Global Structure of Symmetry Groups: The SM and the DFSZ axion

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Four different versions of SM gauge group!

$$G_{\rm SM} = \big(SU(3)_C \times SU(2)_L \times U(1)_Y\big)/\Gamma$$

$$\Gamma = \{1, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_6\}$$

How does phenomenology differ in these various groups?
How can we tell them apart?

BSM physics can distinguish!

1) Fractional Instantons

(Reece; Choi, Forslund, Lam, Shao; Córdova, Hong, Wang)

$$G_{\rm SM} = \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right)/\Gamma$$

$$\Gamma = \{1, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_6\}$$

Quotient gives fractional instanton number

$\Rightarrow$ Quantized axion coupling differs $\quad S \supset i\frac{\ell}{8\pi^2}\int \frac{a}{f_a}\text{Tr}G \wedge G$

Measuring axion-gluon and axion-photon couplings may rule out some versions!

2) Electric Representations

Electric charge definition $$q = \frac{1}{6}Y + \frac{1}{2}T_L^3$$

$\mathbb{Z}_6$ quotient $\Rightarrow$ $$\begin{cases} Y = I_3 \text{ (mod } 3) \\ Y = I_2 \text{ (mod } 2) \end{cases}$$

$$\Rightarrow \quad q = -\frac{1}{3}I_3 \text{ (mod } 1)$$

Integer charges for colorless operators!

Generally $$\Gamma = \mathbb{Z}_n \qquad q_{\min} = \frac{n}{6}e$$

Tight UV connection: FCPs probe GUTs!

Different UV theories demand different versions of the SM gauge group

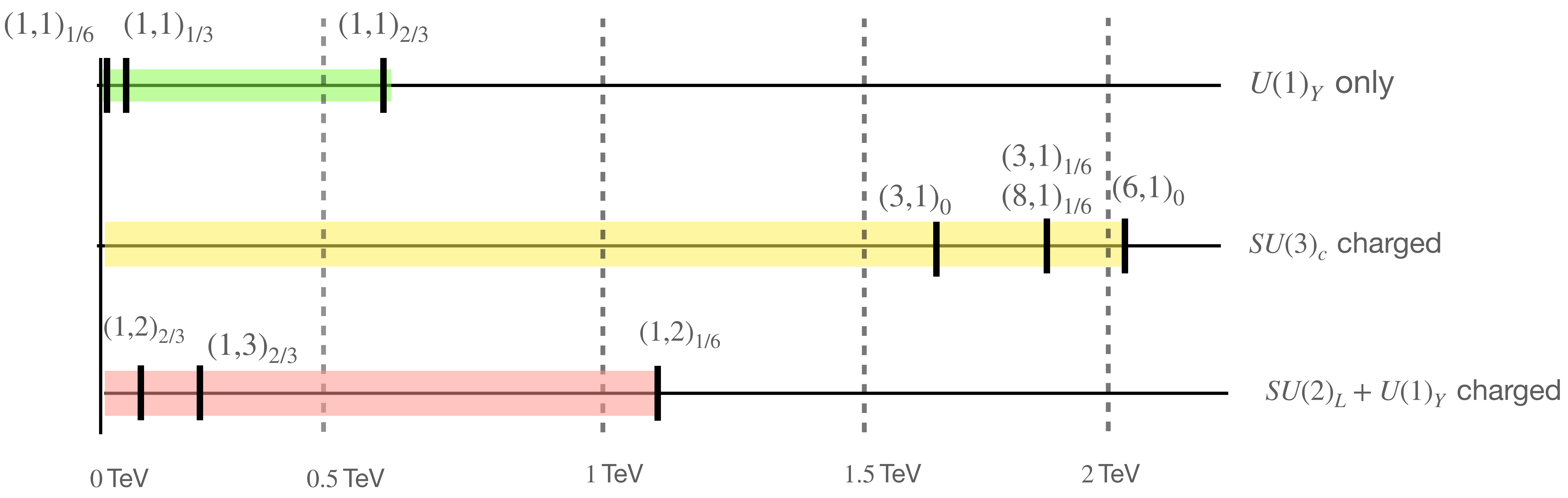
$SU(5), SO(10)$ demand $n = 6$ $\Rightarrow$ Any FCP rules these out!

Pati-Salam demands $n = 3$ $\Rightarrow$ $e/2$ allowed, $e/3$,$e/6$ rules out

Trinification demands $n = 2$ $\Rightarrow$ $e/3$ allowed, $e/2$,$e/6$ rules out

Violates normal notion of ‘decoupling’ by probing *topological* aspects of the physics!

Single particle extension with one Dirac fermion, $(R_C, R_L)_{q_Y}$



Quite low bounds!

Poorly characterized detector response!

Needs further study!

Martin, SK 2024 SciPost Phys

SMEFT study: Li, Xu

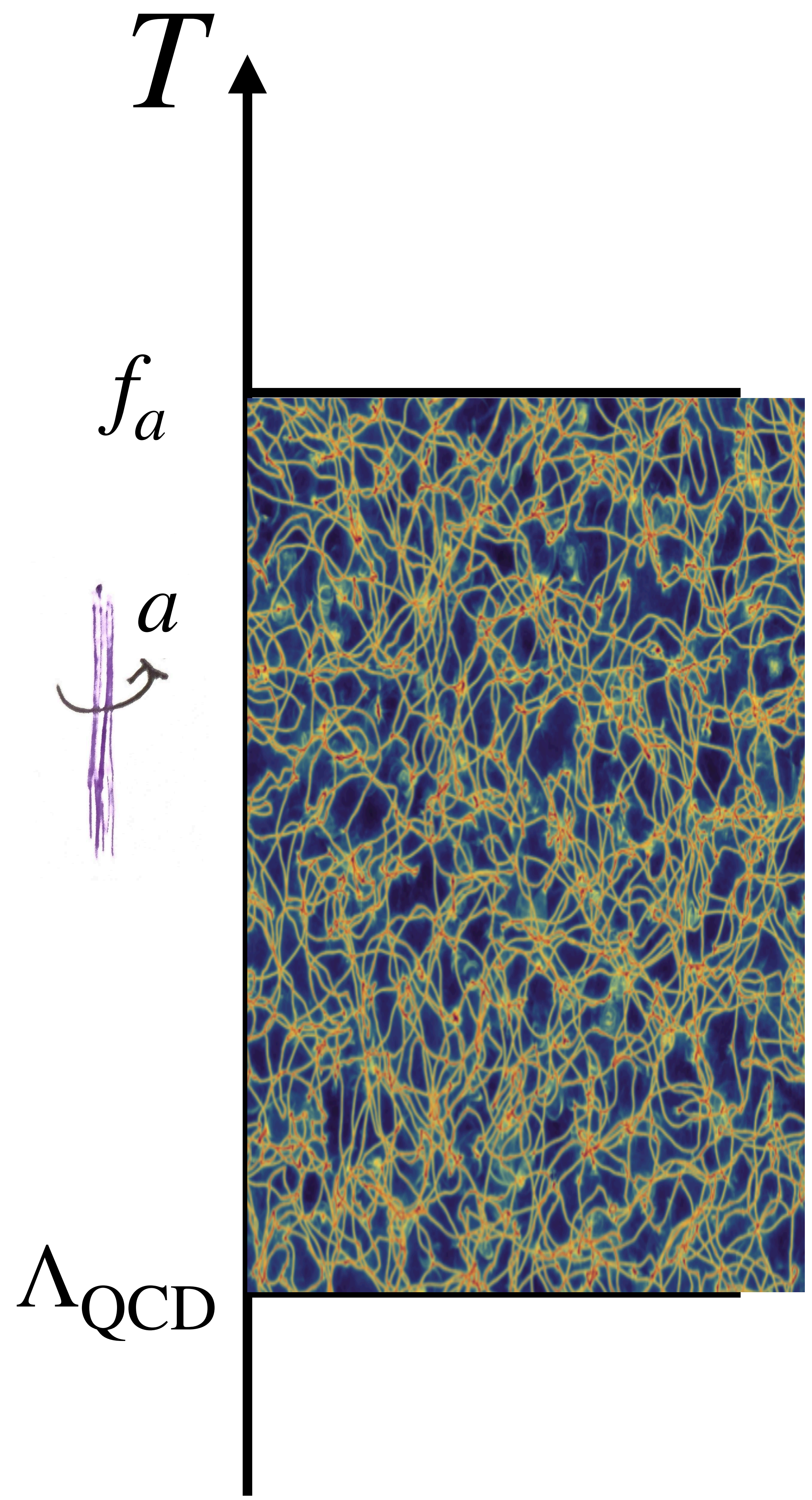
Even smaller charges: Alonso, Dimakou, West

The DFSZ Axion and Global Structure

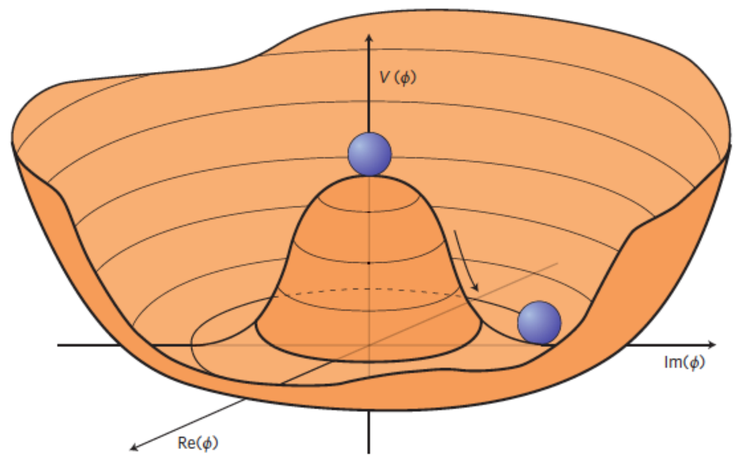
G. Choi, S. Hong, SK
Coming soon

Part of a series using non-invertible symmetry for BSM model building
- C. Córdova, S. Hong, SK, K. Ohmori
  2211.07639; Phys. Rev. X
- C. Córdova, S. Hong, SK
  2402.12453; Phys. Rev. X
- A. Delgado, SK
  2412.05362; JHEP

