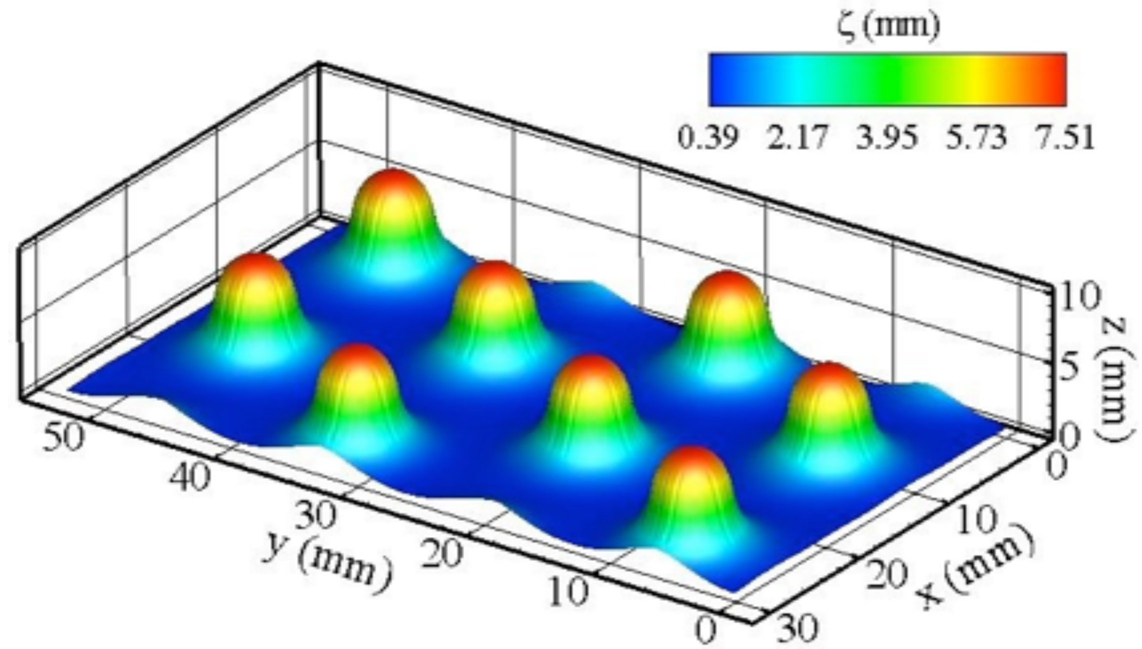
Numerical Simulation of Faraday Waves

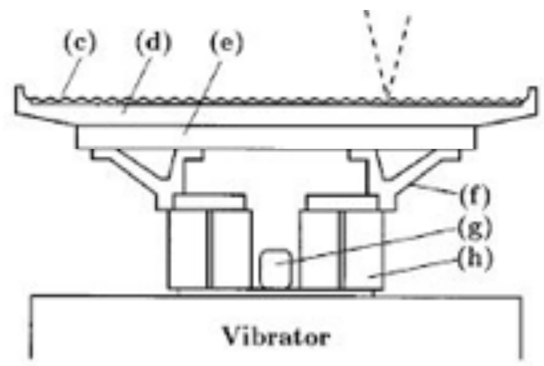


Code BLUE: D Juric, J Chergui & S Shin

Applications: N Périnet, L Kahouadji, A Ebo Adou, L Tuckerman

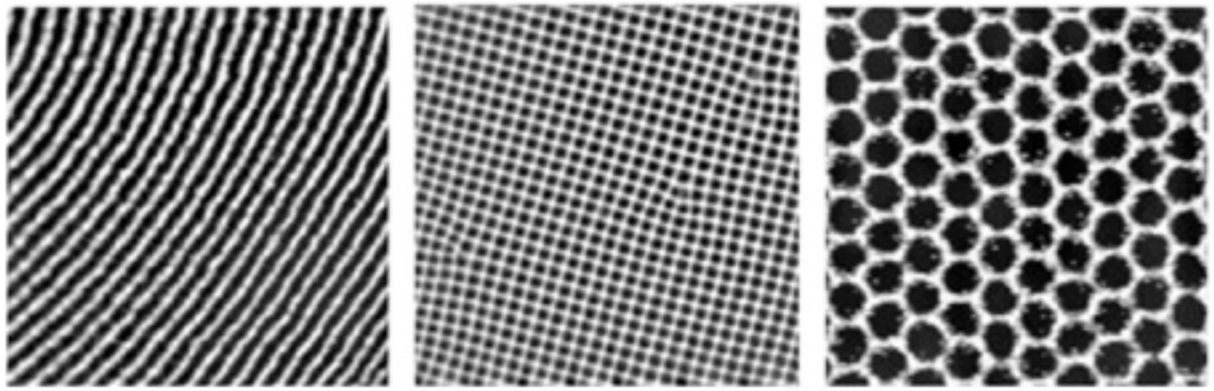
History -- Experiments

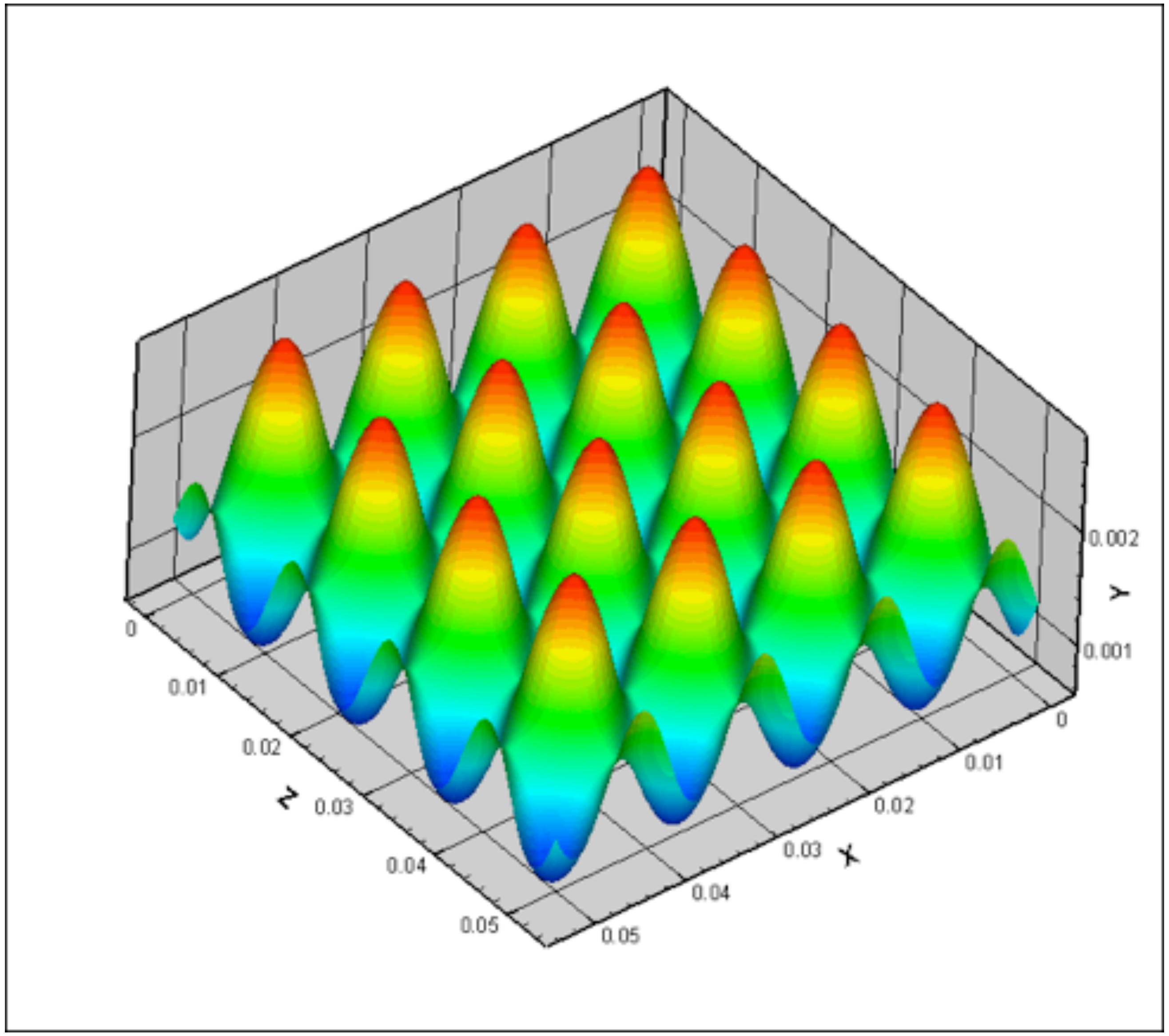
Faraday 1831



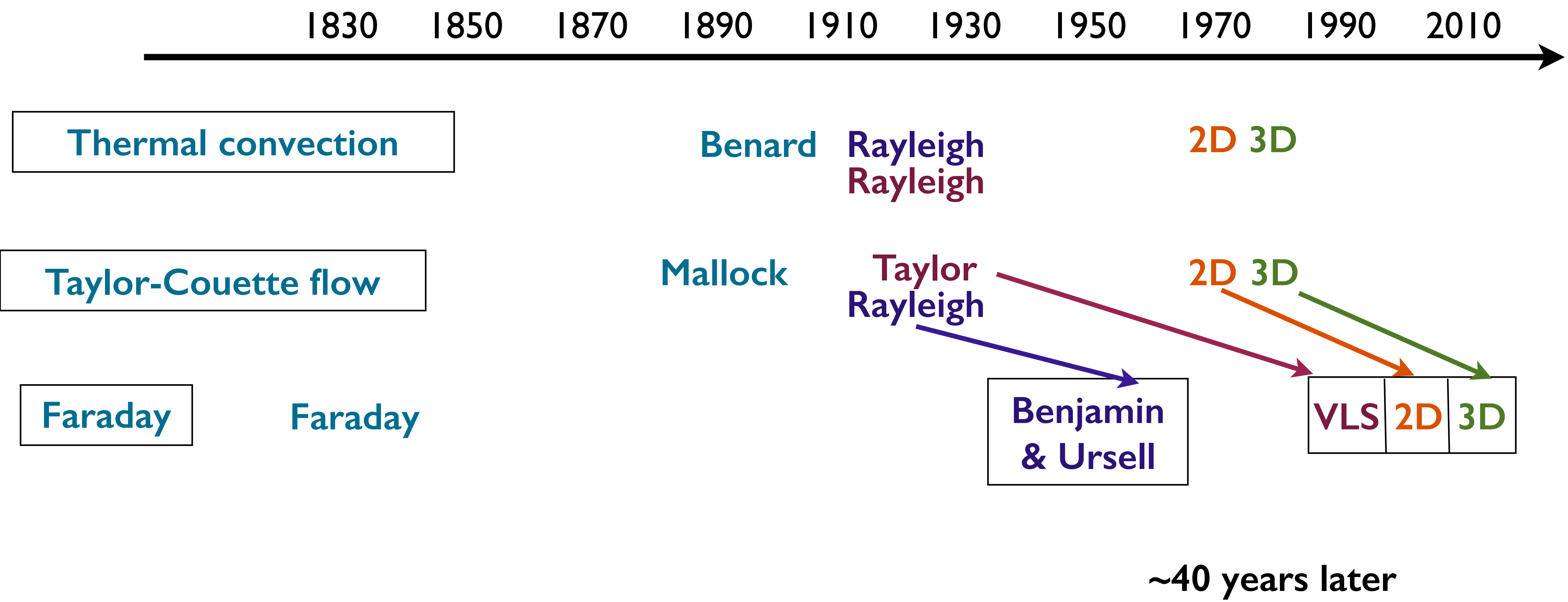
$$g - a\cos(\omega t)$$

Crystalline patterns



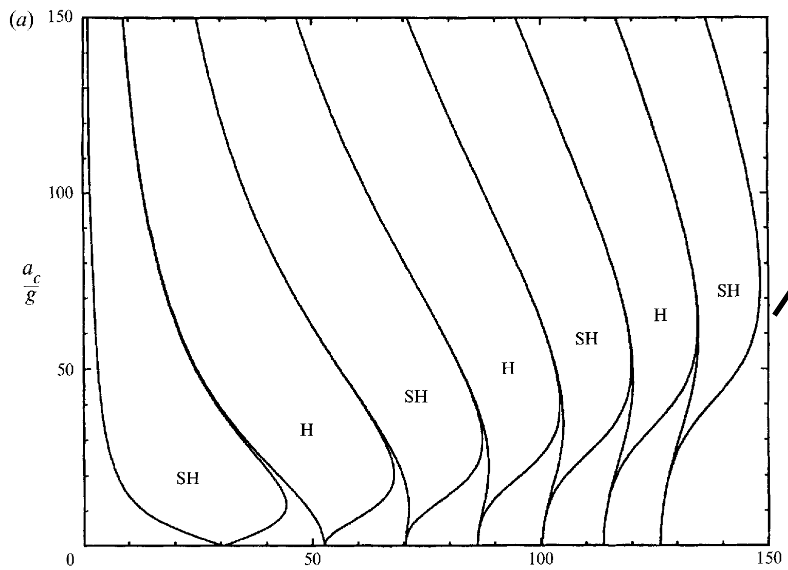


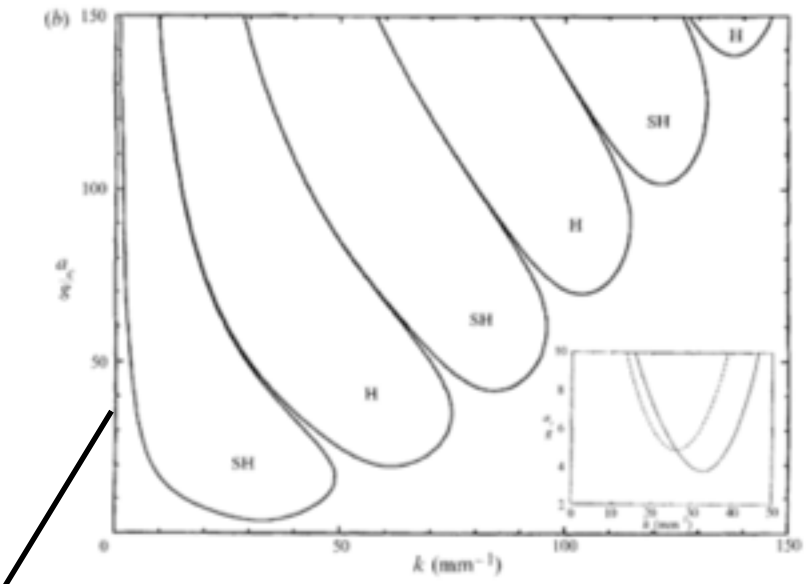
Hydrodynamic Instabilities



History -- Theory & Numerics

Benjamin & Ursell **1954** ***Linear stability analysis (inviscid)***

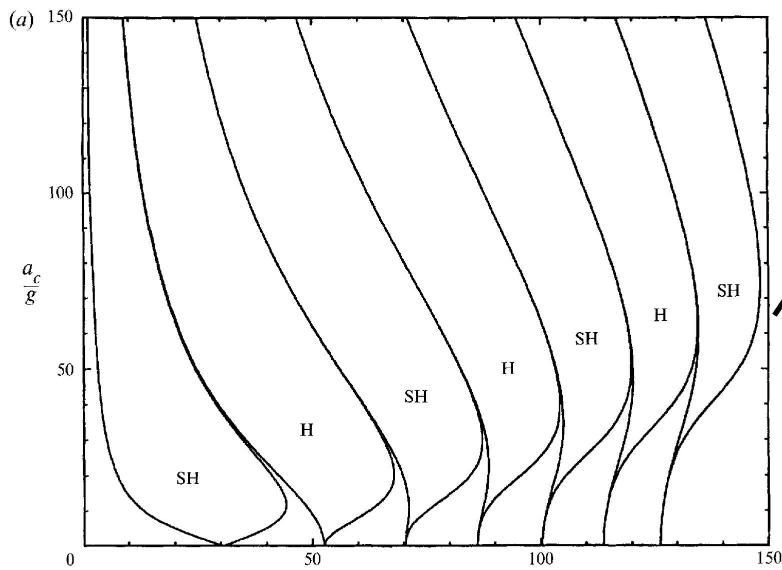


