsymbol{u}^{n+1}) - u^{n+1} \cdot \nabla p^{n+1} = S^{n+1}. \\ p^{n+1} = \mathfrak{p}(\rho^{n+1}, e^{n+1}) \end{array}\right.$$

Second order approximation : MUSCL-like scheme

MUSCL-like scheme -I

[PBHL, A formally second-order cell centred scheme for convection-diffusion equations on general grids. Internat. J. Numer. Methods Fluids 71 (2013), no. 7, 873–890]
[GHLT A MUSCL-type segregated–explicit staggered scheme for the Euler equations. Comput. & Fluids 175 (2018), 91–110.]



- Mass balance $\partial_t \rho + \mathrm{div}(\rho \boldsymbol{u}) = 0$
- FV scheme $\dfrac{|K|}{\delta t}(\rho_K^{n+1} - \rho_K^n) + \displaystyle\sum_{\sigma \in \mathcal{E}(K)} F_{K,\sigma}^n = 0.$

Numerical flux: $F_{K,\sigma}^n = |\sigma| \rho_\sigma^n u_{K,\sigma}^{n+1}, \qquad u_{K,\sigma} = \boldsymbol{u}_\sigma \cdot \boldsymbol{n}_{K,\sigma}$

$$= |\sigma| \rho_\sigma^n u_{K,\sigma}^{n+1} = |\sigma| \rho_\sigma^n (u_{K,\sigma}^{n+1})^+ - \rho_\sigma^n (u_{K,\sigma}^{n+1})^-)$$

Upwind choice: $= |\sigma| \rho_K^n (u_{K,\sigma}^{n+1})^+ - \rho_L^n (u_{K,\sigma}^{n+1})^-)$

$$\rho_K^{n+1} = \rho_K^n - \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} F_{K,\sigma}^n \qquad \rho_\sigma^n = \rho_K^n + (\rho_\sigma^n - \rho_K^n)$$

$$= \rho_K^n - \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^+ \rho_K^n - \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^+ (\rho_\sigma^n - \rho_K^n)$$

$$+ \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^- \rho_K^n + \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^- (\rho_\sigma^n - \rho_K^n)$$

MUSCL-like scheme -II

If $\rho_K^n > 0\ \forall K$, $\qquad \rho_K^{n+1} > 0\ \forall K$?

$$\rho_K^{n+1} = \rho_K^n - \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^+ \rho_K^n - \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^+ \underbrace{(\rho_\sigma^n - \rho_K^n)}_{=\alpha_\sigma(\rho_K^n - \rho_{M_\sigma^n}^n)}$$

$$+ \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^- \rho_K^n + \frac{\delta t}{|K|} \sum_{\substack{\sigma \in \mathcal{E}(K) \\ \sigma = K|L}} |\sigma| (u_{K,\sigma}^{n+1})^- \underbrace{(\rho_\sigma^n - \rho_K^n)}_{=\beta_\sigma(\rho_L^n - \rho_K^n)}$$

$0 \le \alpha_\sigma, \beta_\sigma \le 1$, $\quad M_\sigma^n$ neighbour of K

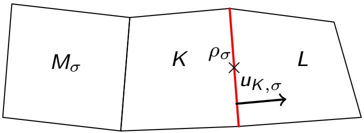
$$\rho_K^{n+1} = \left(1 - \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| u_{K,\sigma}^{n+1})^+ (1 + \alpha_\sigma)\right) \rho_K^n + \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^+ \alpha_\sigma \rho_{M_\sigma}^n$$

$$+ \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^- \beta_\sigma \rho_L^n + \frac{\delta t}{|K|} \sum_{\sigma \in \mathcal{E}(K)} |\sigma| (u_{K,\sigma}^{n+1})^- (1 - \beta_\sigma) \rho_K^n \ge 0$$

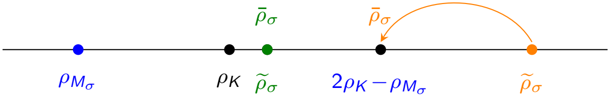
under CFL condition $\displaystyle \delta t \le \frac{|K|}{2 \sum_{\sigma \in \mathcal{E}(K)} |\sigma| |u_{K,\sigma}^{n+1}|}$.

MUSCL-like scheme - III

$\widetilde{\rho}_\sigma$: second order approximation on $\sigma = K|L$, assume $u_{K,\sigma} \geq 0$: K upwind

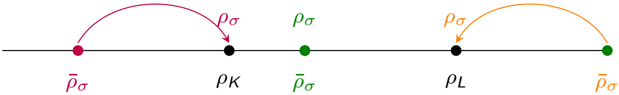


- Limitation "α" ($u_{K,\sigma} > 0$), $\bar{\rho}_\sigma - \rho_K = \alpha_\sigma(\rho_K - \rho_{M_\sigma})$
  - If $\nexists M$; $(\rho_K - \rho_M)(\widetilde{\rho}_\sigma - \rho_K) > 0$ then set $\bar{\rho}_\sigma = \rho_K$ (upwind choice).
  - Else, choose M_σ, and project on admissible interval



- Limitation "β" ($u_{L,\sigma} < 0$), $\bar{\rho}_\sigma - \rho_K = \beta_\sigma(\rho_K^n - \rho_L^n)$

  Project on admissible interval.