Axion cosmology from a general PQ UV completion

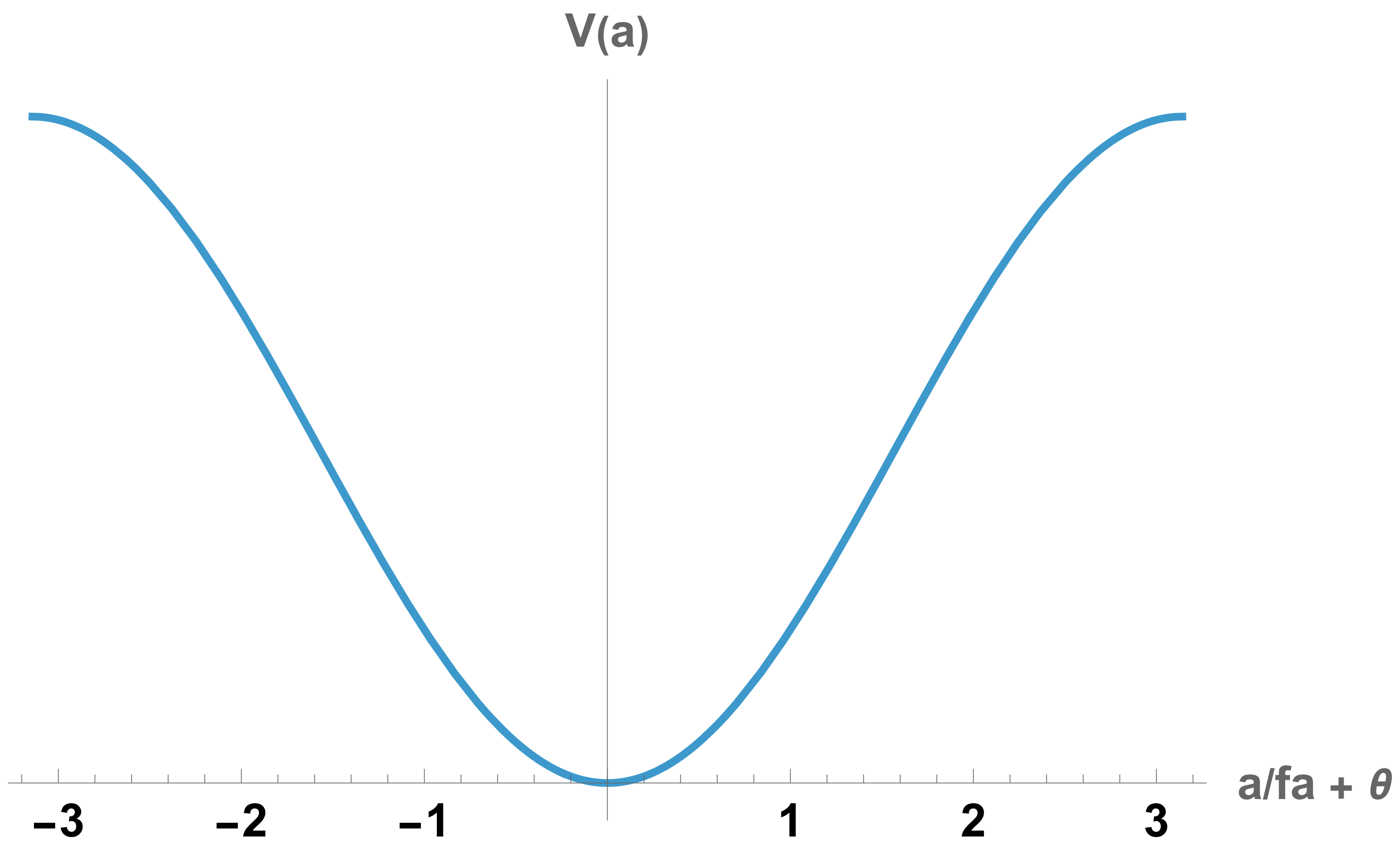


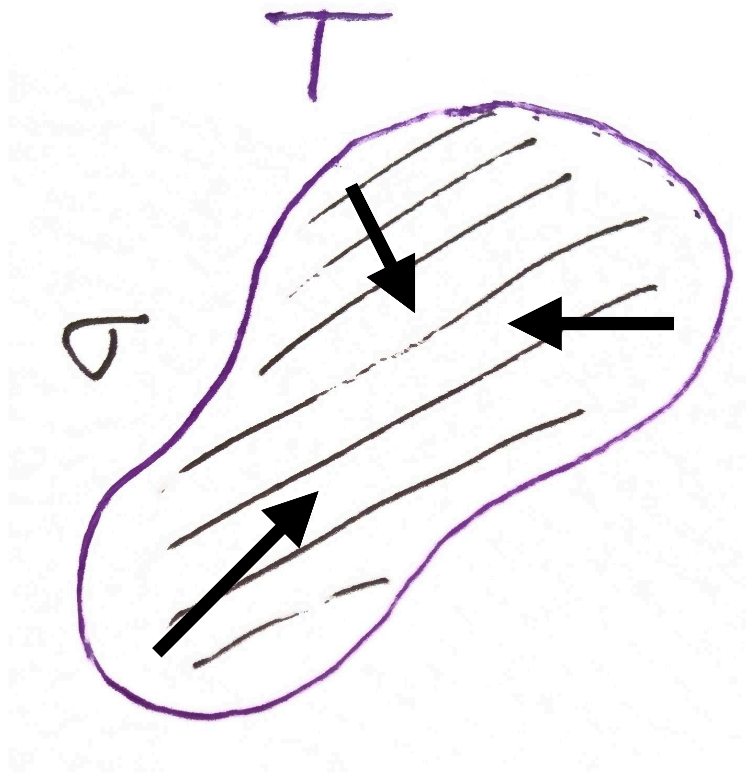
$T \sim f_a$, axion string formation



$T \sim \Lambda_{\rm QCD}$, string-domain wall network formation

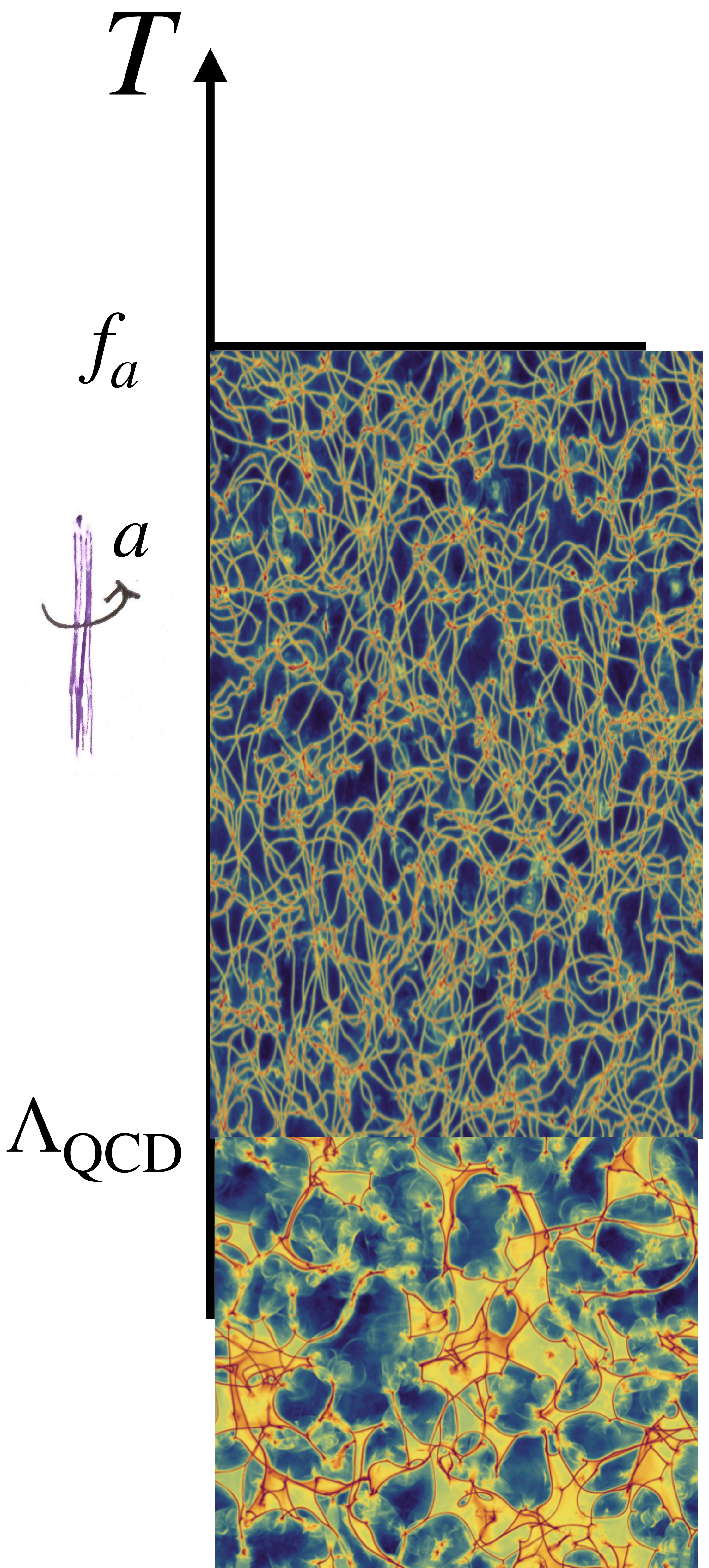
If $N_{\rm DW} = 1$, wall tension collapses them



