Generalized symmetries: useful for model building?

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The horror! The horror!

Conrad, 99

I don't have any good ideas.

Do you?

In QFT, particles are points
(In EFT particles are blobs)

‘Direct detection’ seems out of the question (for now)

’Indirect detection’ …

Generalized symmetries: a way to organize QFTs

with Oscar Randal-Williams & Joe Tooby-Smith

Slogan 1: Don’t do QFT, do QFTs.

Slogan 2: Don’t do ‘generalized symmetries’, do ‘generalized generalized symmetries’.

Slogan 3: The title of this workshop is tautological.

IHES
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Symmetry ~~*and*~~ Topology *in* Particle Physics !

Example. Toy-model building: 2-d gauge TQFTs

Everything in this talk is invertible

E

V E R

Y T H I N

G I N T H I S

T A L K I S T O P

O L O G I C A L

Slogan 1: Don’t do QFT, do QFTs.

A lesson from history:

QFTs in 2026 vs. geography in 1459


ETHYOPIA OCCI
AFRICA
ROSSIA
EVROPA
SEPTENTRIO
POLVS ARTICVS

A miracle: local TQFTs assemble into a ‘space’!

Why is this a miracle? The ‘c’-word …

PARENTAL

ADVISORY

EXPLICIT CONTENT

category

The ‘f’-word …

functor

What is a local TQFT?

Physics: a complex number $Z(= \int e^{iS})$

Maths: An ∞-functor $Z : \mathrm{Cob} \to \mathrm{C}$

The miracle: the ∞-category of TQFTs is in fact a ‘space’

a.k.a. ‘the cobordism hypothesis’

Baez-Dolan, Lurie

cf. Grothendieck

By ‘space’, I mean ‘homotopy type’.

Roughly, a topological space made even simpler by forgetting open sets, remembering only points, paths between points, paths between paths, ...

Upshot: TQFTs assemble into arguably the simplest type of space there is.

(QFTs assemble into a ‘smooth space’: a sheaf assigning a space to every $\mathbb{R}^n$.)

n.b. no cheating: this is lossless!

What does this have to do with generalized symmetries?

Slogan 2: Don’t do ‘generalized symmetries’, do ‘generalized generalized symmetries’.

A homotopy type X has a group associated to each point $x \in X$: the fundamental group $\pi_1(X, x)$ (i.e. classes of loops under concatenation).

Claim: this is the 0-form symmetry group of the theory x.

Proof: cobordism hypothesis

A homotopy type X also has abelian groups associated to each point $x \in X$: the higher homotopy groups $\pi_{n>1}(X, x)$

Claim: this is the $(n-1)$-form symmetry group of the theory x.

There is more, but not much more.

A homotopy type X is classified by

1. a set $\pi_0(X)$
2. groups $\pi_1(X, x)$
3. abelian groups $\pi_{n>1}(X, x)$
4. actions of $\pi_1(X, x)$ on $\pi_{n>1}(X, x)$
5. Postnikov invariants, e.g. $k \in H(B\pi_1, \pi_2)$

Upshot: There is more to symmetry than ‘generalized symmetries’; we have ‘generalized generalized symmetries’ of QFT x. It’s natural to define the generalized generalized symmetry to be the space $\Omega_x(X)$.

Slogan 3: The title of this workshop is nearly tautological.

The generalized generalized symmetry $\Omega_x X$ is a group in the sense of homotopy theory.

Such groups are equivalent (via the functors Ω and B) to pointed connected spaces.

So symmetry = (pointed, connected) topology

(For TQFTs, we have $X \cong \coprod_{[x]} B\Omega_x(X)$ so symmetry tells you (almost) everything. This is far from true for QFTs.)

Example. Toy-model building

2-d gauge TQFTs

Consider 2-d $\mathbb{Z}_p \times \mathbb{Z}_p$ gauge TQFT
There is a theta-term action $q \in H^2(B\mathbb{Z}_p^2, U(1)) = \mathbb{Z}_p$

Dijkgraaf & Witten, 90

Claim: there are 0- and 1-form symmetries $\mathbb{Z}^2_{\gcd(p,q)}$.

Gaiotto & al, 1412.5148

In fact we have $S_{\gcd(p,q)^2}$ and $U(1)^{\gcd(p,q)^2}$

To see this, consider the space of 2-d TQFTs. It has

- $\pi_0 = \mathbb{Z}^{\geq 1}$
- $\pi_1(m \in \pi_0) = S_m$
- $\pi_2(m \in \pi_0) = (U(1))^m$
- the action of pi_1 on π_2 permutes the factors
- $k = 0$.

Many different (2-d gauge) TQFTs are dual: roughly, quantization remembers only the number m of (projective) irreps.

The symmetry of any one theory is consequently much larger.

You're not going to see this by studying classical actions!

Is this relevant for pheno?
You could spend G€ on a collider to try to distinguish these theories
…