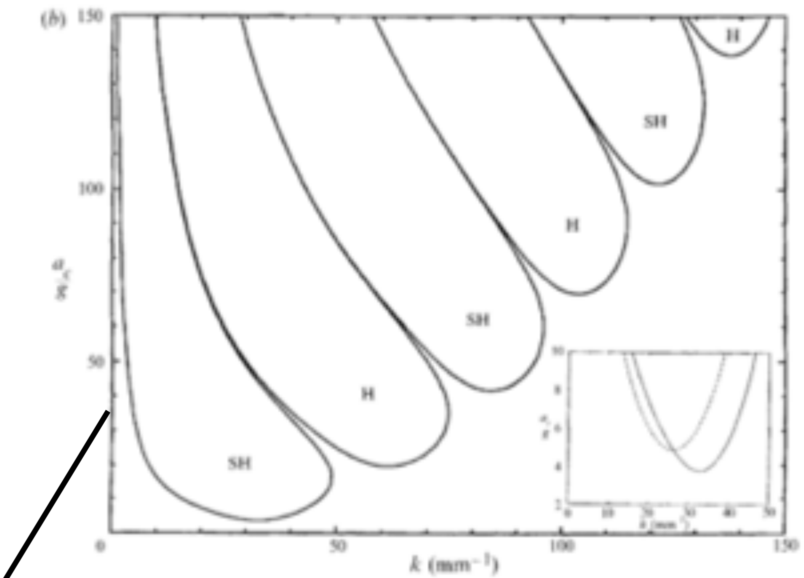


Kumar & Tuckerman **1994** ***Linear stability analysis (viscous)***

History -- Theory & Numerics

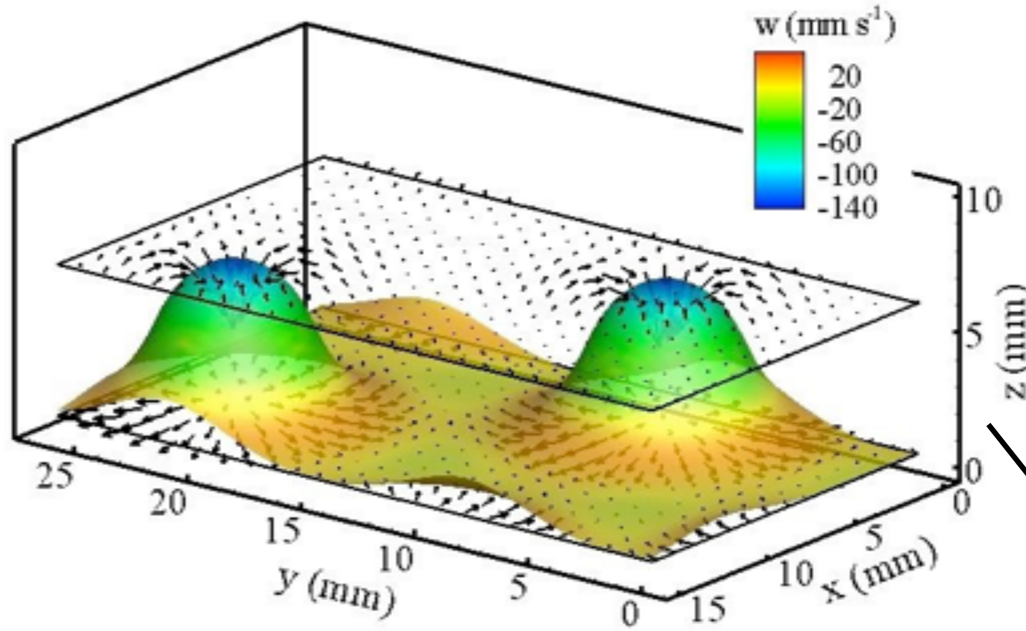
Benjamin & Ursell **1954** ***Linear stability analysis (inviscid)***





Kumar & Tuckerman **1994** ***Linear stability analysis (viscous)***

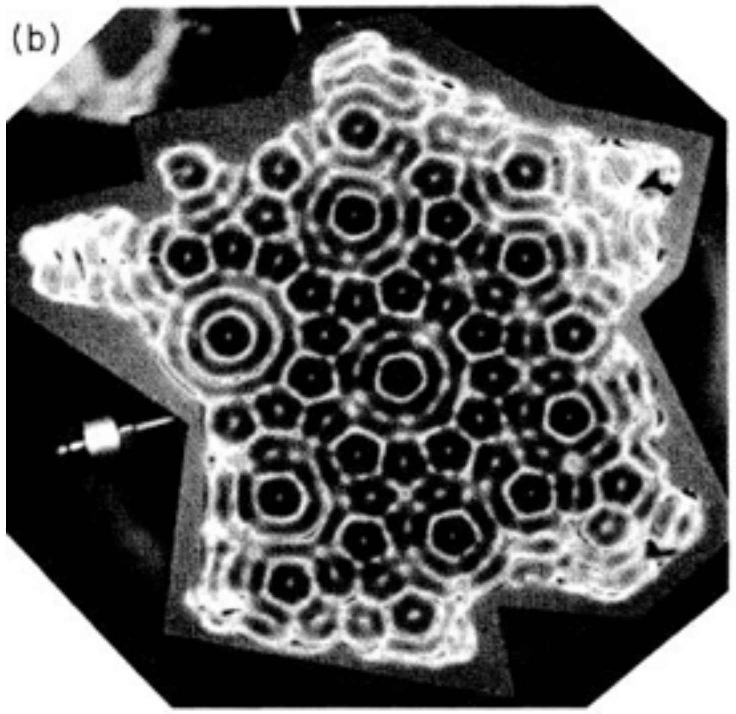
Chen & Wu **2000** ***2D numerical simulation***



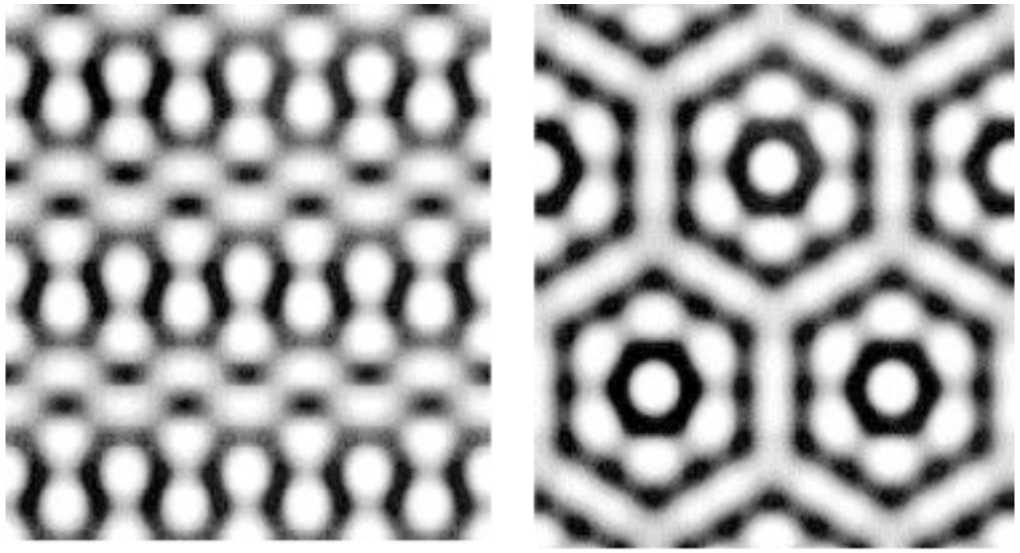
Périnet, Juric & Tuckerman **2009** ***3D numerical simulation***

Exotic Patterns

Edwards & Fauve JFM, 1994 *Quasipatterns*

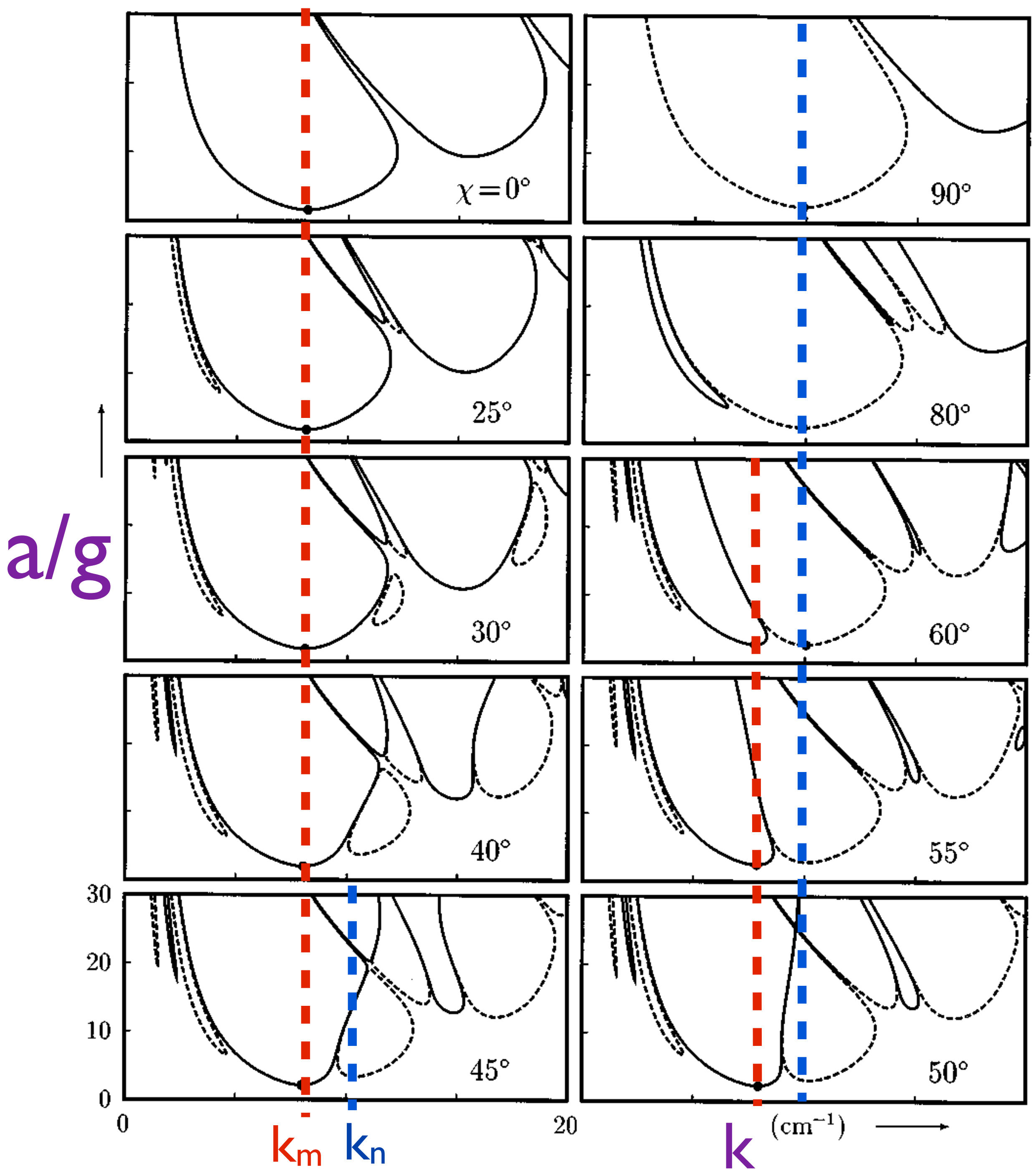


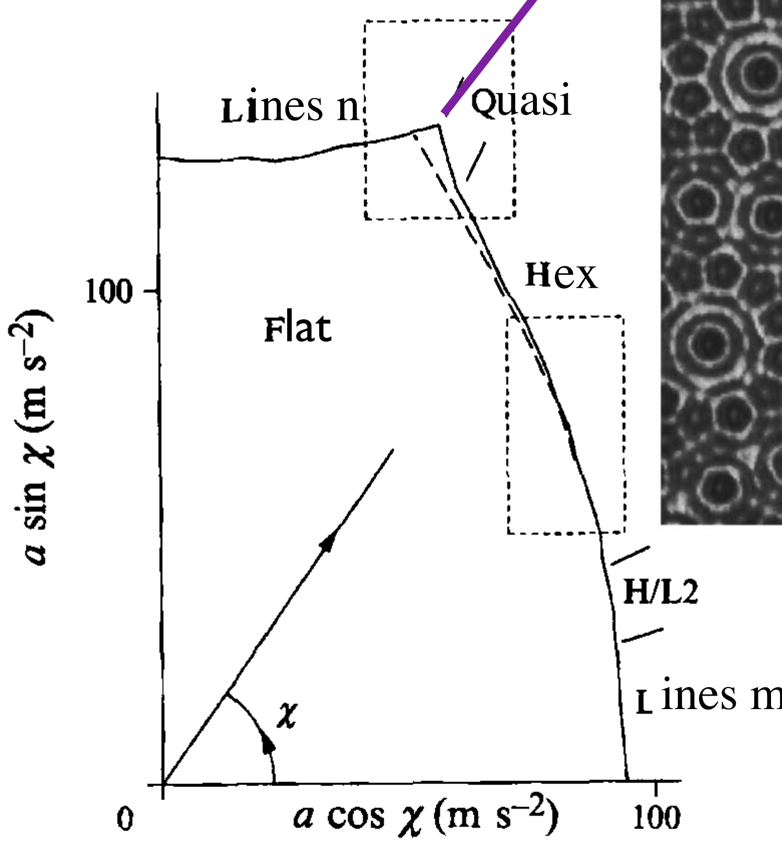
Kudrolli, Pier, Gollub Phys D, 1998 *Superlattices*