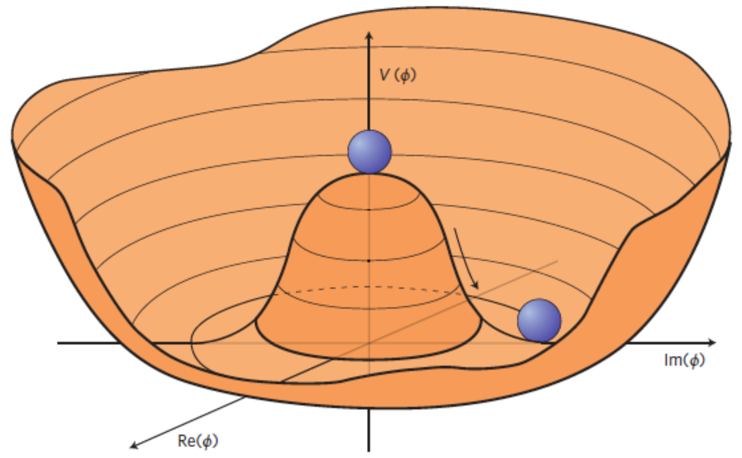


Simulations: 2112.05117
O'Hare, Pierobon, Redondo, Wong

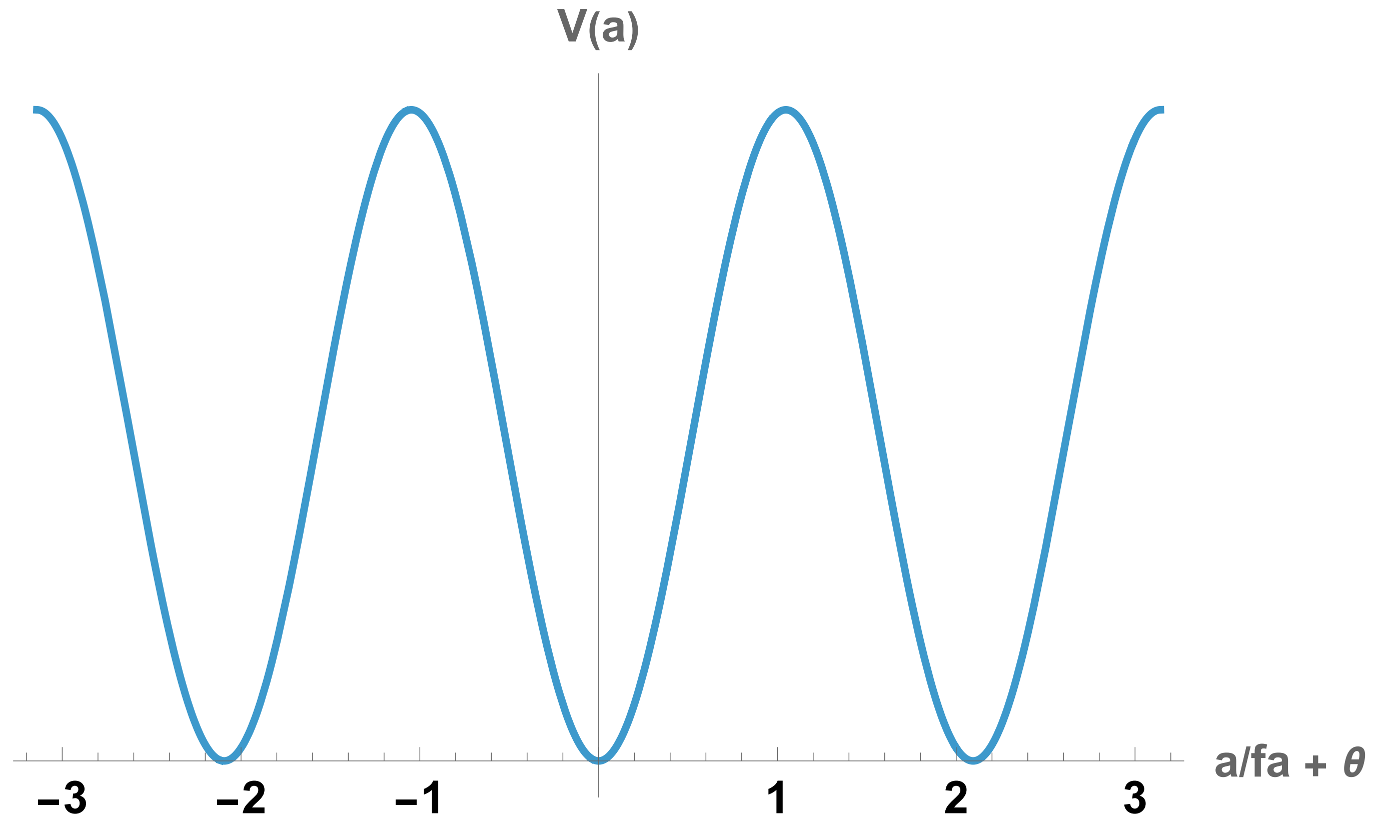
Axion cosmology from a general PQ UV completion



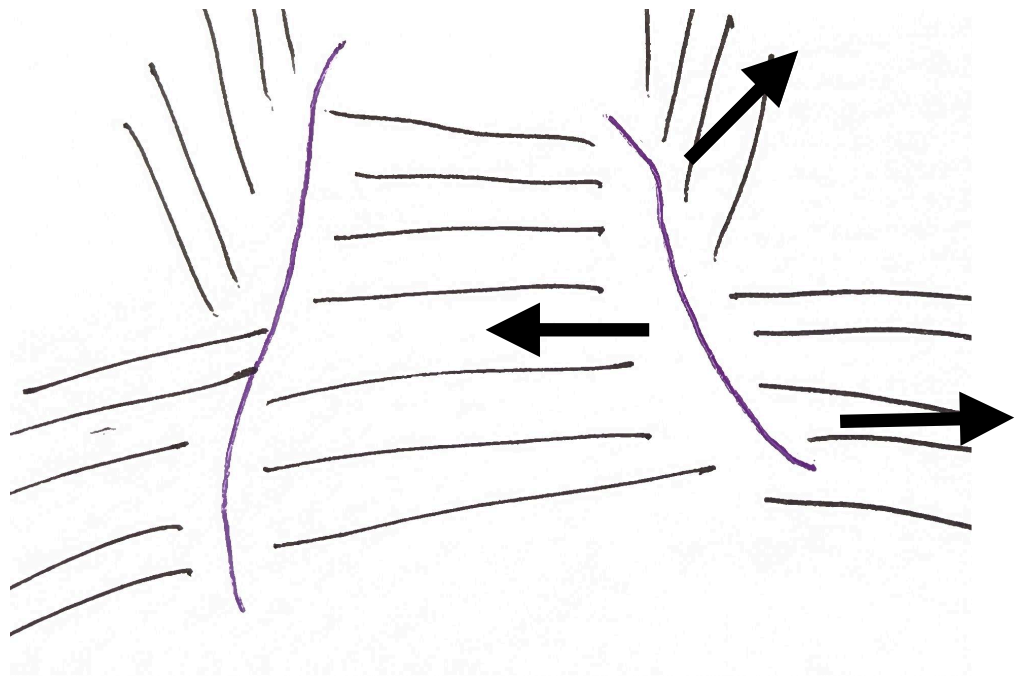
$T \sim f_a$, axion string formation



$T \sim \Lambda_{\rm QCD}$, string-domain wall network formation



If $N_{\rm DW} > 1$ complicated tangle of strings/DWs



THE DOMAIN WALL PROBLEM

Most elegant *invisible* axion model is DFSZ

$$\mathscr{L} \supset y_u H_u Q\bar{u} + y_d H_d Q\bar{d} + \lambda_3 H_u H_d \phi + \text{h.c.}$$

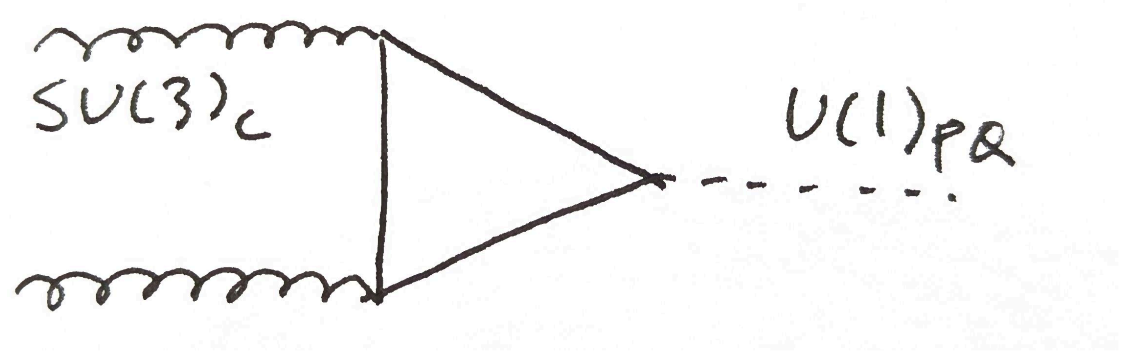
Dine, Fischler, Srednicki 1981; Zhitnitsky 1980

Global symmetry budget: 6 fields - 3 interactions
= 1 gauged $U(1)_Y$ + 2 global e.g. B-L and Peccei-Quinn

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{\text{B-L}}$	$U(1)_{\mathrm{PQ}_3}$
Q	3	2	$+1$	$+1$	$+1$
$\bar{u}$	$\bar{3}$	$-$	-4	-1	0
$\bar{d}$	$\bar{3}$	$-$	$+2$	-1	0
H_u	$-$	2	$+3$	0	-1
H_d	$-$	2	-3	0	-1
ϕ	$-$	$-$	0	0	$+2$

First look at DFSZ axion

Axion is PNGB of *anomalous* Peccei-Quinn



$$\partial_\mu J^\mu_{\rm PQ} = \frac{A}{16\pi^2}\text{Tr}(G\tilde{G}) \qquad A = (+1)\times 2 \times N_g = 2N_g$$

So anomaly explicitly breaks $U(1)_{\rm PQ} \to \mathbb{Z}_{2N_g}$

$$\mathscr{L} \supset 2N_g \frac{a}{f_a}\frac{\text{Tr}G\tilde{G}}{16\pi^2} \quad \to \quad V(a) \sim \Lambda^4_{\rm QCD}\cos\left(2N_g\frac{a}{f_a}\right)$$

If we wind around $a \to a + 2\pi f_a$, pass $2N_g$ maxima so seemingly $N_{\rm DW} = 2N_g$

Require more careful analysis

Neutral pseudoscalar phases

$$H_u = \begin{pmatrix} 0 \\ \frac{v_u}{\sqrt{2}} \end{pmatrix} e^{i\theta_u}, \qquad H_d = \begin{pmatrix} \frac{v_d}{\sqrt{2}} \\ 0 \end{pmatrix} e^{i\theta_d}, \qquad \phi = \frac{v_\phi}{\sqrt{2}} e^{i\theta_\phi}$$

Want canonically normalized $(v_u\theta_u, v_d\theta_d, v_\phi\theta_\phi) \mapsto a, z, A$

In DFSZ we have
- Gauged direction
- Anomalous direction
- Explicitly broken direction