– Advantages: no slope reconstruction, OK for any mesh

– Drawback: limited to linear flux

Staggered Discretisation — MAC

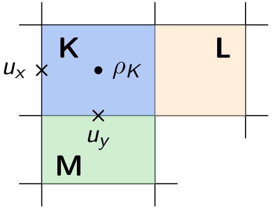
Rectangular meshes

Scalar unknowns (p, ρ, e) at cell centers

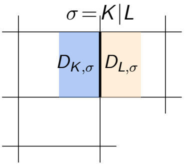
u_x on vertical faces,
u_y on horizontal faces

$D_{K,\sigma}$: half-cell adjacent to σ

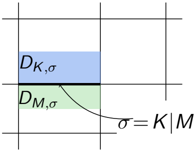
Dual cell $D_\sigma = D_{K,\sigma} \cup D_{L,\sigma}$



(a) primal mesh



(b) dual u_x



(c) dual u_y

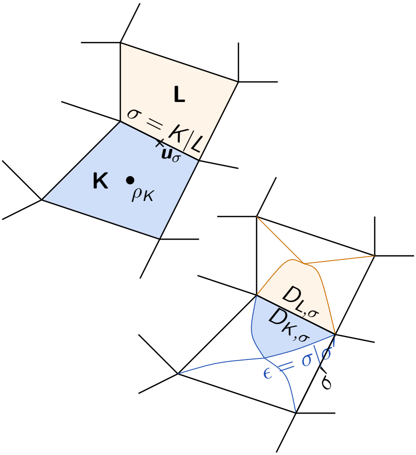
Staggered Discretisation — Rannacher–Turek

Scalar unknowns (p, ρ, e) at cell centers

Velocity $\boldsymbol{u}$ at face centers

Dual cells $D_\sigma = D_{K,\sigma} \cup D_{L,\sigma}$

RT/CR: mixed quadrangular and triangular meshes



Space-time discrete kinetic energy

Discrete kinetic energy equation on dual mesh, requires discrete mass balance on dual mesh, and compatible discretization of momentum balance.

- Mass balance on primal mesh

$$\frac{|K|}{\delta t}(\rho_K^n - \rho_K^{n-1}) + \sum_{\sigma \in \mathcal{E}(K)} F_{K,\sigma}^n = 0.$$

- Momentum balance on dual mesh

$$\frac{|D_\sigma|}{\delta t}(\rho_{D_\sigma}^n \widetilde{\boldsymbol{u}}_\sigma^{n+1} - \rho_{D_\sigma}^{n-1} \boldsymbol{u}_\sigma^n) + \sum_{\epsilon \in \mathcal{E}(D_\sigma)} F_{\sigma,\epsilon}^{n-1} \boldsymbol{u}_\epsilon^{n+1} + (\overline{\nabla}^n p)_\sigma = 0$$

Choose ρ_{D_σ} and $F_{\sigma,\epsilon}$ such that

$$\frac{|D_\sigma|}{\delta t}(\rho_{D_\sigma}^n - \rho_{D_\sigma}^{n-1}) + \sum_{\epsilon \in \mathcal{E}(D_\sigma)} F_{\sigma,\epsilon}^{n-1} = 0$$

Density : $\begin{cases} |D_\sigma|\rho_{D_\sigma}^n = |K|\rho_K^n + |L|\rho_L^n \text{ if } \sigma = K|L, \\ \rho_{D_\sigma}^n = \rho_K^n \text{ if } \sigma \subset \partial\Omega \end{cases}$

Dual flux $F_{\sigma,\epsilon}$: linear combination of the primal fluxes.

Total Energy Estimate

- Summing internal energy equation on all primal meshes
- Summing kinetic energy equation on all dual meshes
- Using discrete grad-div duality (discretization of pressure gradient)
- Summing in time

$\rightsquigarrow$ total energy estimate.

By topological degree argument, existence of a solution.

Discrete entropy inequality also holds.

Lax–Wendroff Consistency

Under assumptions

- CFL, mesh regularity,
- L^∞ bounds, BV bounds on $\rho, e, p, \boldsymbol{u}$

LW consistency

If $\rho^{(m)} \to \bar\rho$, $\boldsymbol{u}^{(m)} \to \bar{\boldsymbol{u}}$, $e^{(m)} \to \bar e$, $p^{(m)} \to \bar p$ in L^1, then $(\bar\rho, \bar{\boldsymbol{u}}, \bar e, \bar p)$ satisfies weakly:

$$
\begin{aligned}
&\partial_t \bar\rho + \mathrm{div}(\bar\rho \bar{\boldsymbol{u}}) = 0 \\
&\partial_t(\bar\rho \bar{\boldsymbol{u}}) + \mathrm{div}(\bar\rho \bar{\boldsymbol{u}} \otimes \bar{\boldsymbol{u}}) + \nabla \bar p = 0 \\
&\partial_t(\bar\rho \bar E) + \mathrm{div}((\bar\rho \bar E + \bar p)\bar{\boldsymbol{u}}) = 0
\end{aligned}
$$

and the entropy inequality $\partial_t \eta(\bar\rho, \bar e) + \mathrm{div}[\eta(\bar\rho, \bar e)\bar{\boldsymbol{u}}] \le 0$ (under additional assumption $\frac{\delta t_m}{h_m} \to 0$ as $m \to +\infty$)

Convergence on Traveling Vortex (Manufactured Solution)

Navier-Stokes, MAC scheme, Ma$= 0.002$ and 0.2

Scheme	ρ order	$\boldsymbol{u}$ order
Upwind	≈ 0.85	≈ 1.2
MUSCL(ρ, e)	≈ 1.5	≈ 1.5
MUSCL(ρ, p)	≈ 1.5	≈ 1.5

- Quasi second order in 1D; preserved in 2D (less restrictive limiter)
- Both Mach numbers: consistent behaviour

Jean-Claude...

était un grand scientifique, un collègue formidable,

un pyrénéen convaincu (*chut... c'est une photo des Alpes*),

et un ami très cher.

Je n'ai pu donner ici qu'un tout petit aperçu des nombreuses idées qu'il nous a données.

Toutes continueront à nous inspirer.