Two-frequency forcing

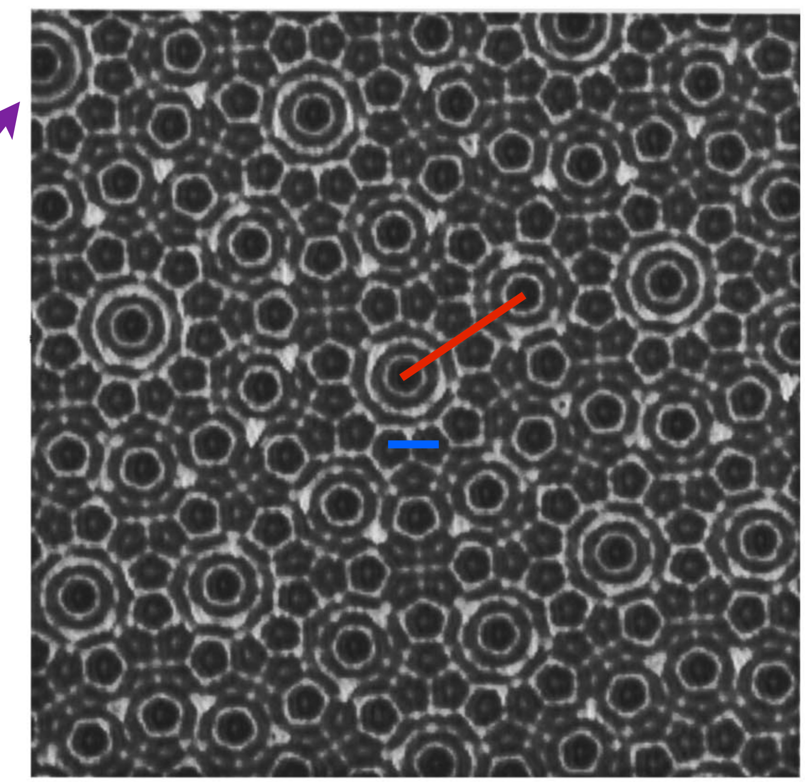
$$f(t) = a[\cos(\chi)\boxed{\cos(m\omega t)} + \sin(\chi)\boxed{\cos(l\omega t + \phi)}]$$





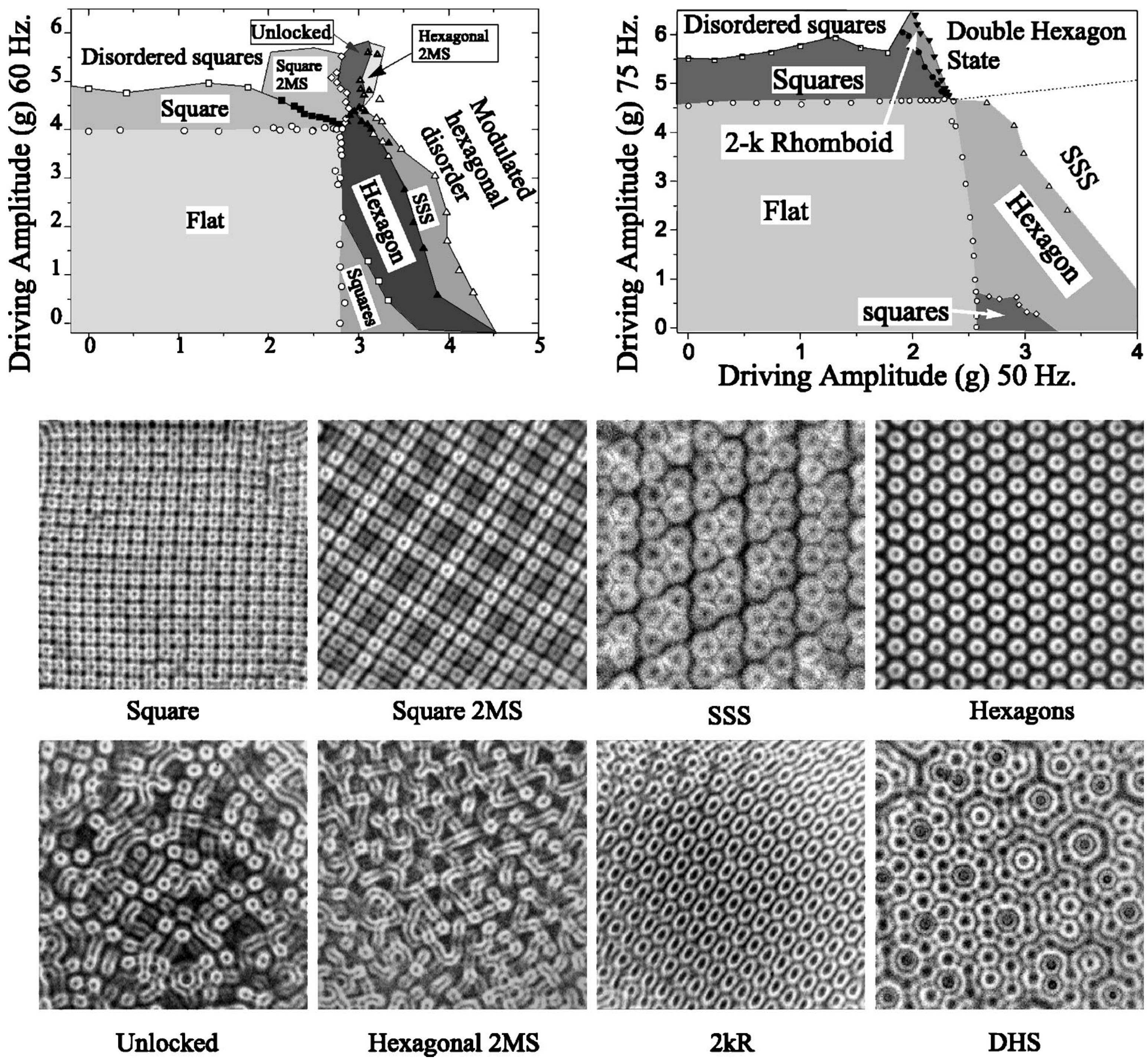
(m,n)=(4,5)

bicritical point: two length scales



Besson, Edwards, Tuckerman, 1996; Edwards & Fauve, 1993

Two-frequency forcing



(m,n)=(2,3)

Arbell & Fineberg, (2002)

Theory: Silber, Skeldon, Rucklidge, Proctor, Vinals, van de Water... (1995-2012)

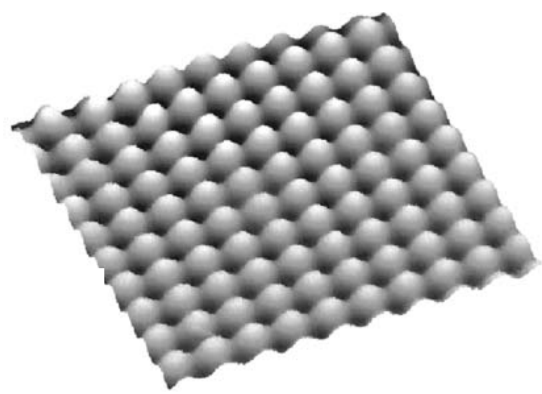
Back to single-frequency forcing

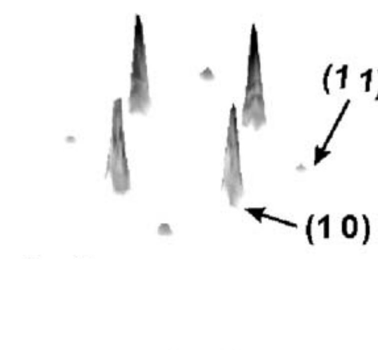
Kityk, Embs, Menkhonoshin & Wagner **2005**

Quantitative measurement of surface height

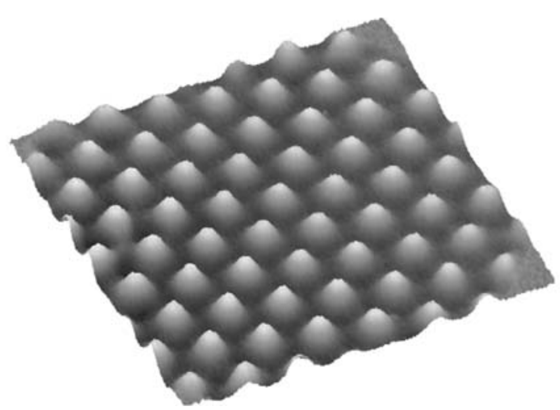
upper and lower fluids with nearly equal densities
(to match refraction indices) and high viscosities