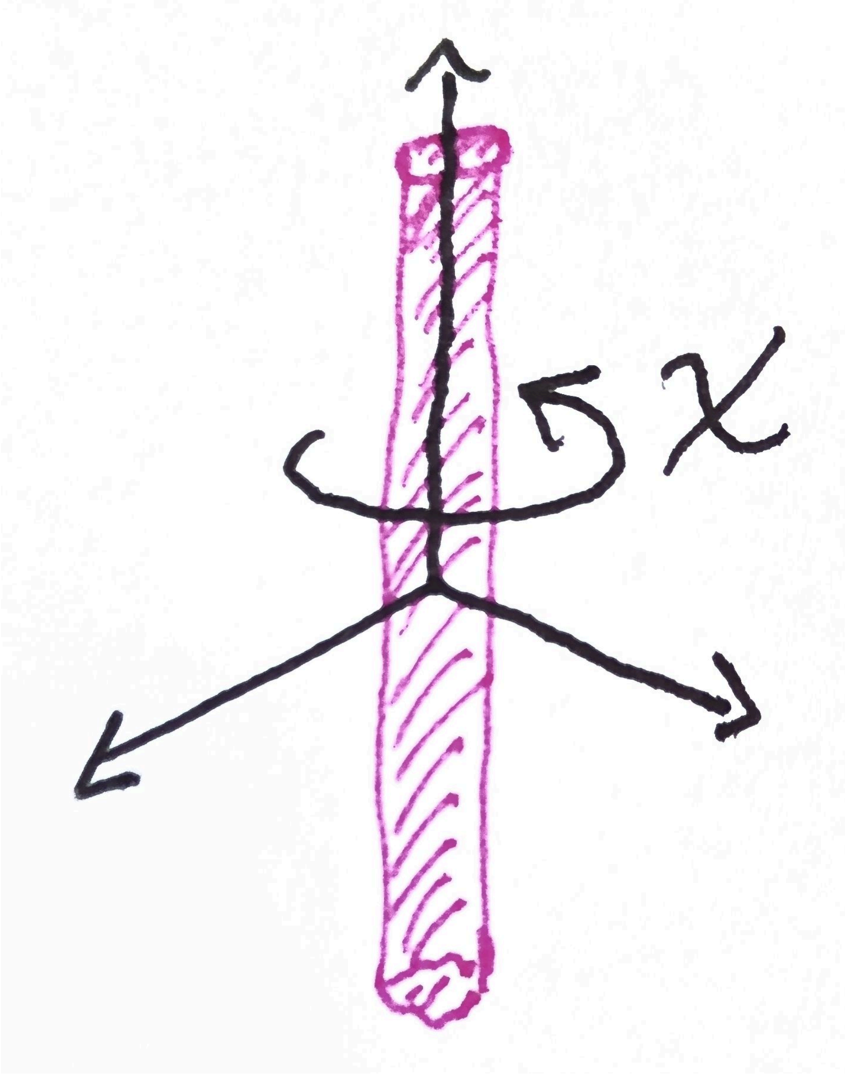
Nontrivial to find!

Space of global symmetries is really T^3/S^1, that choice of basis for $U(1)_{\text{PQ}}$ can't be special

DFSZ Domain Wall problem

How many domain walls end on a minimal-winding axion string?

Around a string, go on a nontrivial *closed* path through field space along a symmetry direction.



The IR expectation $N_{\mathrm{DW}} = A$ comes from full $U(1)_{\mathrm{PQ}}$ rotation around string

$$a(\chi) = a(0) + \alpha_{\mathrm{PQ}} f_a, \quad \alpha_{\mathrm{PQ}} = \chi$$

Is this really the minimal string, or is there a closed path around which the axion rotates less?

Must account for global structure $\frac{\text{gauge} \times \text{global}}{\Gamma}$!

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathrm{PQ}_3}$
Q	3	2	$+1$	$+1$
$\bar{u}$	$\bar{3}$	$-$	-4	0
$\bar{d}$	$\bar{3}$	$-$	$+2$	0
H_u	$-$	2	$+3$	-1
H_d	$-$	2	-3	-1
ϕ	$-$	$-$	0	$+2$

PQ = I_2 (mod 2) = Y (mod 2)

$$G_{\mathrm{EW}} = \frac{SU(2)_L \times U(1)_Y}{\Gamma} \qquad \Gamma = \{1, \mathbb{Z}_2\}$$

$$\Rightarrow \quad \frac{G_{\mathrm{EW}} \times U(1)_{\mathrm{PQ}}}{\mathbb{Z}_2}$$

Minimal winding axion string in $G_{\text{EW}} \times U(1)_{\text{PQ}}$

Wind around PQ direction only

$$\alpha_{\text{PQ}} = 2\pi$$

$$\begin{pmatrix} \theta_u \\ \theta_d \\ \theta_\phi \end{pmatrix} \to \begin{pmatrix} \theta_u - 2\pi \\ \theta_d - 2\pi \\ \theta_\phi + 4\pi \end{pmatrix}$$

$$a \to a + 2\pi f_a$$

But this is *not* the minimal string in $(G_{\text{EW}} \times U(1)_{\text{PQ}})/\mathbb{Z}_2$, e.g. consider

Wind halfway around PQ direction and around $U(1)_Y$

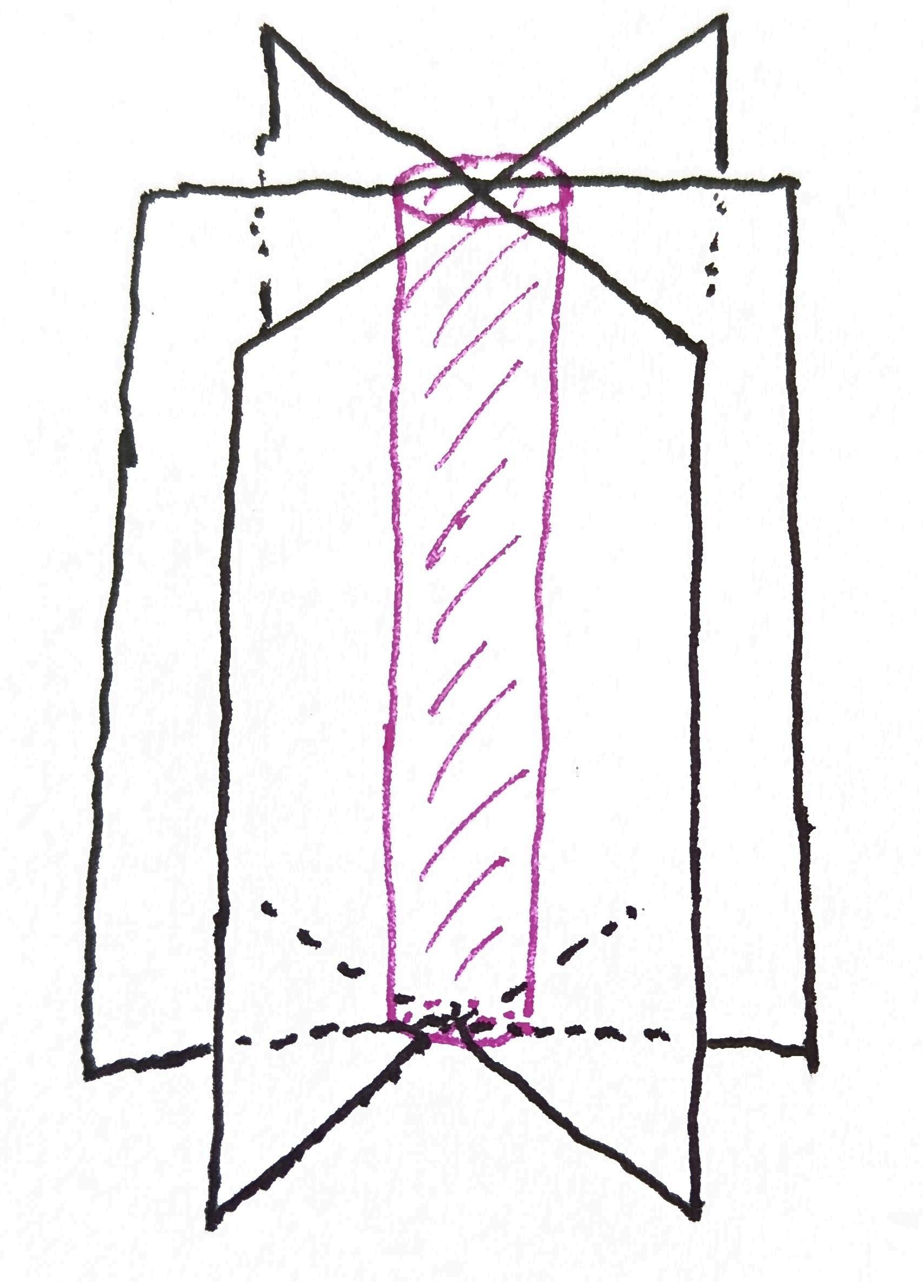
$$\alpha_{\text{PQ}} = \pi, \alpha_Y = \pi$$

$$\begin{pmatrix} \theta_u \\ \theta_d \\ \theta_\phi \end{pmatrix} \to \begin{pmatrix} \theta_u + 2\pi \\ \theta_d - 4\pi \\ \theta_\phi + 2\pi \end{pmatrix}$$