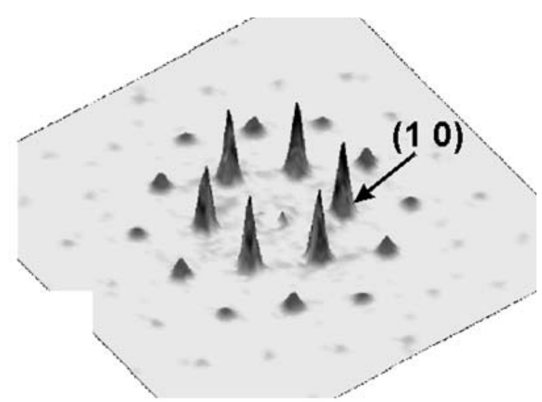
low acceleration:
squares





high acceleration:
hexagons

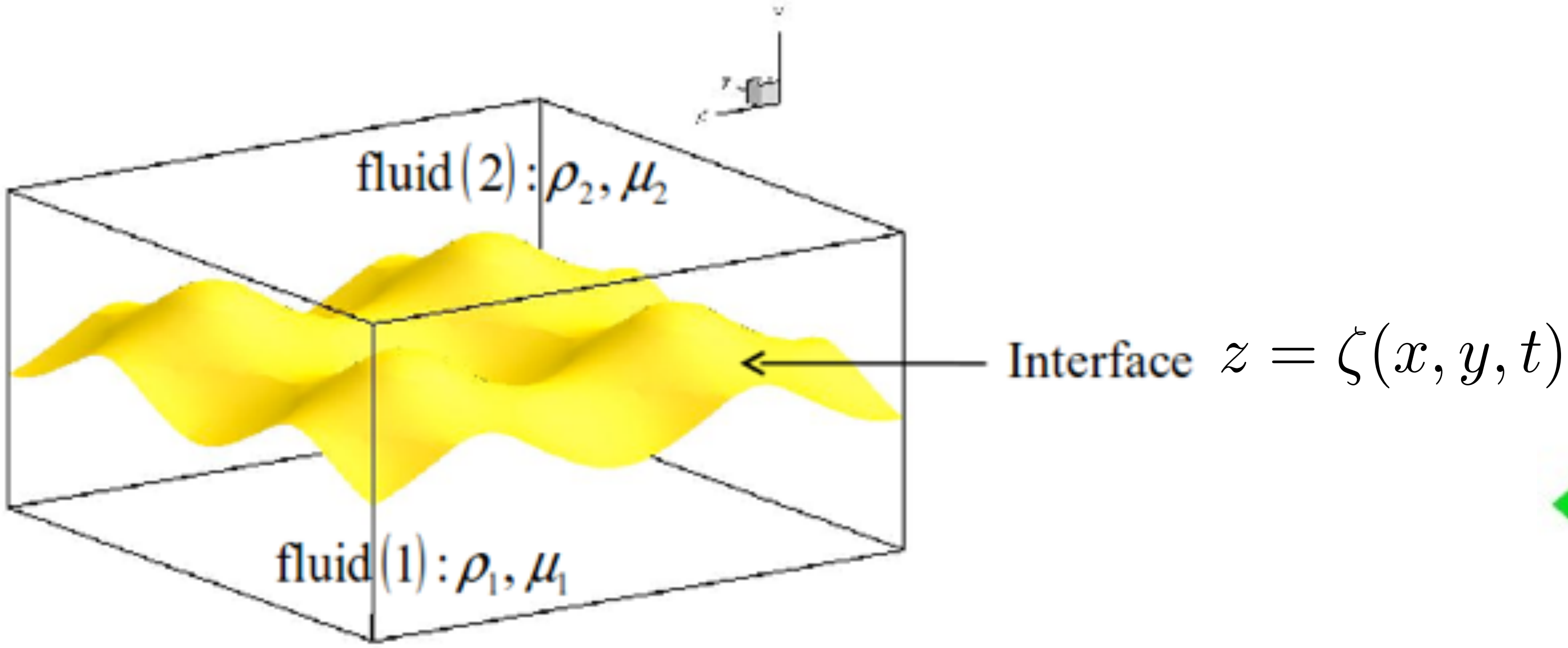


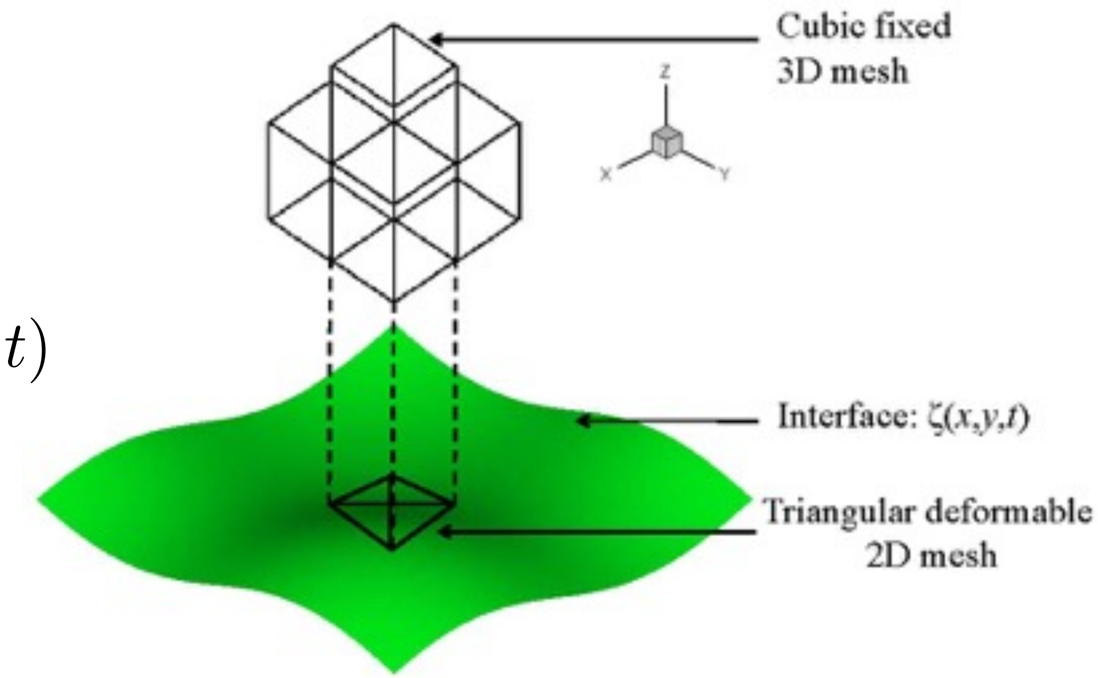


Interface

2D spatial Fourier spectrum

Equations and numerical methods



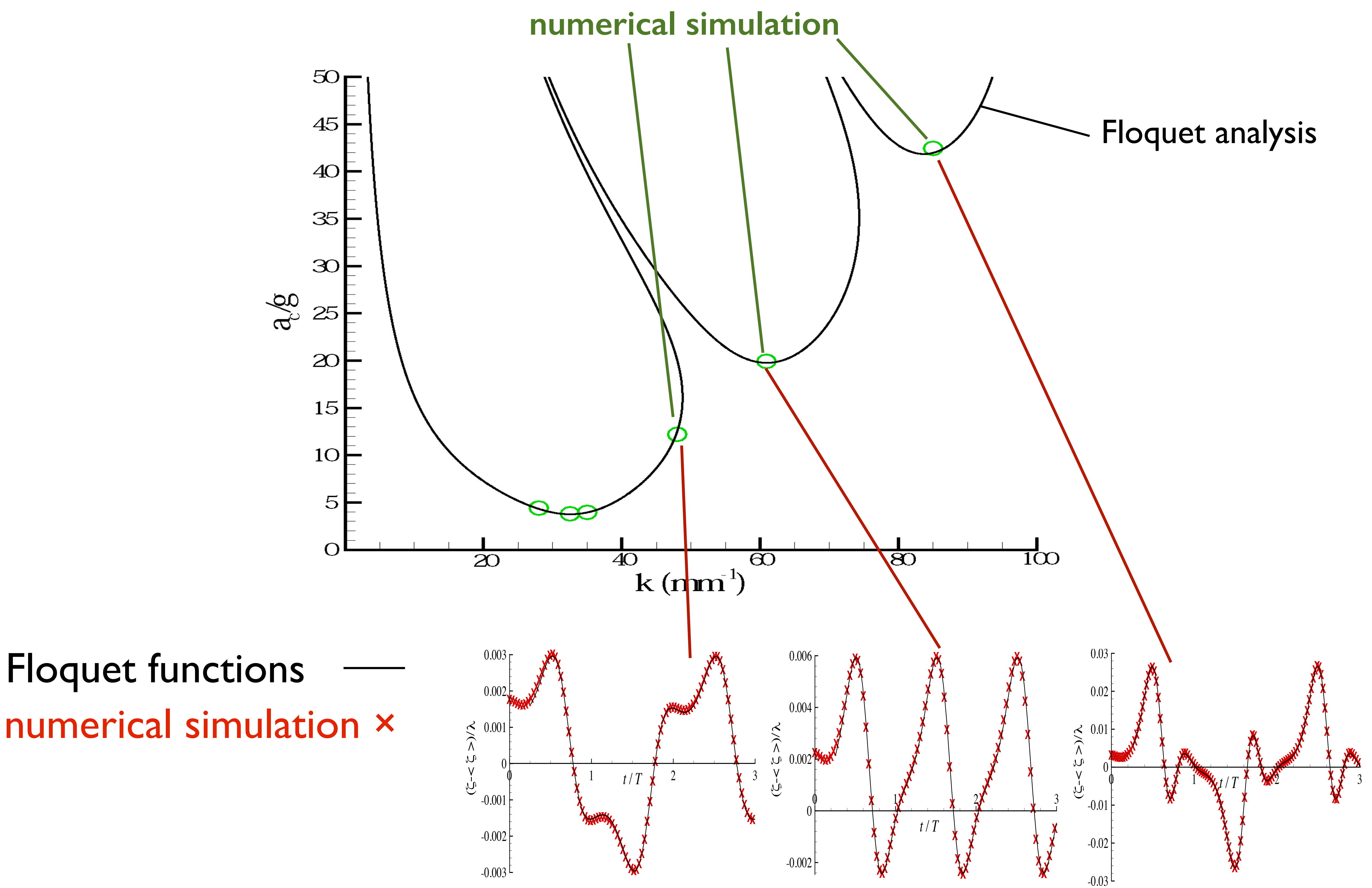


$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla \cdot \mu \nabla \mathbf{u} - [g + a\cos(\omega t)]\,\mathbf{e}_z + \int \delta(\mathbf{x} - \mathbf{x}')\,\sigma\,\kappa\,\mathbf{n}\,dV$$

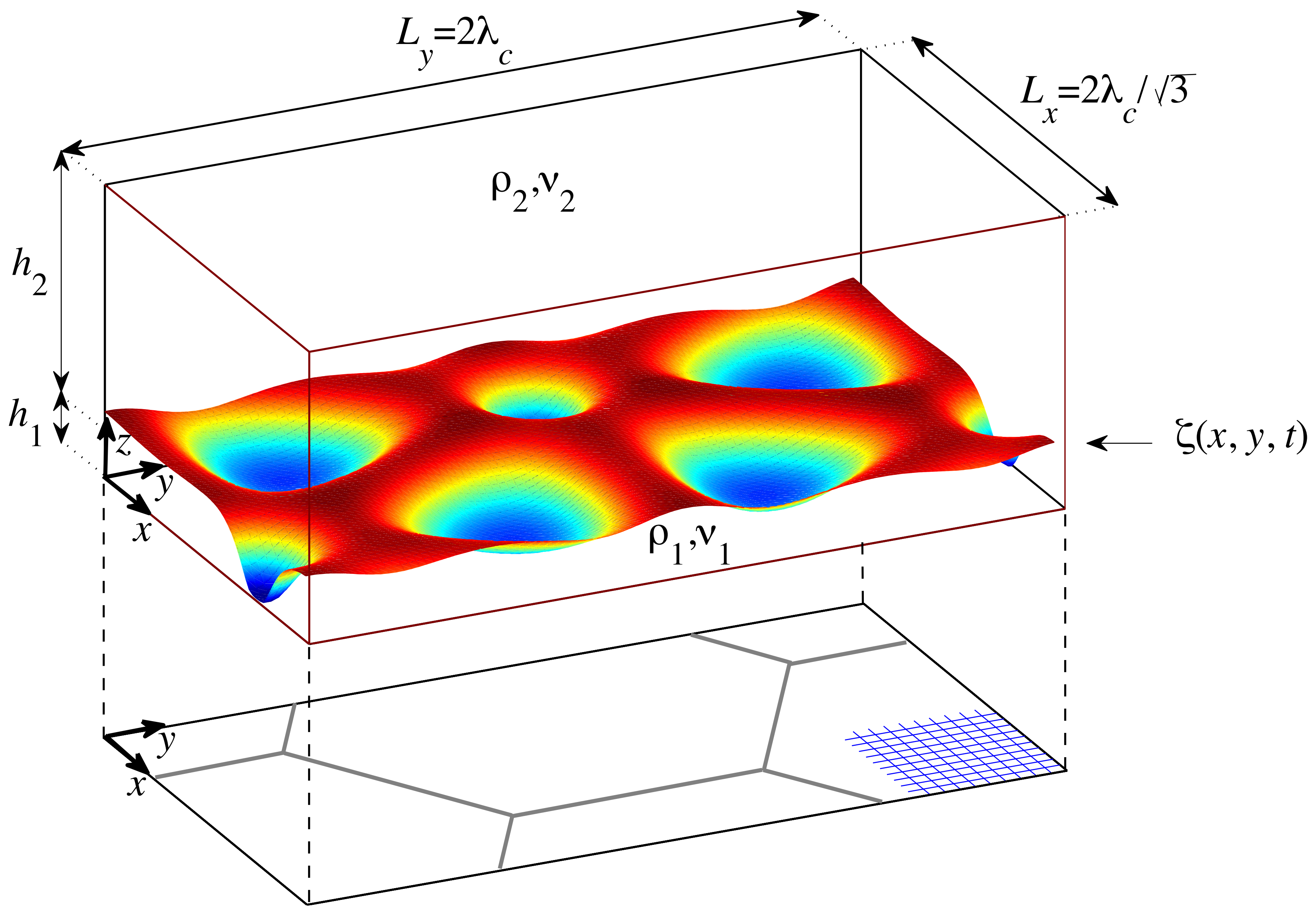
Front-tracking method (Tryggvason et al., Peskin)

Navier-Stokes in single domain with varying ρ, ν

Compare numerical simulation with Floquet analysis

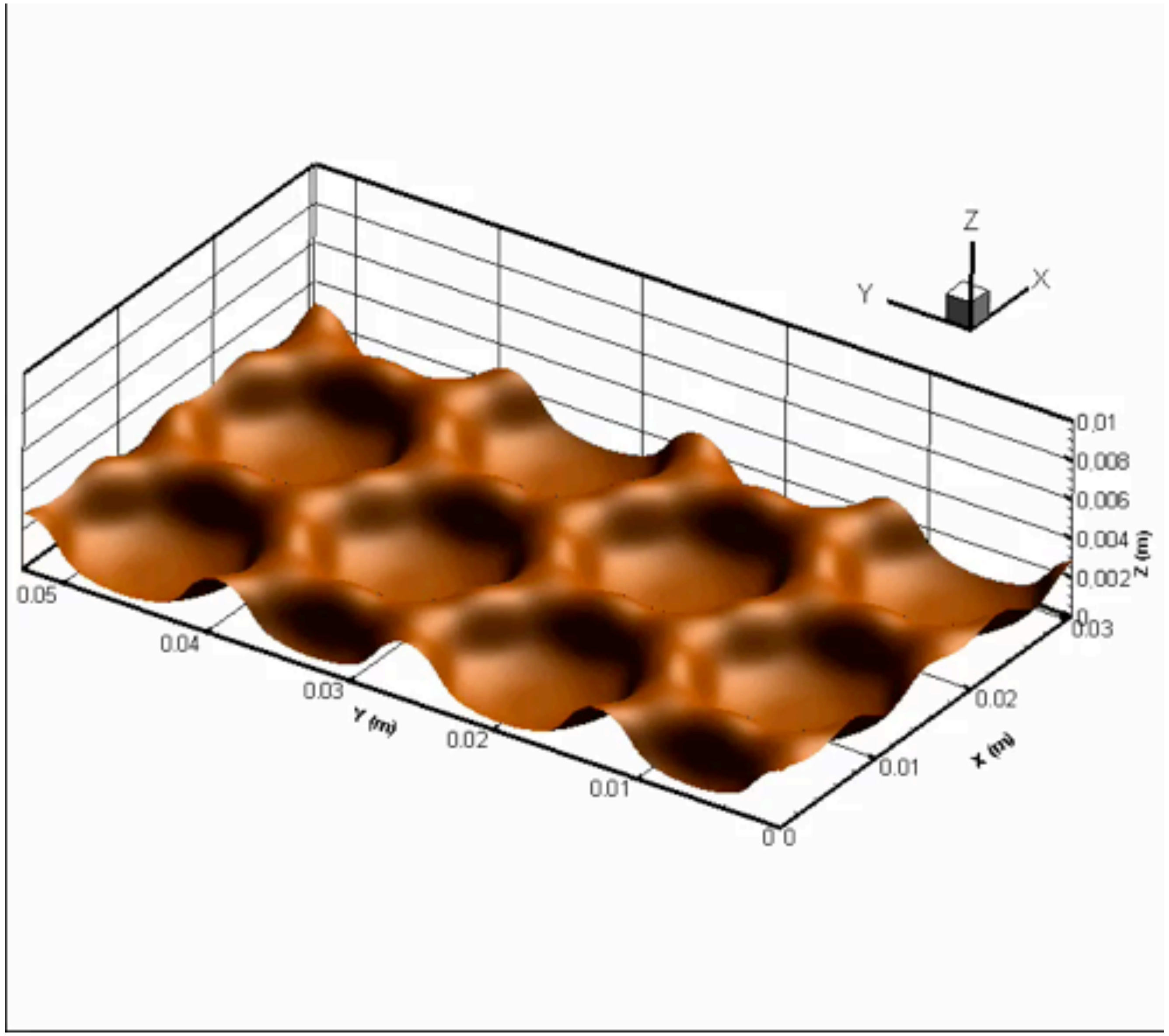


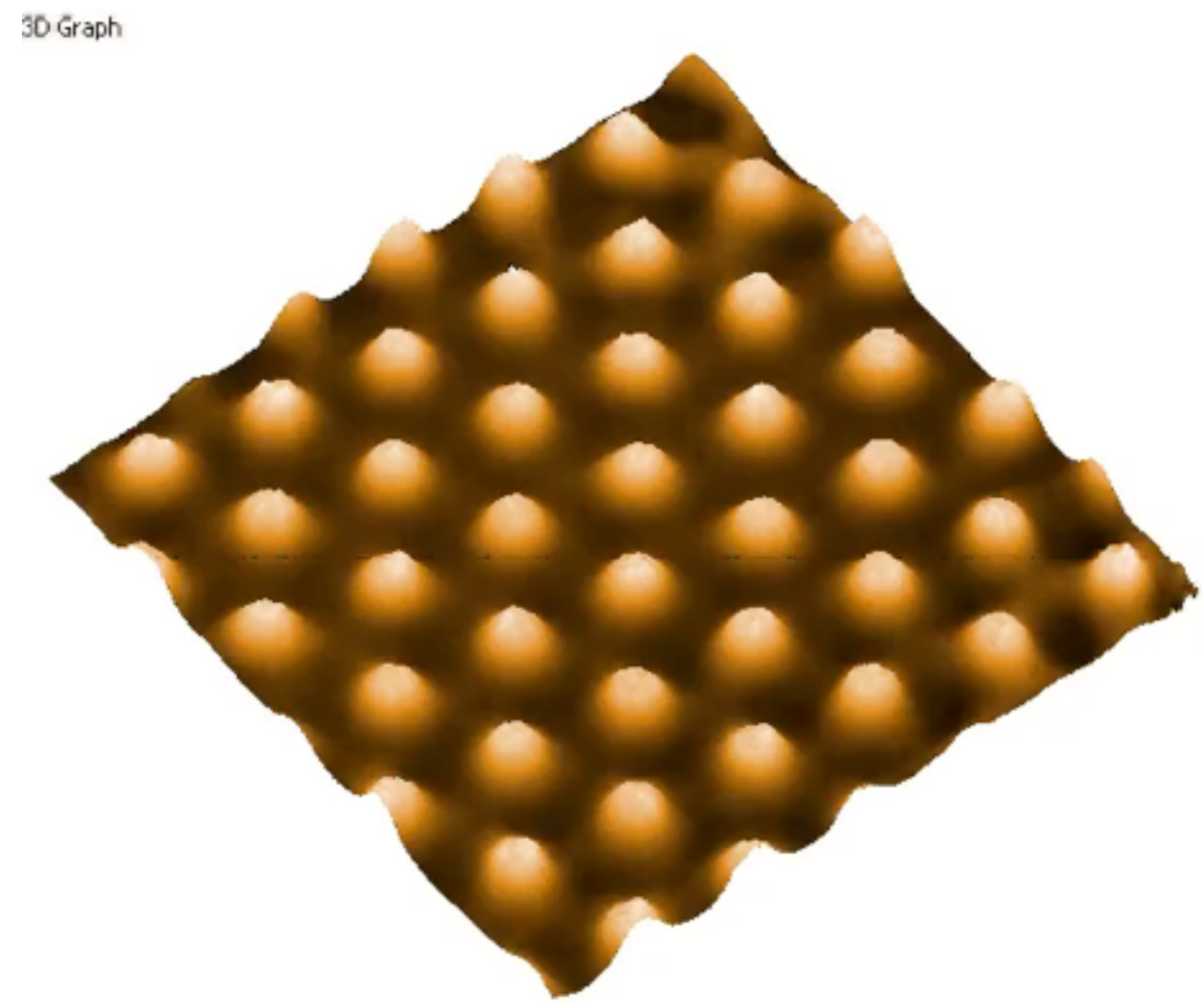
Computations carried out in *minimal hexagonal domain:* smallest rectangular domain that can accomodate hexagons

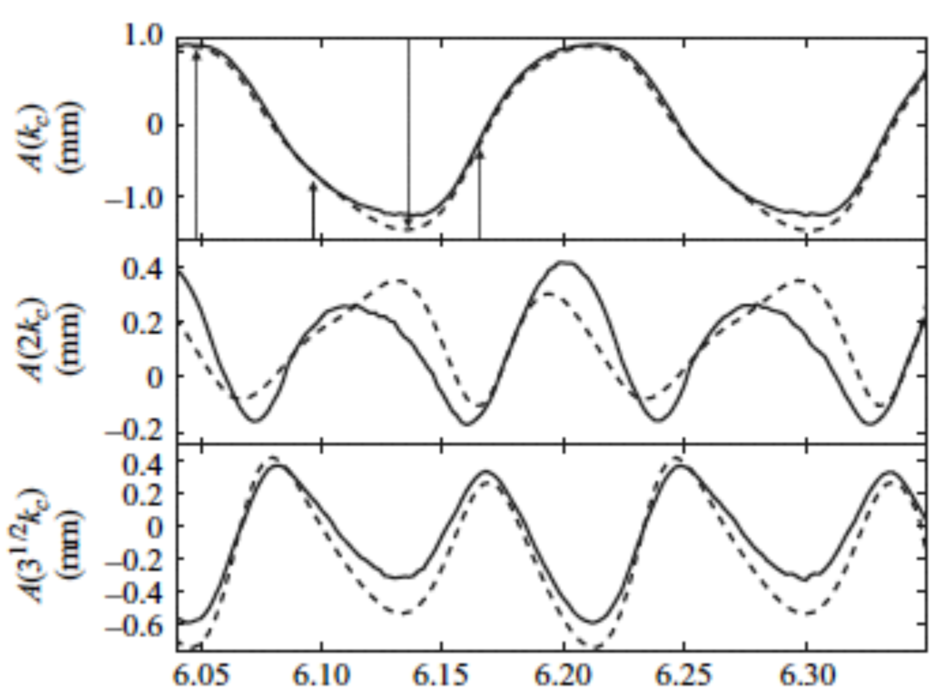


Resolution $N_x \times N_y \times N_z = 75 \times 125 \times 225$

Faraday waves







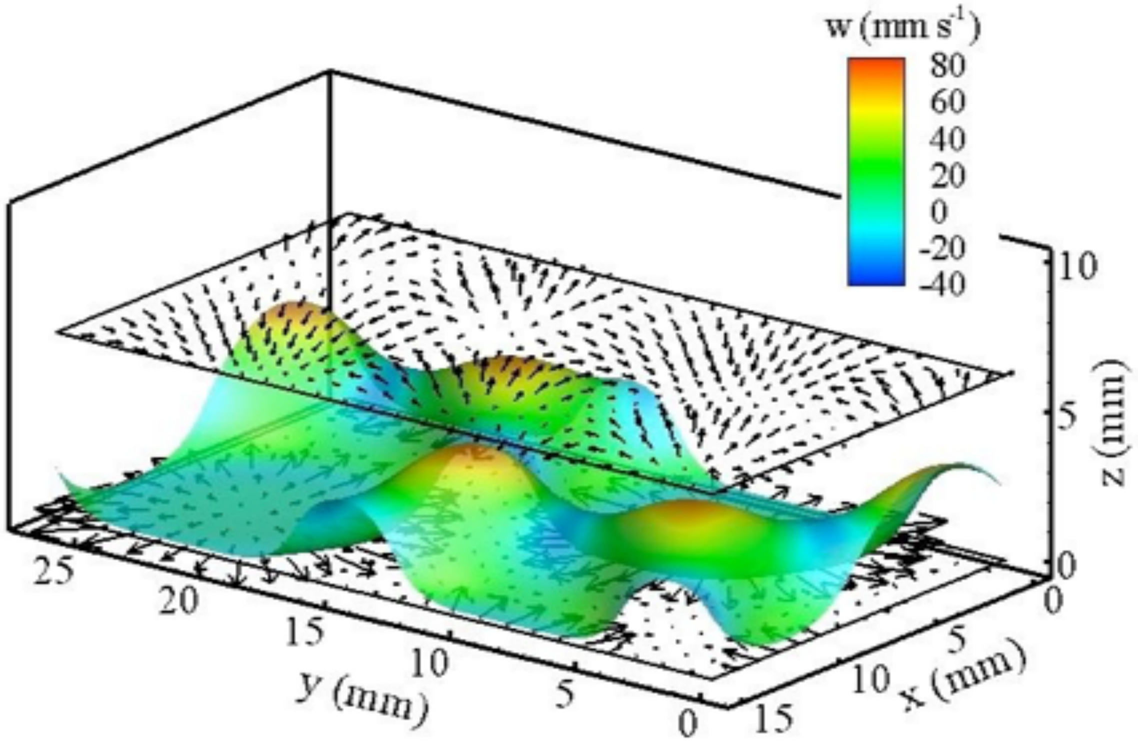
Simulation:

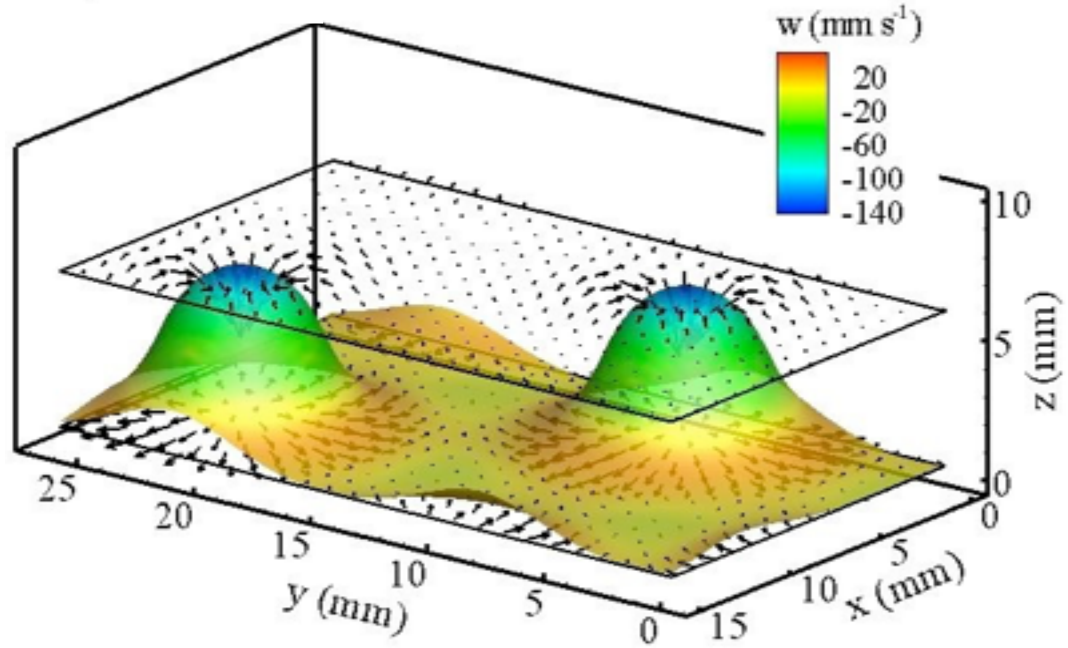
Perinét, Juric & Tuckerman
JFM (2009)

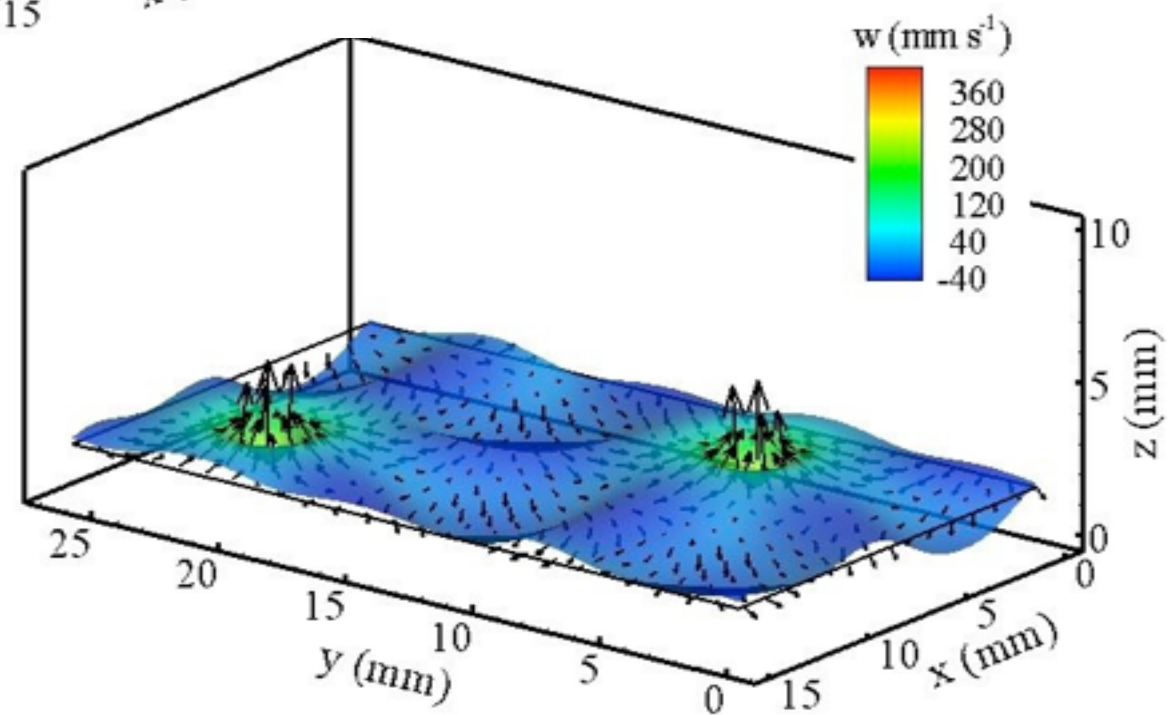
Experiment:

Kityk, Embs,
Mekhonoshin & Wagner
Physical Review E (2005)