Now $a \to a + \pi f_a, z \to z + \pi f_z$

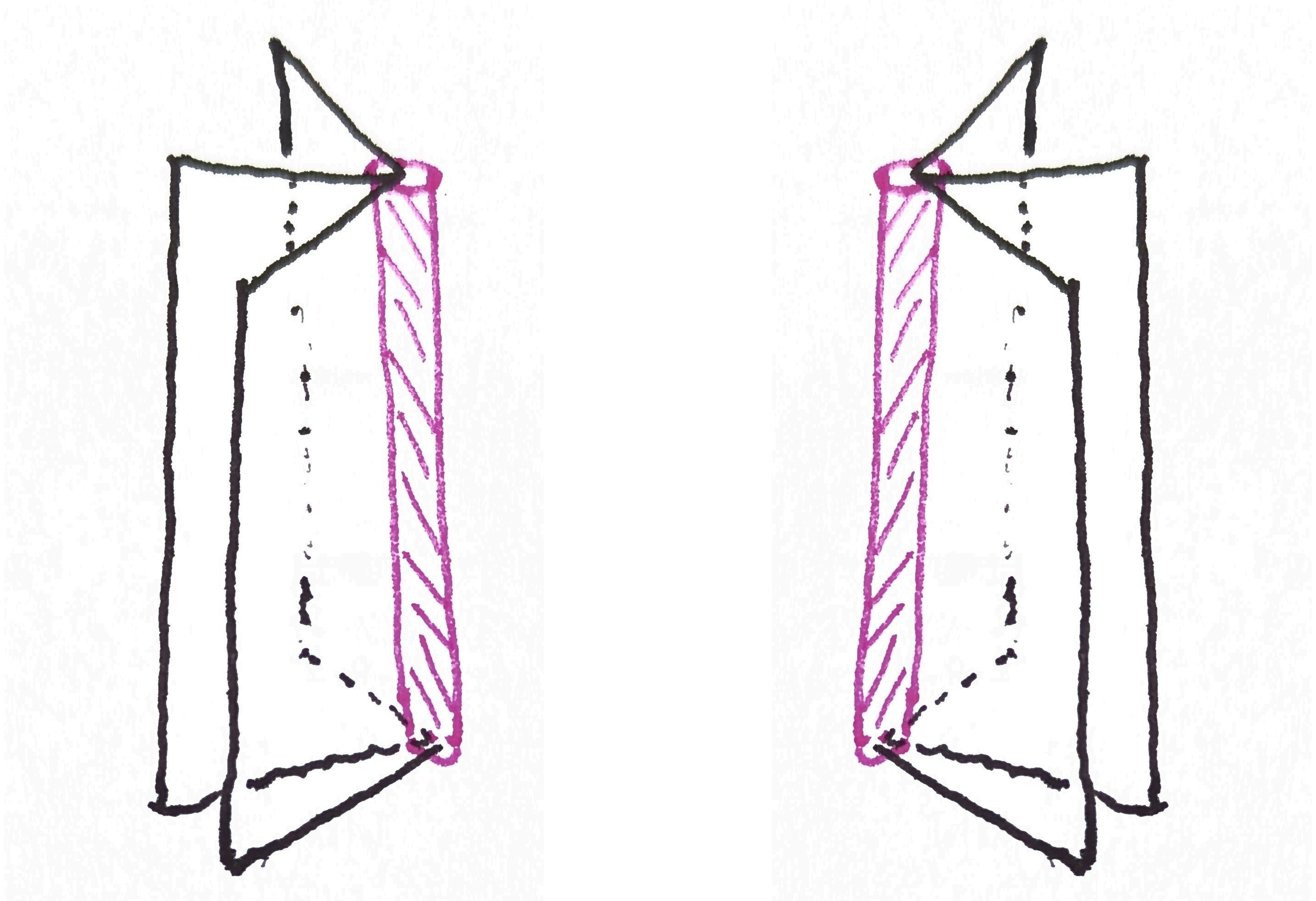
So how many domain walls end on a minimal axion string?

$$G_{\mathrm{EW}} \times U(1)_{\mathrm{PQ}}$$



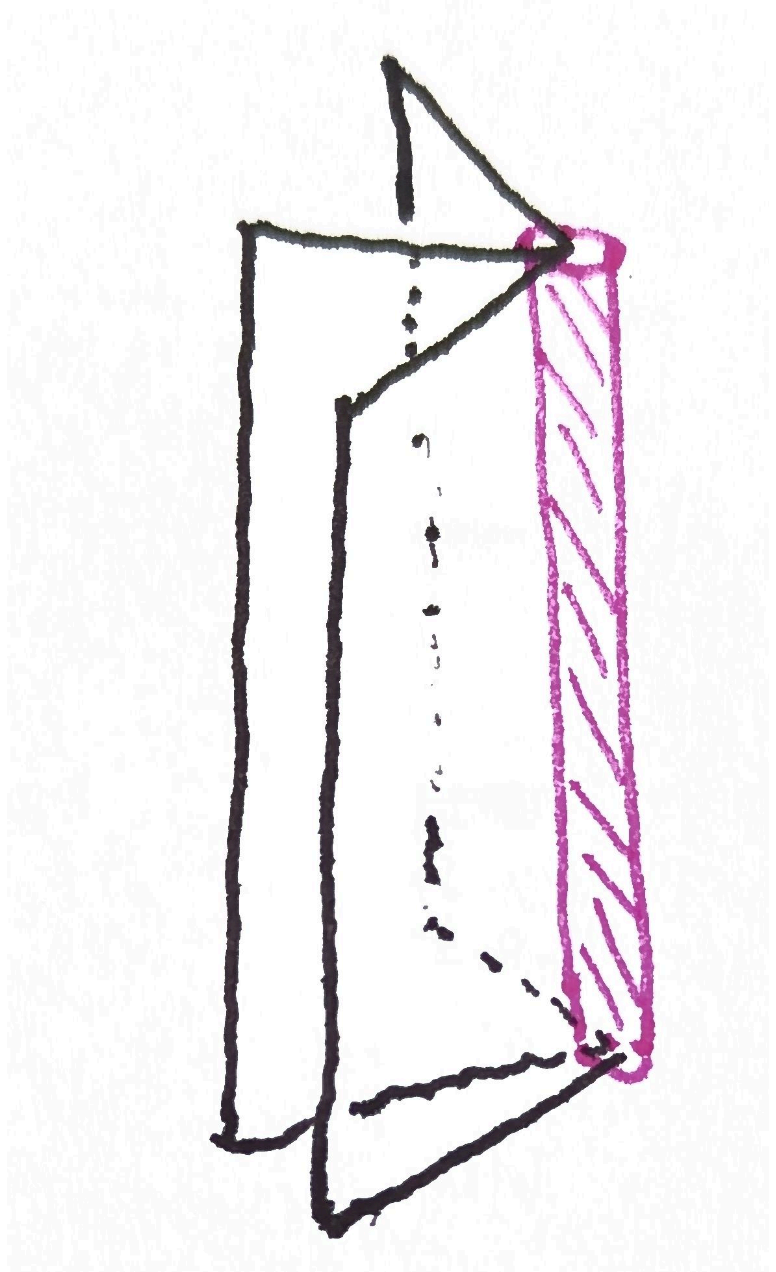
$\mathbb{Z}_{2N_g}$ domain wall problem

$$\frac{G_{\mathrm{EW}} \times U(1)_{\mathrm{PQ}}}{\mathbb{Z}_2}$$



$\mathbb{Z}_{N_g}$ domain wall problem

Still $\mathbb{Z}_{N_g}$ domain wall problem remains



In DFSZ, this degeneracy is enforced by a normal $\mathbb{Z}_{N_g}$ Peccei-Quinn symmetry

If we go to $(SU(3)_C \times SU(3)_H)/\mathbb{Z}_3$

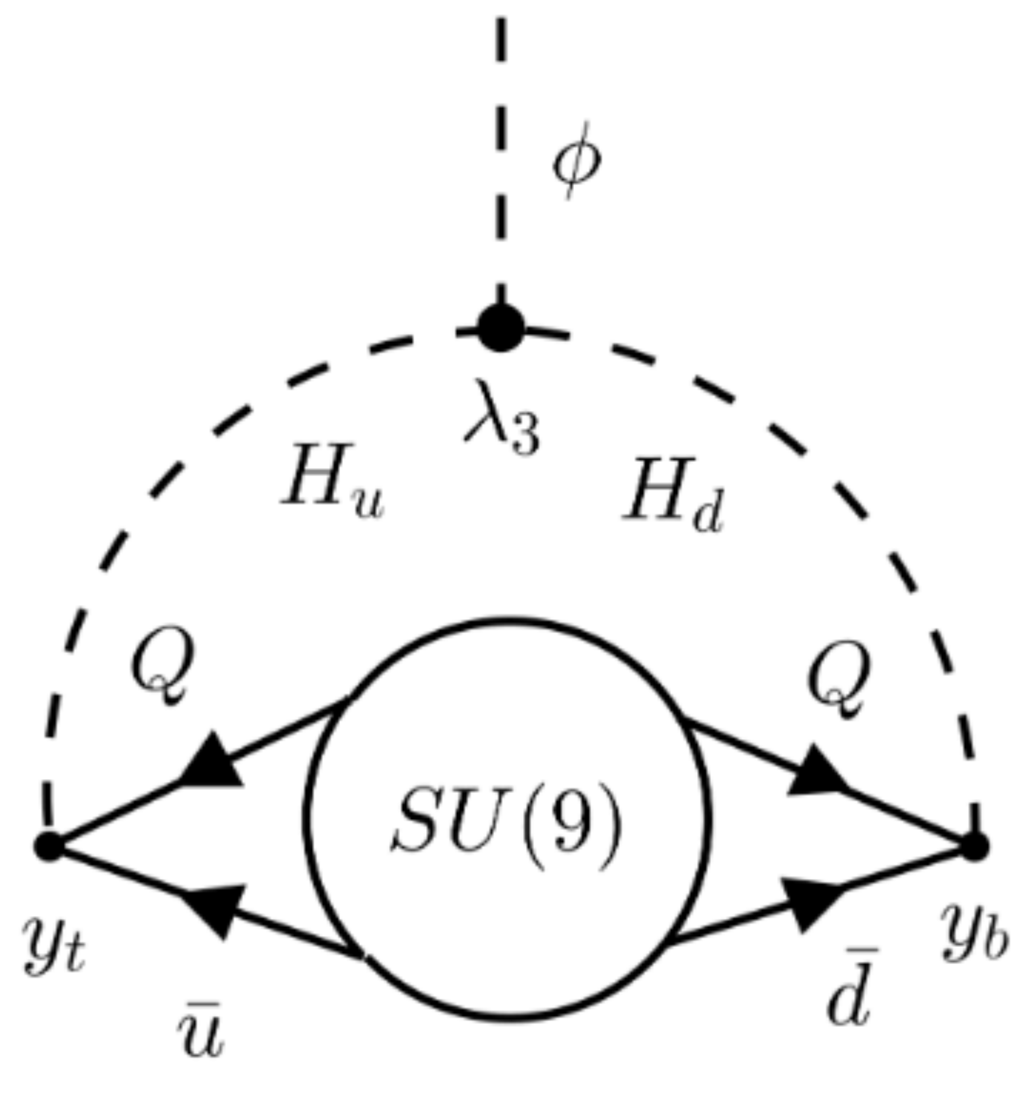
$$\bar{u} = \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \end{pmatrix}$$

Due to $\dfrac{\text{gauge} \times \text{gauge}}{\Gamma}$, this $\mathbb{Z}_{N_g}$ Peccei-Quinn symmetry becomes noninvertible!