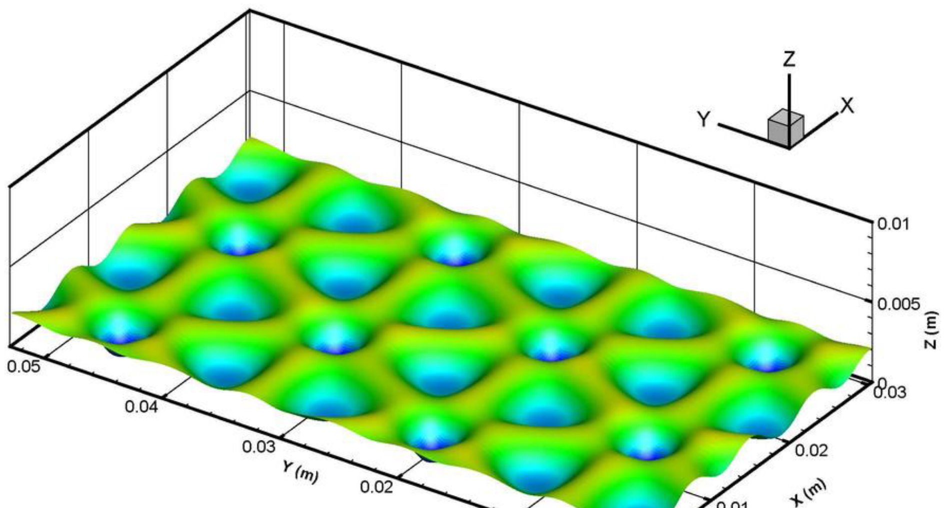
Velocity field at various instants



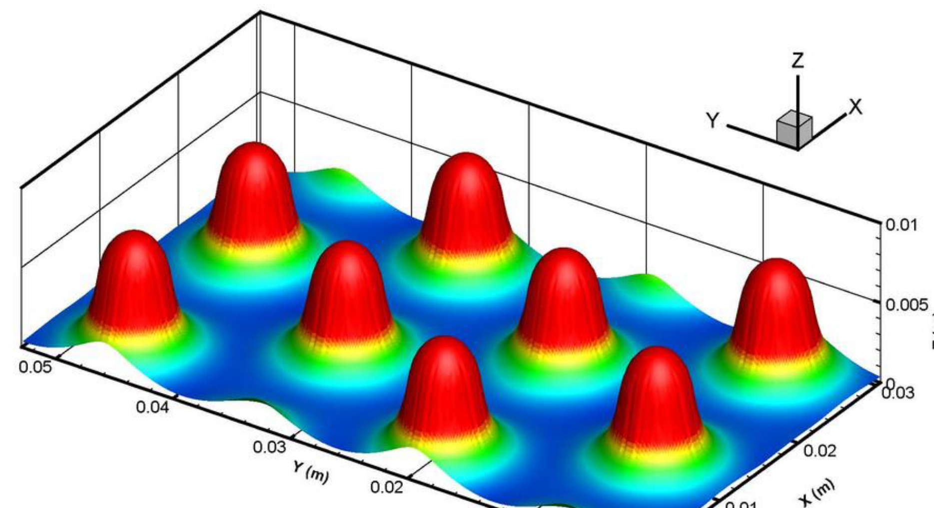




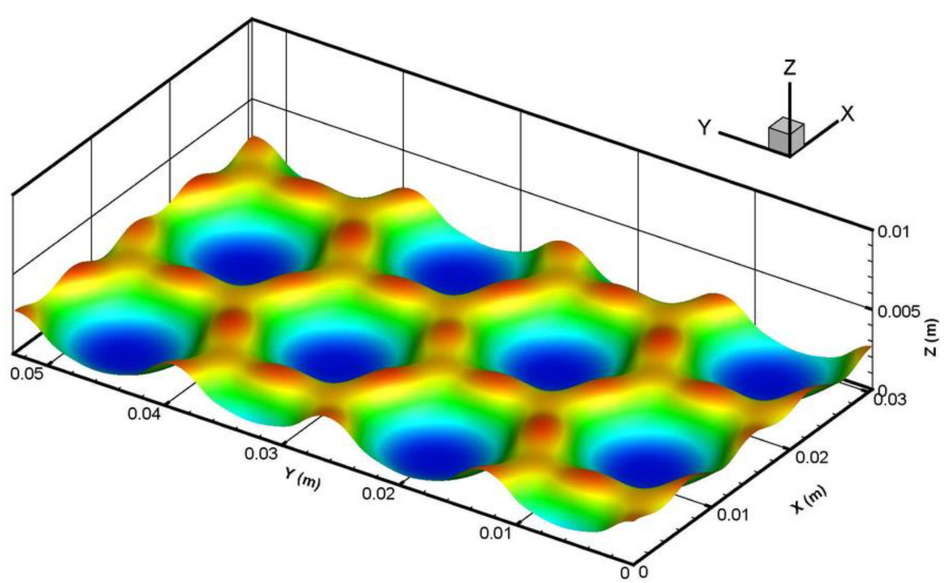
Hexagonal pattern over one subharmonic oscillation period