Noninvertible spurion analysis: go to UV theory that breaks $\mathbb{Z}_{3,m}^{(1)}$ to find nonperturbative explicit breaking!

Embed in $SU(9)$ color-flavor unification

	$SU(9)$	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathrm{PQ}_3}$
Q	9	2	+1	+1
$\bar{u}$	$\bar{9}$	–	−4	0
$\bar{d}$	$\bar{9}$	–	+2	0
H_u	–	2	−3	−1
H_d	–	2	+3	−1
ϕ	–	–	–	+2

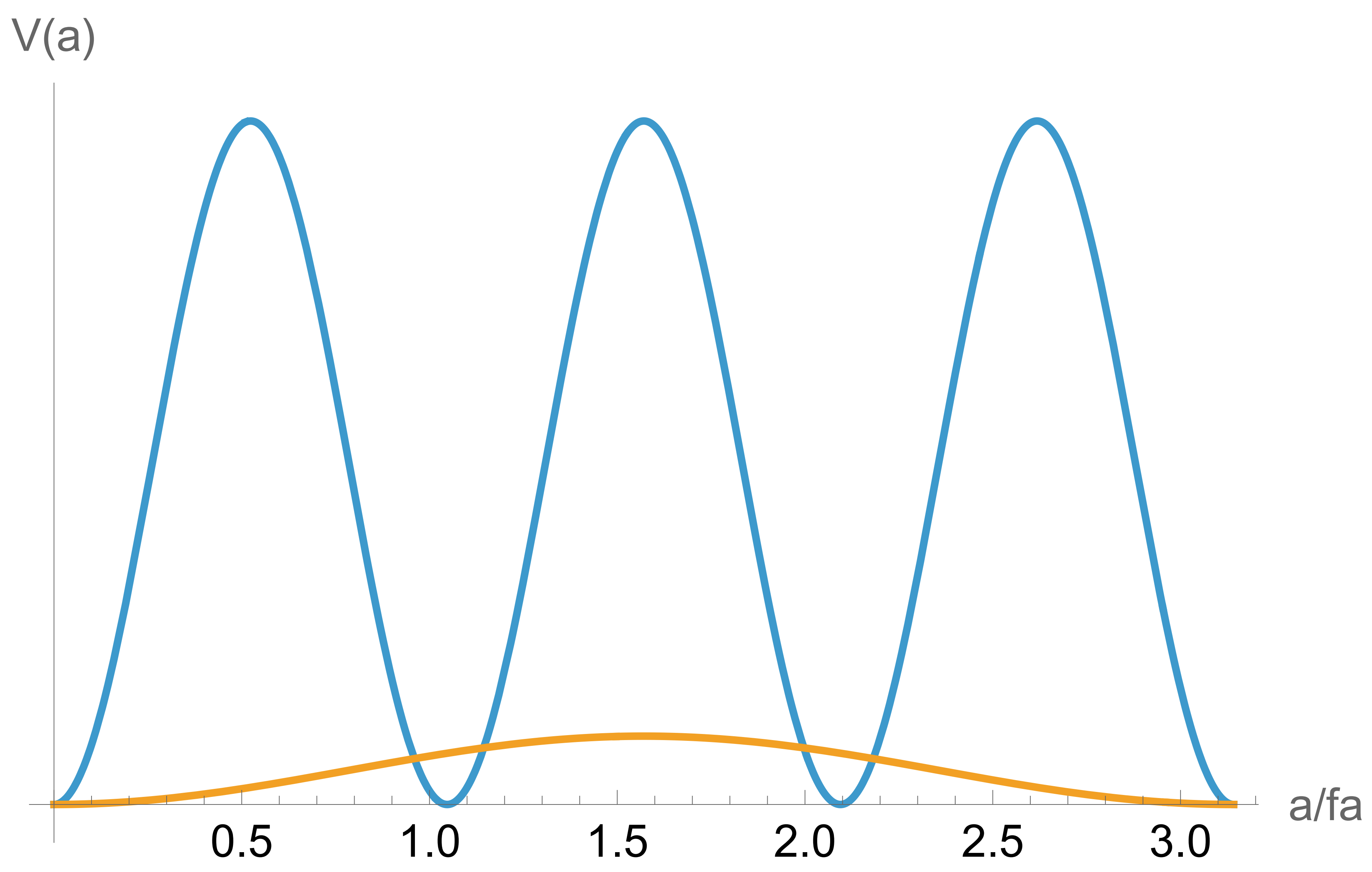


$$V(a) \sim \Lambda_{\mathrm{QCD}}^4 \cos\left(6\frac{a}{f_a}\right) + \epsilon \cos\left(2\frac{a}{f_a}\right)$$

$$\epsilon \sim \frac{y_t y_b}{(4\pi)^2} v_9^2 v_\phi^2 \exp - \frac{8\pi^2}{g^2(v_9)}$$

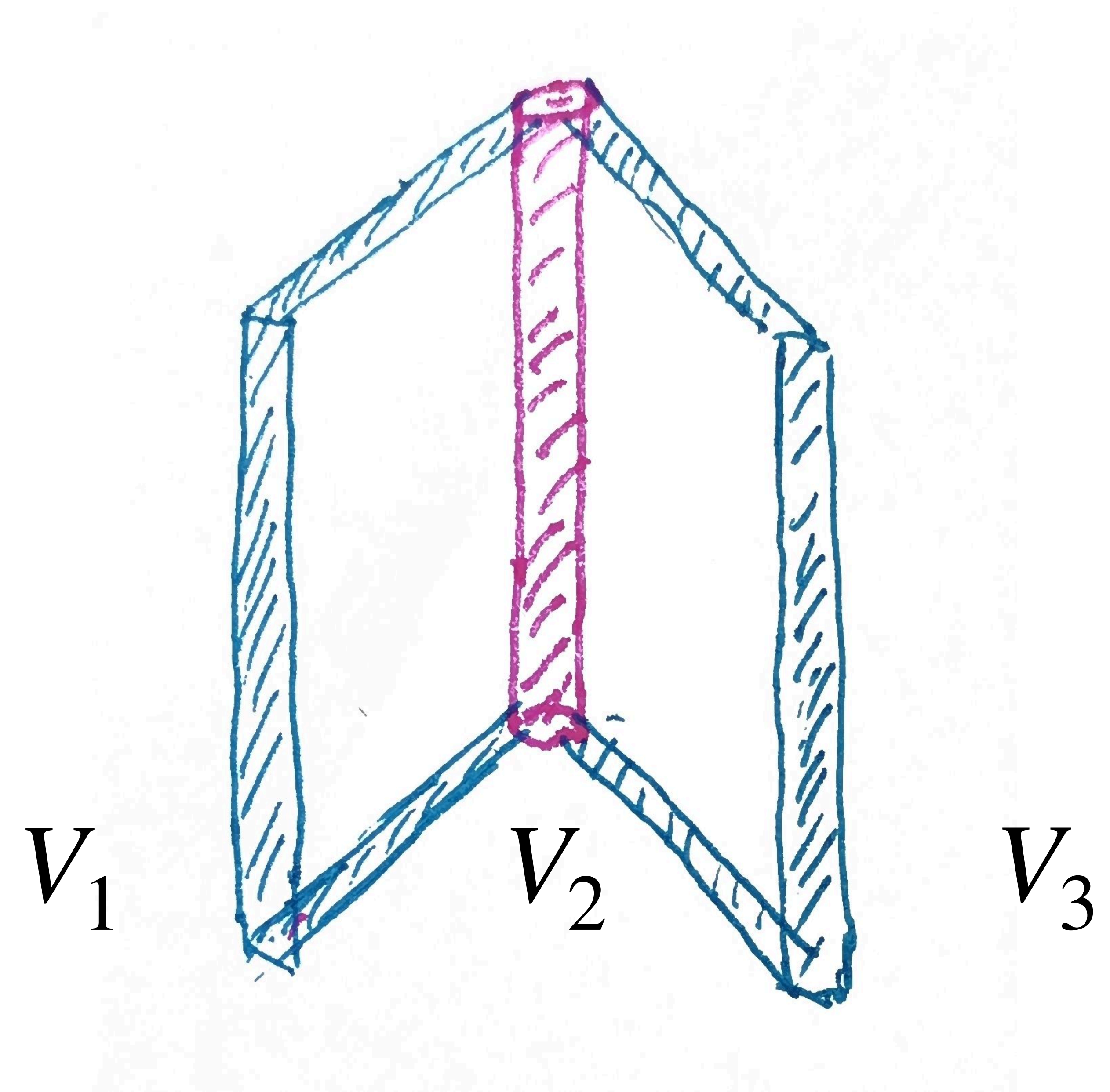
Small instantons bias domain walls

$$V(a) \sim \Lambda_{\rm QCD}^4 \cos\left(6\frac{a}{f_a}\right) + \epsilon \cos\left(2\frac{a}{f_a}\right)$$



Small instantons bias domain walls

$$V_{\text{QCD}} \propto \cos\left(N_{\text{DW}}\frac{a}{f_a}\right)$$

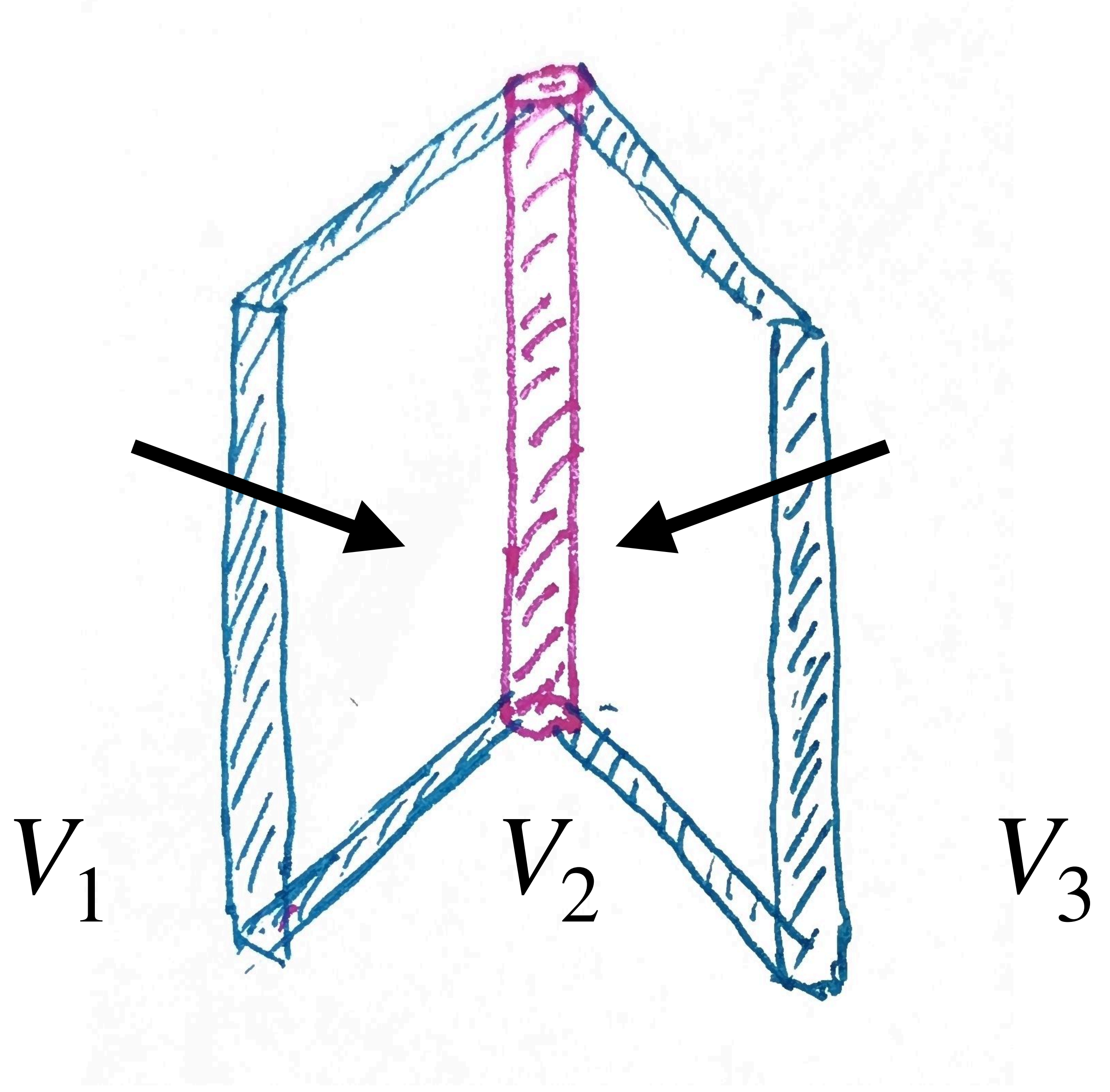


$$V_1 = V_2 = V_3$$

Small instantons bias domain walls

$$V_{\mathrm{QCD}} \propto \cos\left(N_{\mathrm{DW}}\frac{a}{f_a}\right)$$

$$\Delta V \propto \cos\left(\frac{a}{f_a}\right)$$



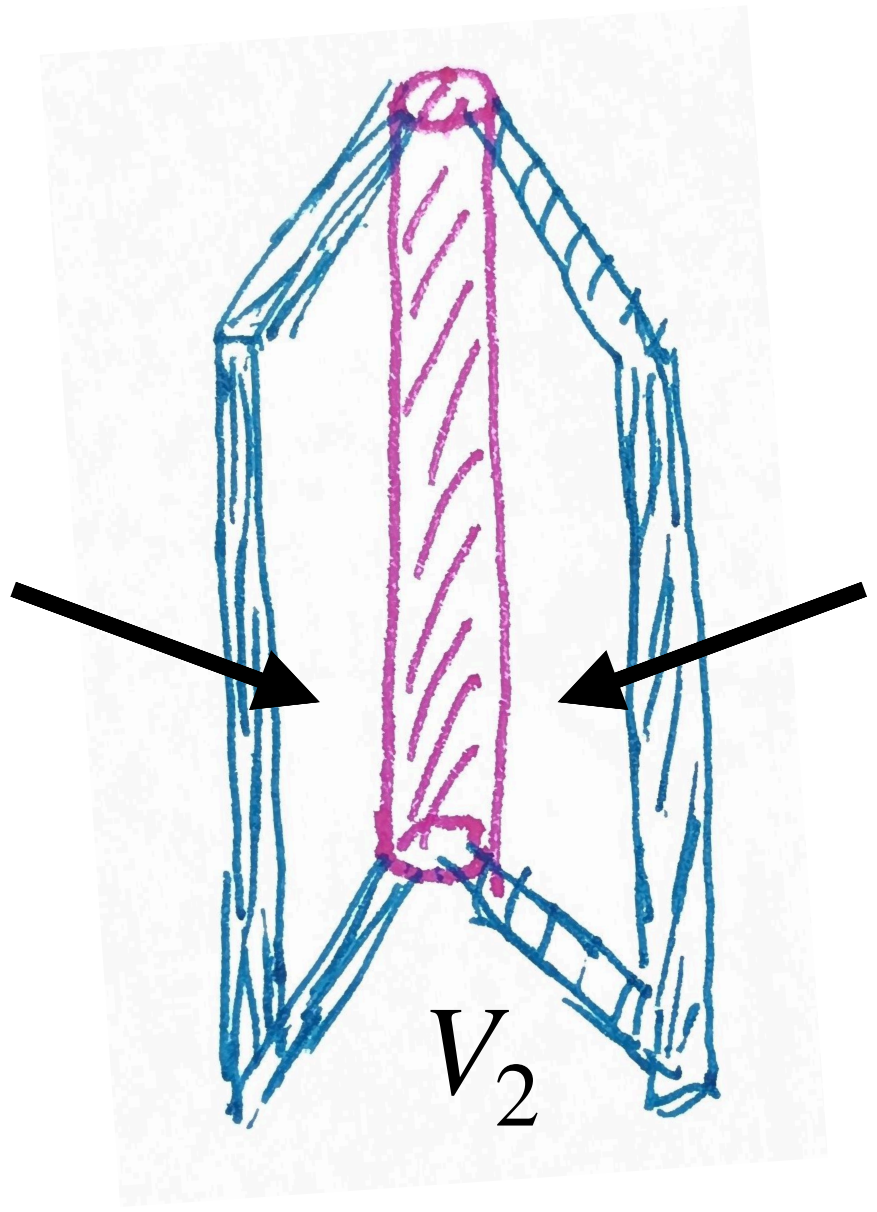
Pressure

$P \sim \Delta V$

Small instantons bias domain walls

$$V_{\text{QCD}} \propto \cos\left(N_{\text{DW}}\frac{a}{f_a}\right)$$

$$\Delta V \propto \cos\left(\frac{a}{f_a}\right)$$



Pressure
$P \sim \Delta V$

Small instantons bias domain walls

$$V_{\mathrm{QCD}} \propto \cos\left(N_{\mathrm{DW}}\frac{a}{f_a}\right)$$

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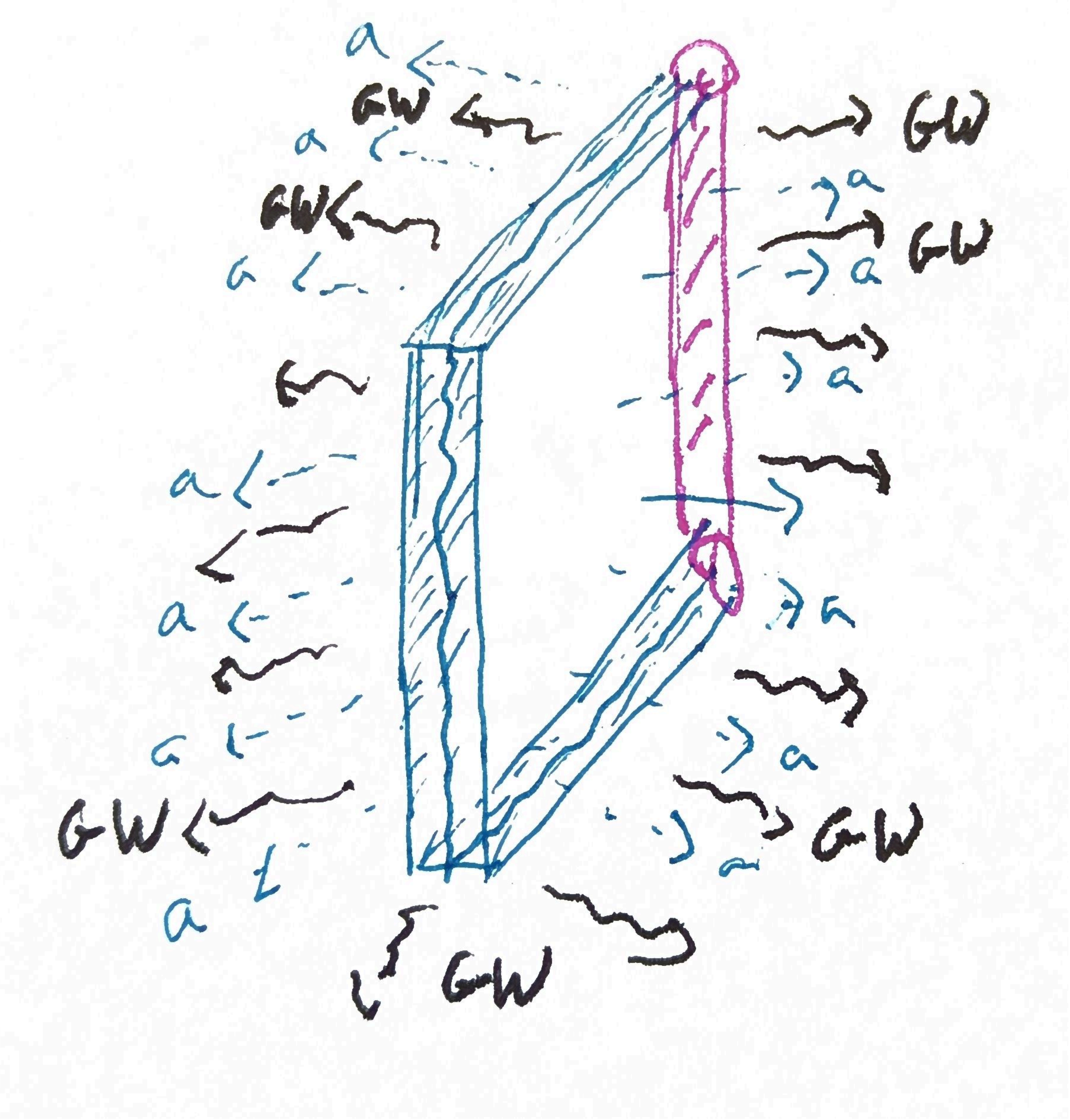
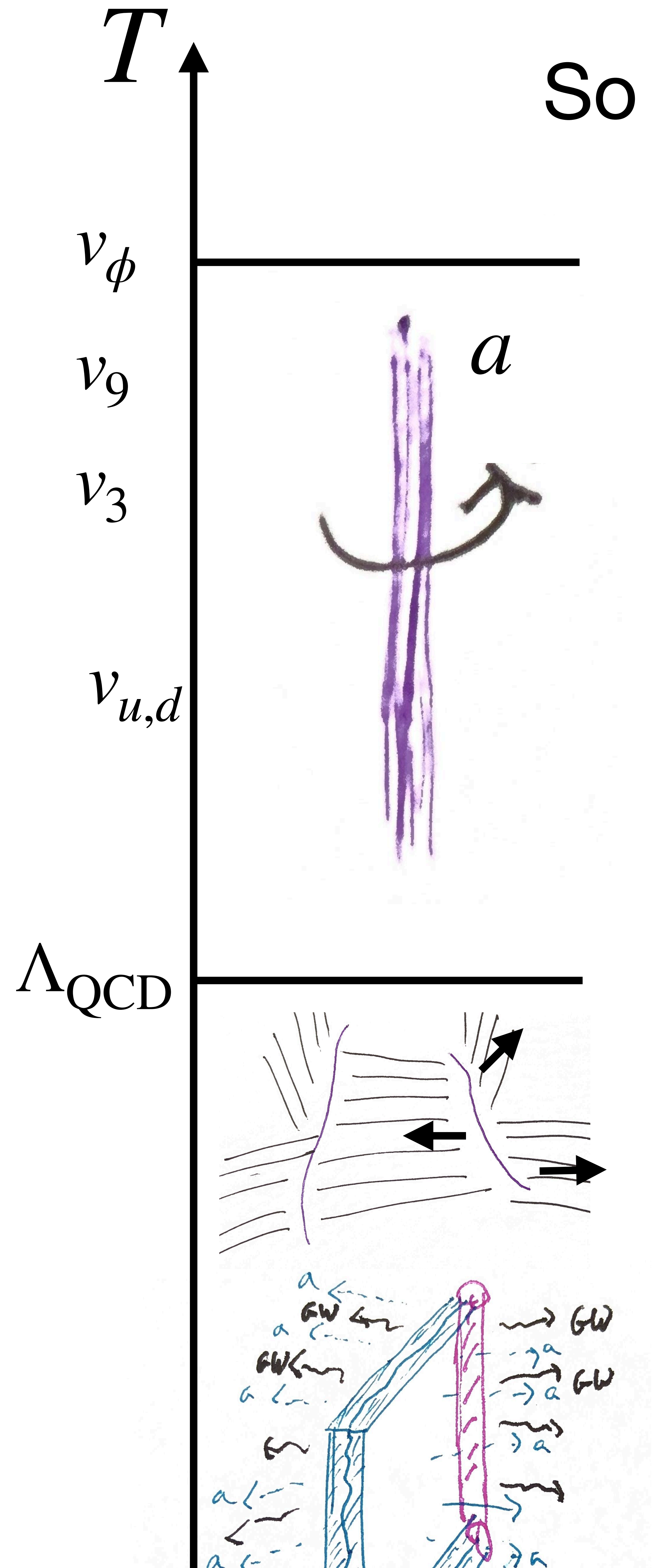


$$t_{\mathrm{ann}} \sim \frac{\sigma}{\Delta V}$$



So altogether we have solved the DFSZ domain wall problem

Step -1: Axion direction in field space

Step 0: $\dfrac{\text{gauge} \times \text{global}}{\Gamma}$ in vanilla DFSZ

Step 1: $\dfrac{\text{gauge} \times \text{gauge}}{\Gamma}$ makes $\mathbb{Z}_{N_g}$ noninvertible

Step 2: Noninvertible naturalness

Now $\mathbb{Z}_{N_g}$ domain wall network collapses at

$$T_{\rm ann} \sim \frac{v_9}{\Lambda_{\rm QCD}} \sqrt{v_\phi M_{\rm pl}}\, e^{-\frac{4\pi^2}{g^2(v_9)}} \gtrsim T_{\rm BBN}$$

What about the quartic model?

$$\mathscr{L} \supset y_u H_u Q \bar{u} + y_d H_d Q \bar{d} + \lambda_4 H_u H_d \phi^2 + \text{h.c.}$$

Same charges except $[\phi] = 2 \mapsto [\phi] = 1$,
so again $A = 2N_g$ but now overlap is gone

Here it's genuinely a $\mathbb{Z}_{2N_g}$ DW problem and NIS will not solve the issue

It's model building time!

Embed quartic model in LR