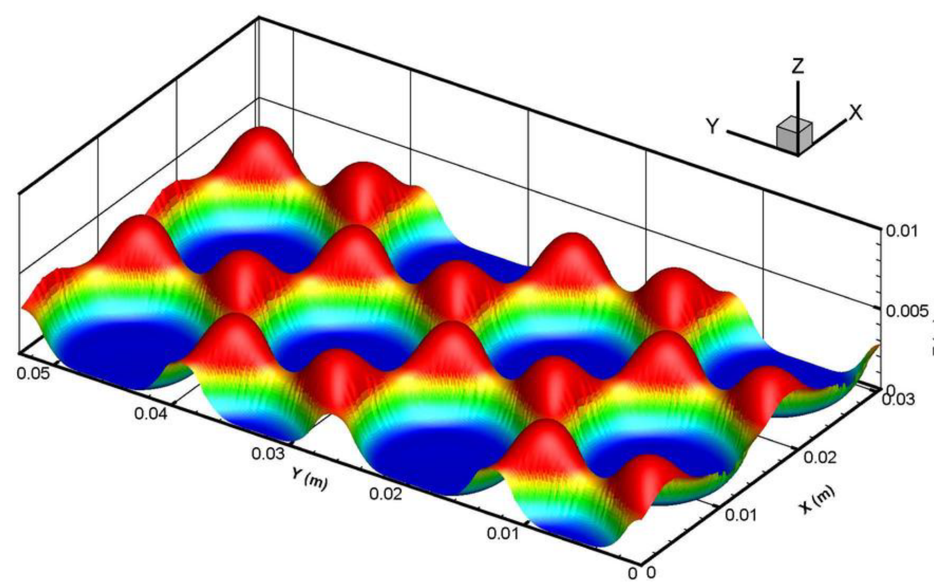
t=0



t=T/4

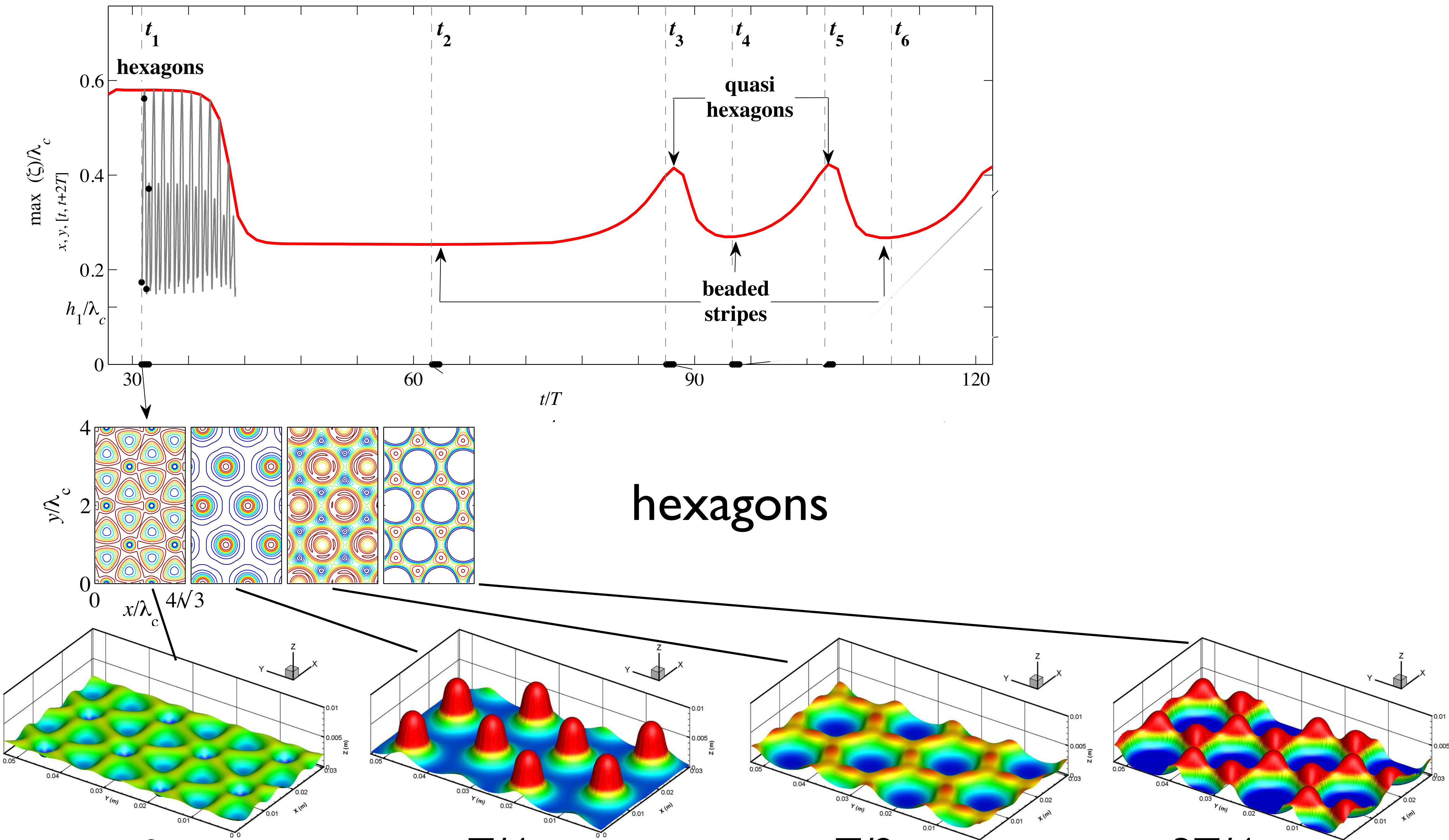


t=T/2



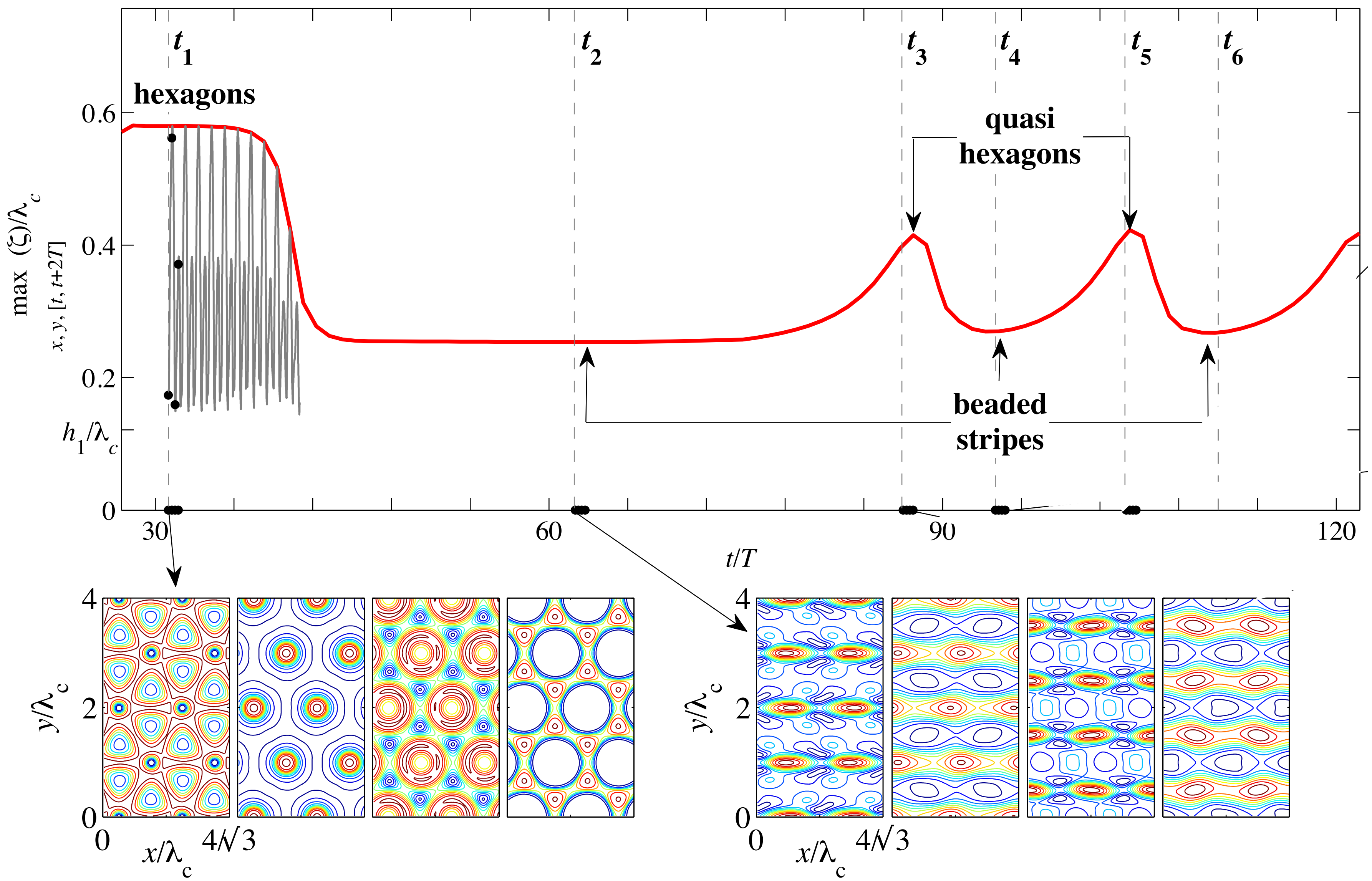
t=3T/4

Long-time evolution



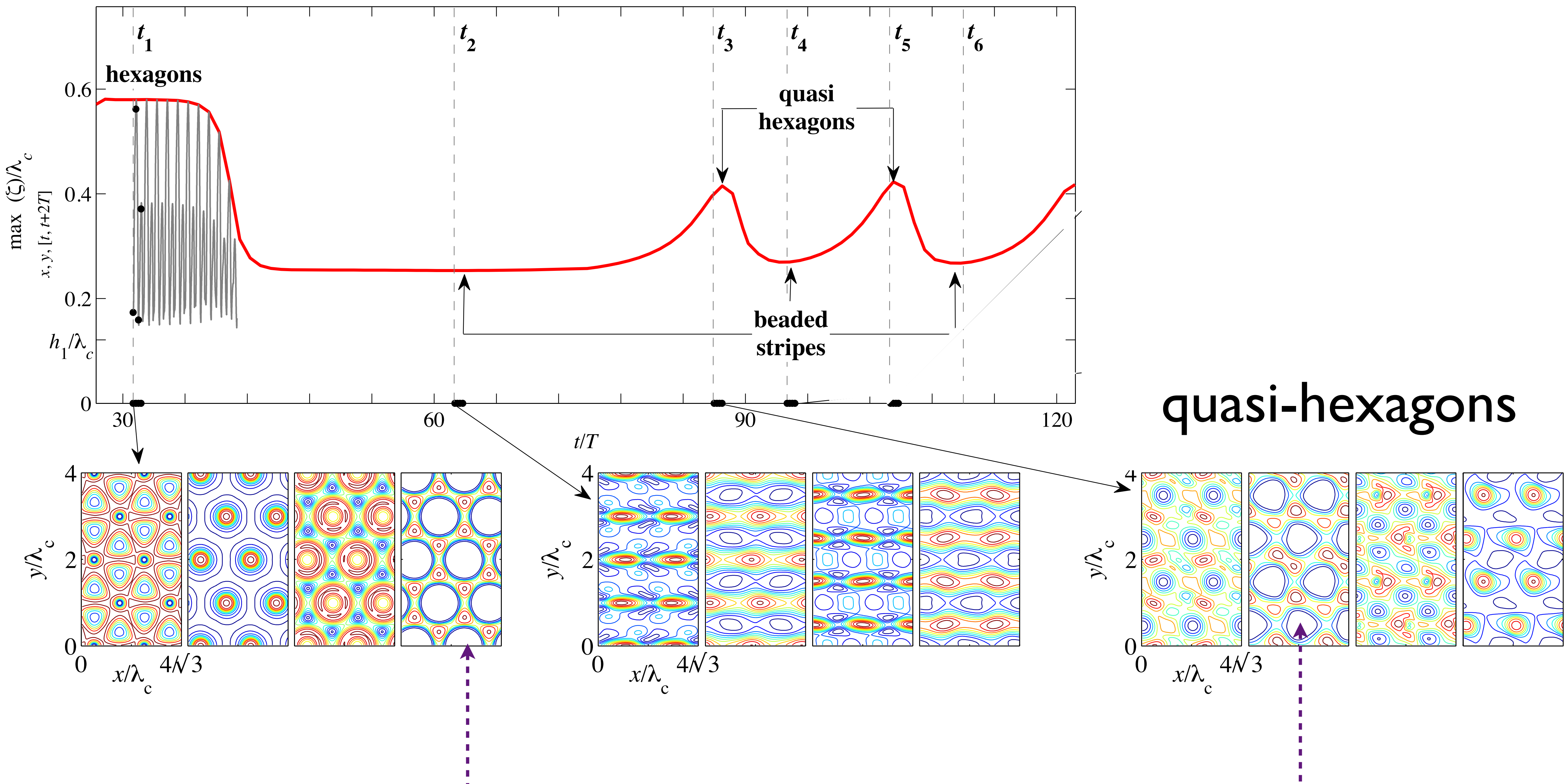
t=0 t=T/4 t=T/2 t=3T/4

Long-time evolution



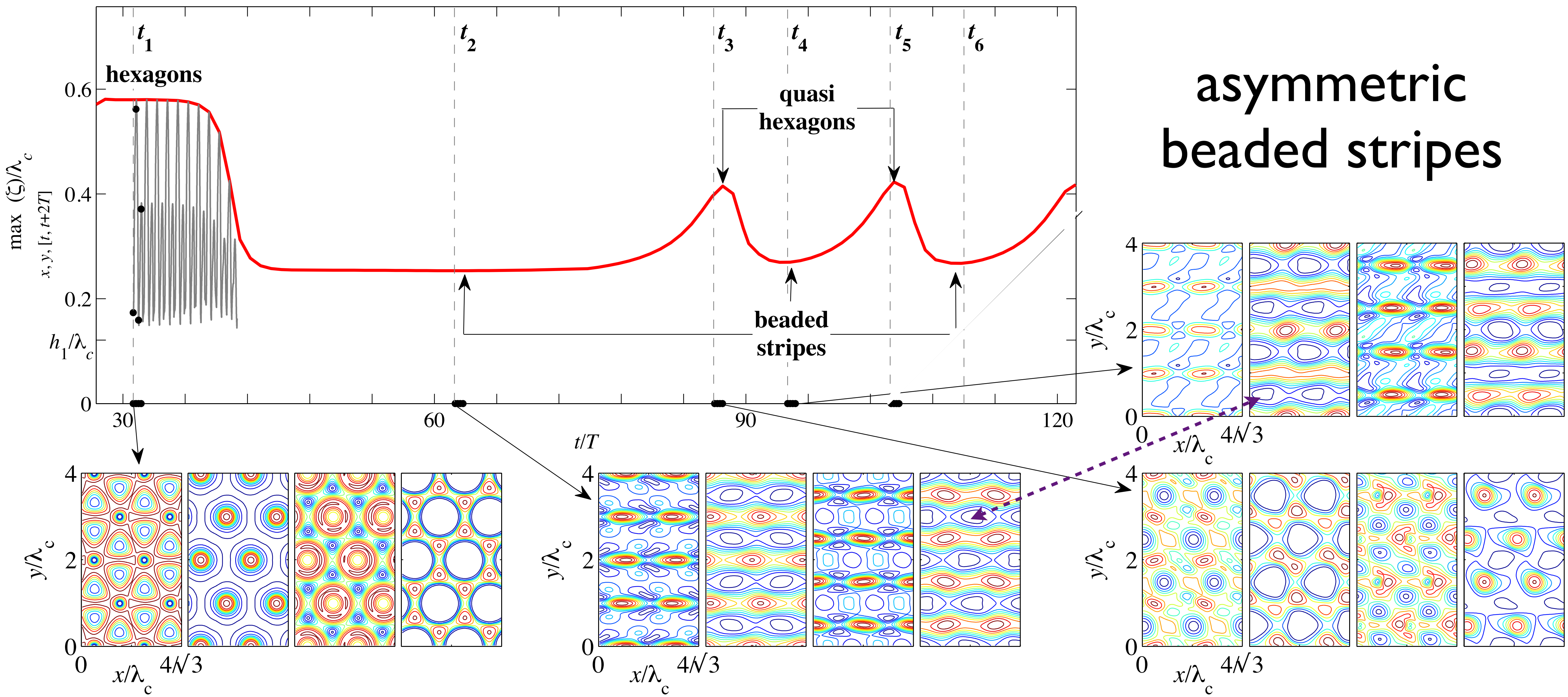
beaded stripes

Long-time evolution



quasi-hexagons

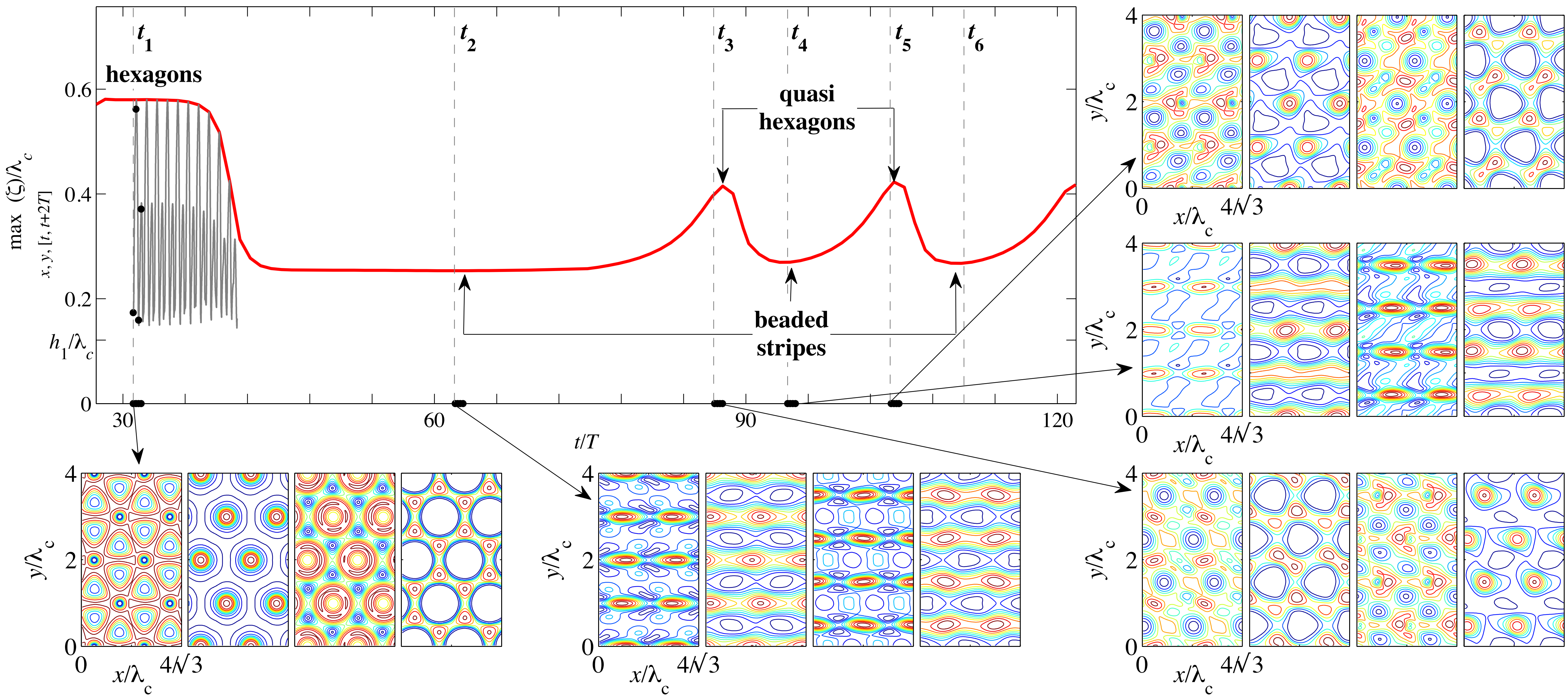
Long-time evolution



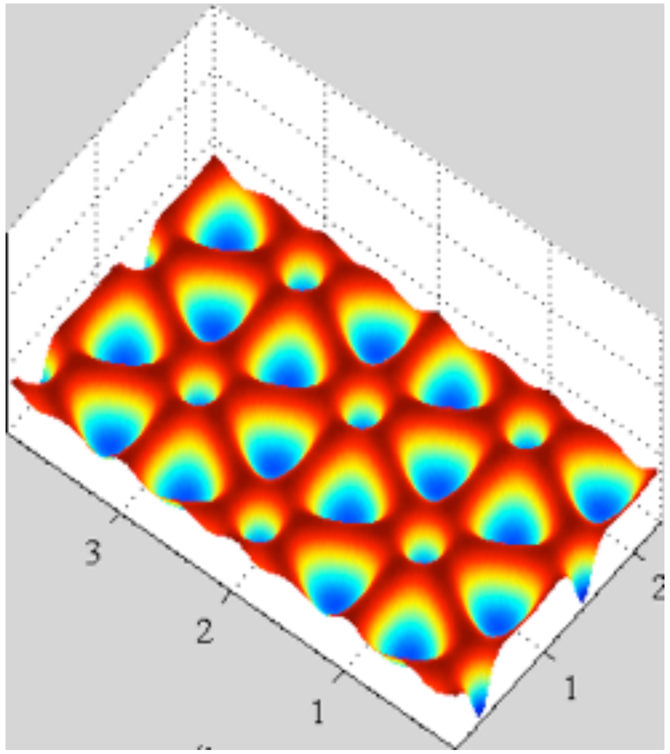
asymmetric beaded stripes

Long-time evolution

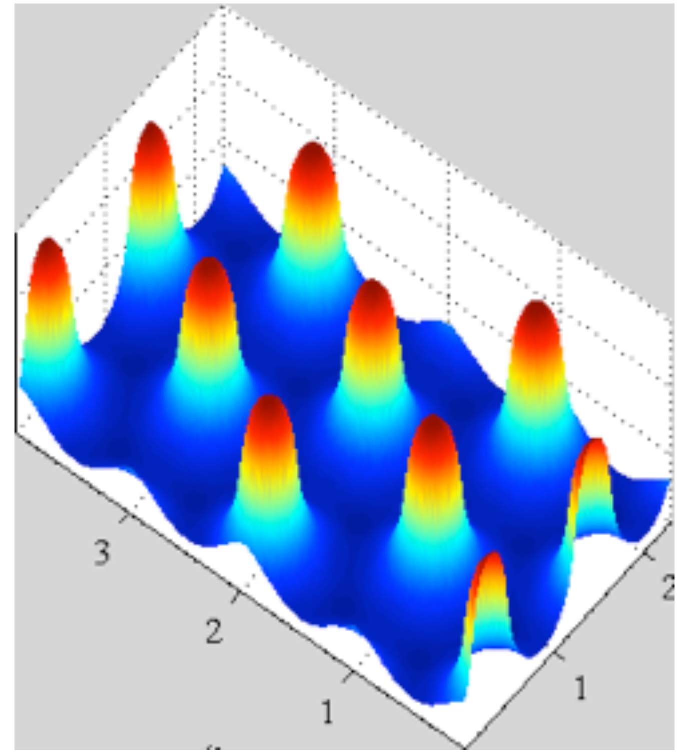
quasi-hexagons



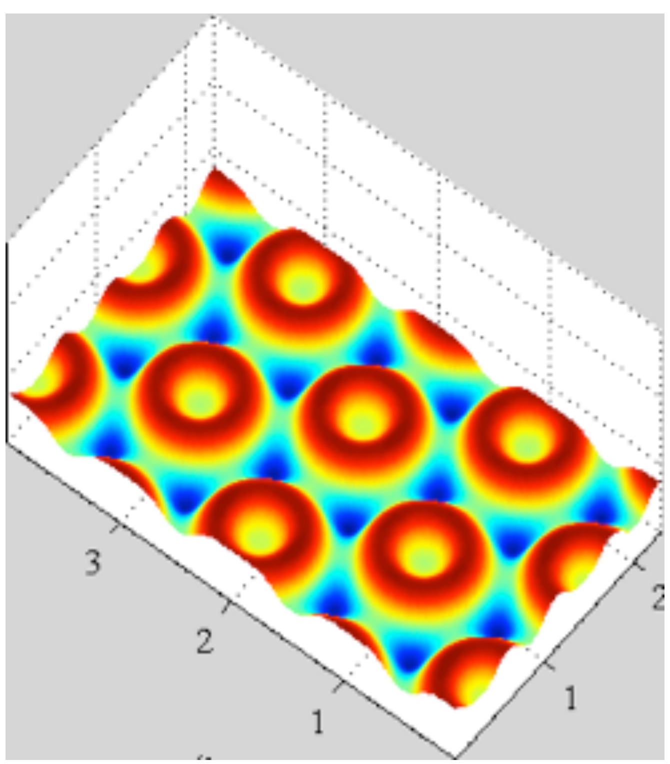
Stroboscopic films show long-time behavior