$$G_{\rm LR} = \frac{SU(2)_L \times SU(2)_R \times U(1)_{B-N_cL}}{\Gamma_{\rm LR}}$$

$$\Gamma_{\rm LR} = \{1, \mathbb{Z}_2\}$$

	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-N_cL}$	$U(1)_{\rm PQ}$
Q	2	–	+1	0
$\bar{Q} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$	–	2	−1	+1
$H = \begin{pmatrix} H_d \\ H_u \end{pmatrix}$	2	2	0	−1
φ	–	2	−3	+1
φ'	–	2	+3	+1

$$\mathscr{L} \supset yHQ\bar{Q} + \lambda_4 HH\varphi\varphi'$$

$$\Rightarrow \frac{G_{\rm LR} \times U(1)_{\rm PQ}}{\mathbb{Z}_2}$$

Wind halfway around PQ direction and halfway around $SU(2)_R$

$$\begin{pmatrix} \theta_u \\ \theta_d \\ \theta_\varphi \\ \theta_{\varphi'} \end{pmatrix} \to \begin{pmatrix} \theta_u - 2\pi \\ \theta_d + 0 \\ \theta_\varphi + 2\pi \\ \theta_{\varphi'} + 0 \end{pmatrix}$$

Now $A = 2N_g$

but $N_{\rm DW} = N_g$

Some Questions

How to generally determine the axion mode?

Multiple Higgsed gauge groups, multiple axion potentials, many scalars, …
Formalism to do this from the full field space?

Other important overlooked physics of global structure of symmetry groups?

Where is $(\text{global} \times \text{global})/\Gamma$ important?

More subtle ways to probe global structure? e.g Po-Shen's talk

What about other discrete action on group? e.g. $U(1)$ vs. $O(2)$

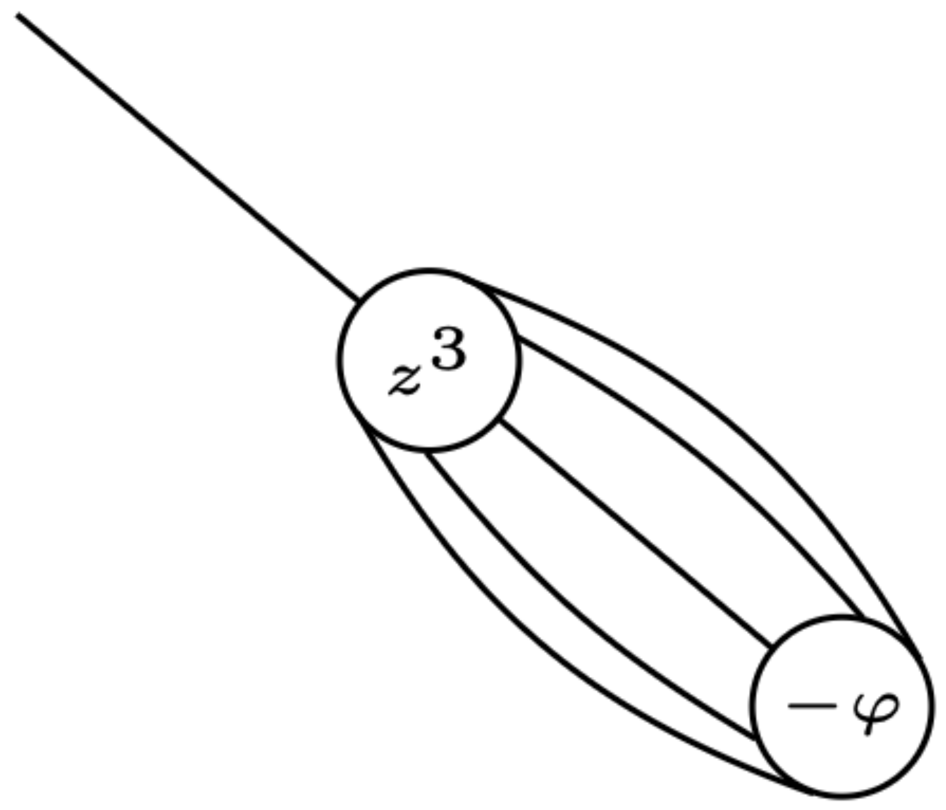
Does this really require $(G \times U(1)_{\text{PQ}})/\Gamma$ or only effectively?

Notes

- Lazarides-Shafi (1982) discover this feature with different language

$$\frac{Spin(10) \times U(1)_{\mathrm{PQ}}}{\mathbb{Z}_4} \text{ with } A = 4N_g$$

- Lu, Reece, Sun (2024) observe IR $\mathbb{Z}_N$ not enough, as the string-wall network behavior is UV sensitive.



- Suzuki, Yokosuka (2026) propose IR Lagrangian

$$S = \frac{1}{2\pi} \int N_1 \phi dC + N_2 B \wedge dA + \mathrm{lcm}(N_1, N_2)\, MC \wedge A$$