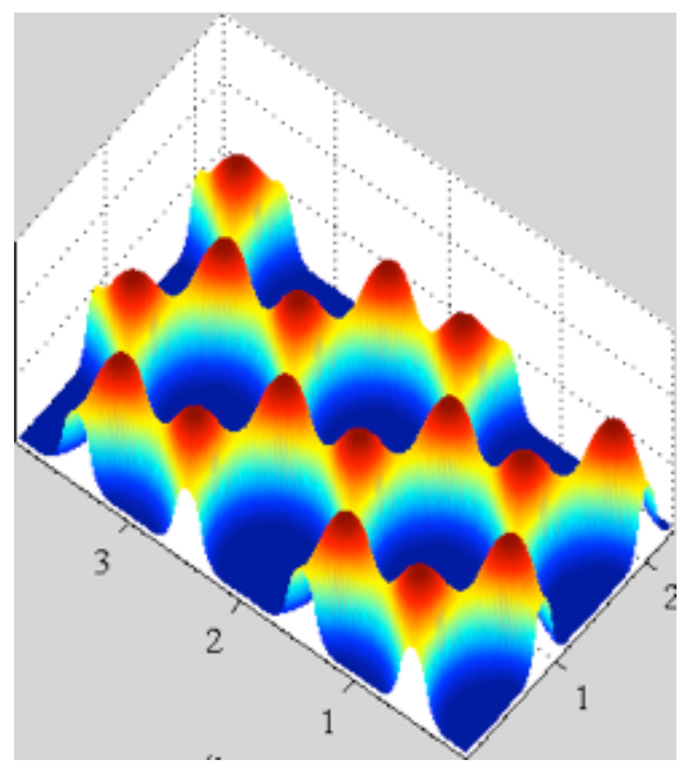
t=0 mod T



t=1/4 mod T

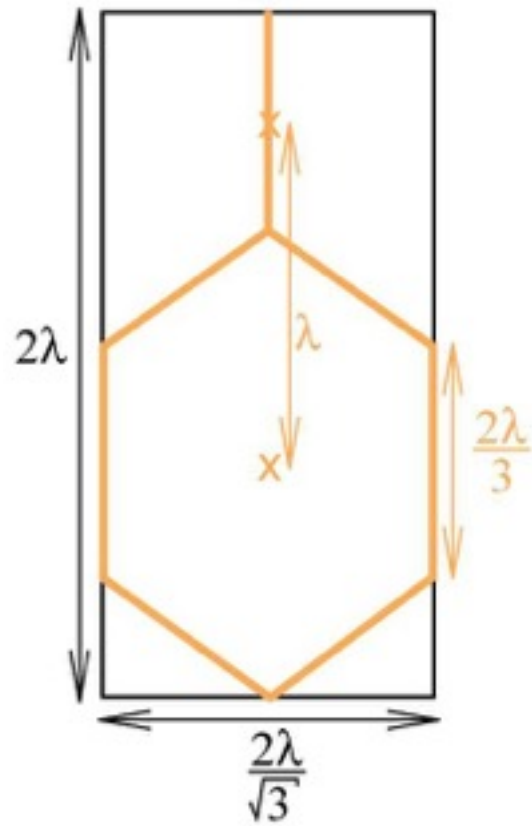


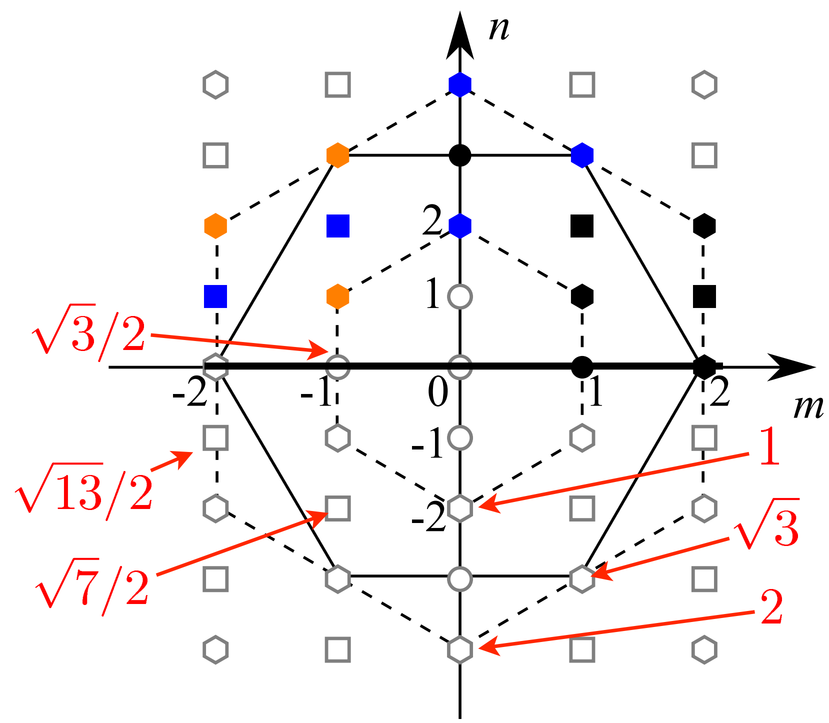
t=1/2 mod T

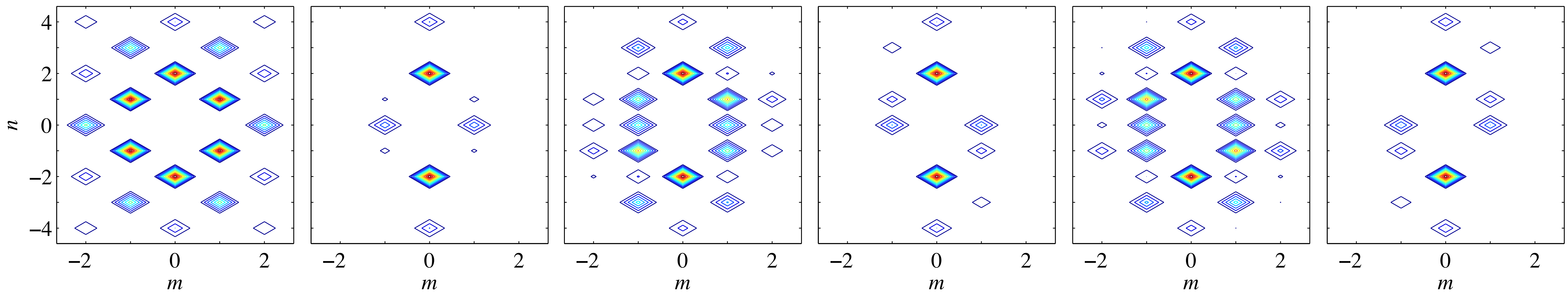


t=3/4 mod T

Fourier spectra

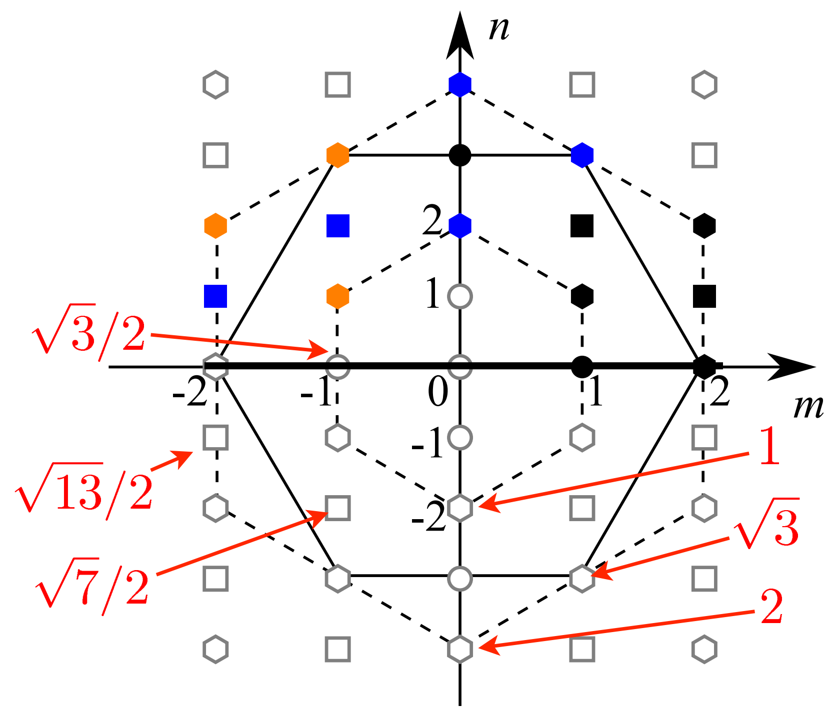


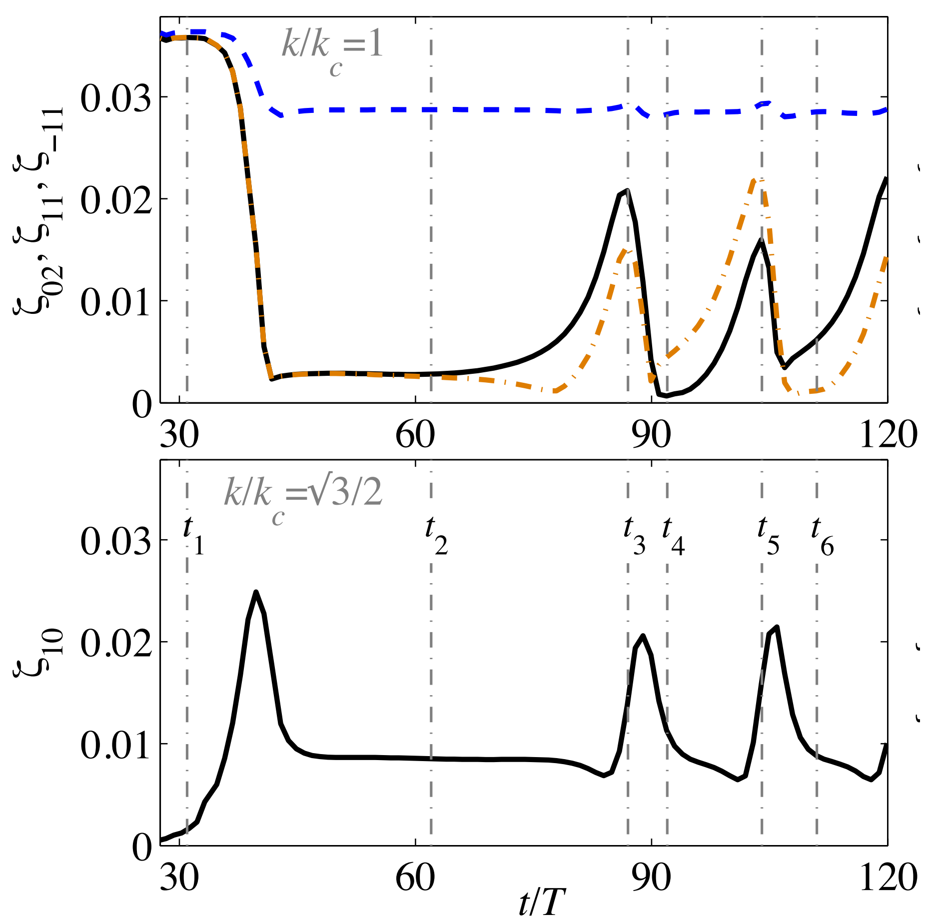


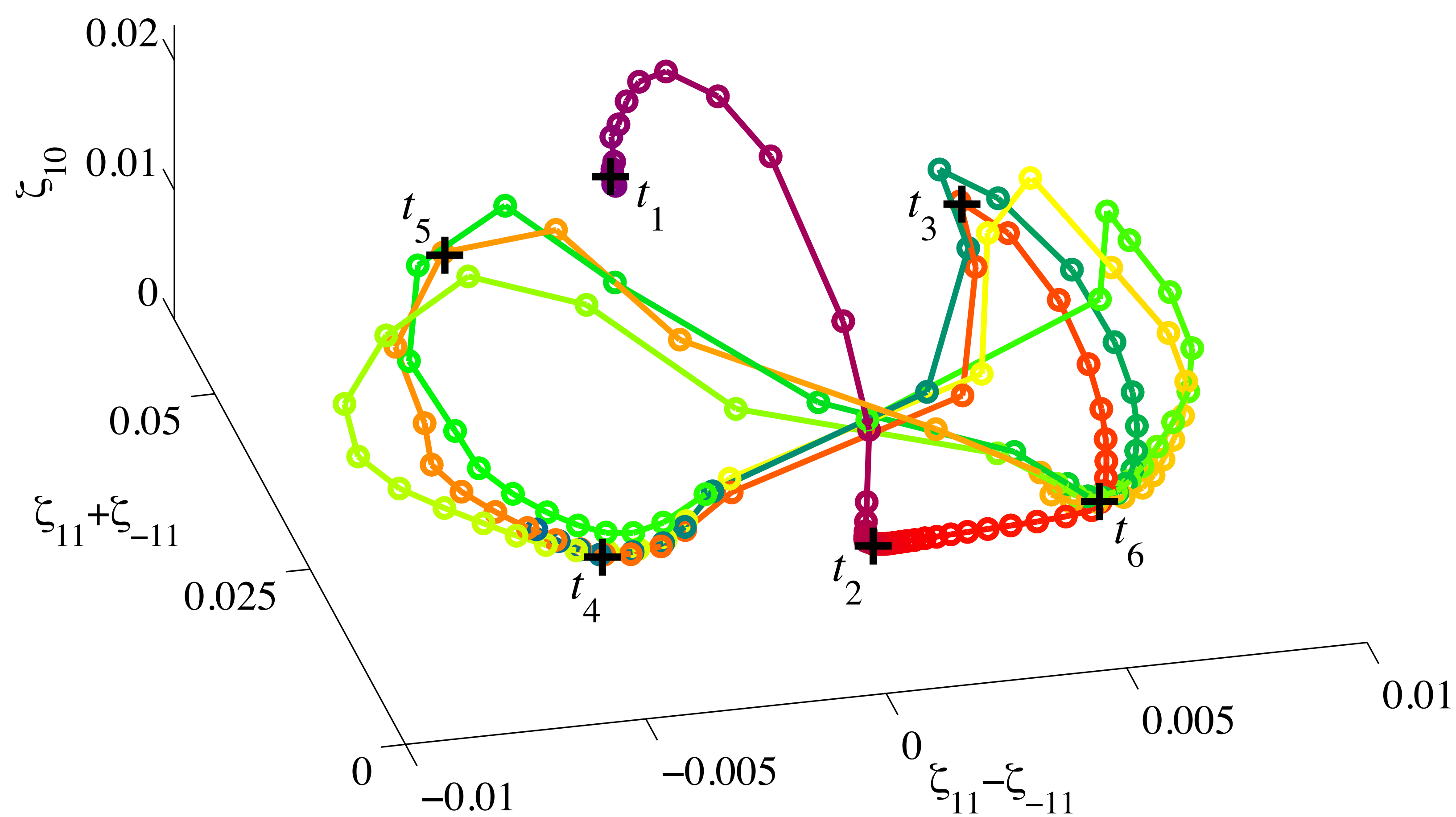


t_1	t_2	t_3	t_4	t_5	t_6
hexagons	beaded stripes	quasi-hexagons	asymmetric beaded stripes	quasi-hexagons	asymmetric beaded stripes

Fourier spectra : time evolution

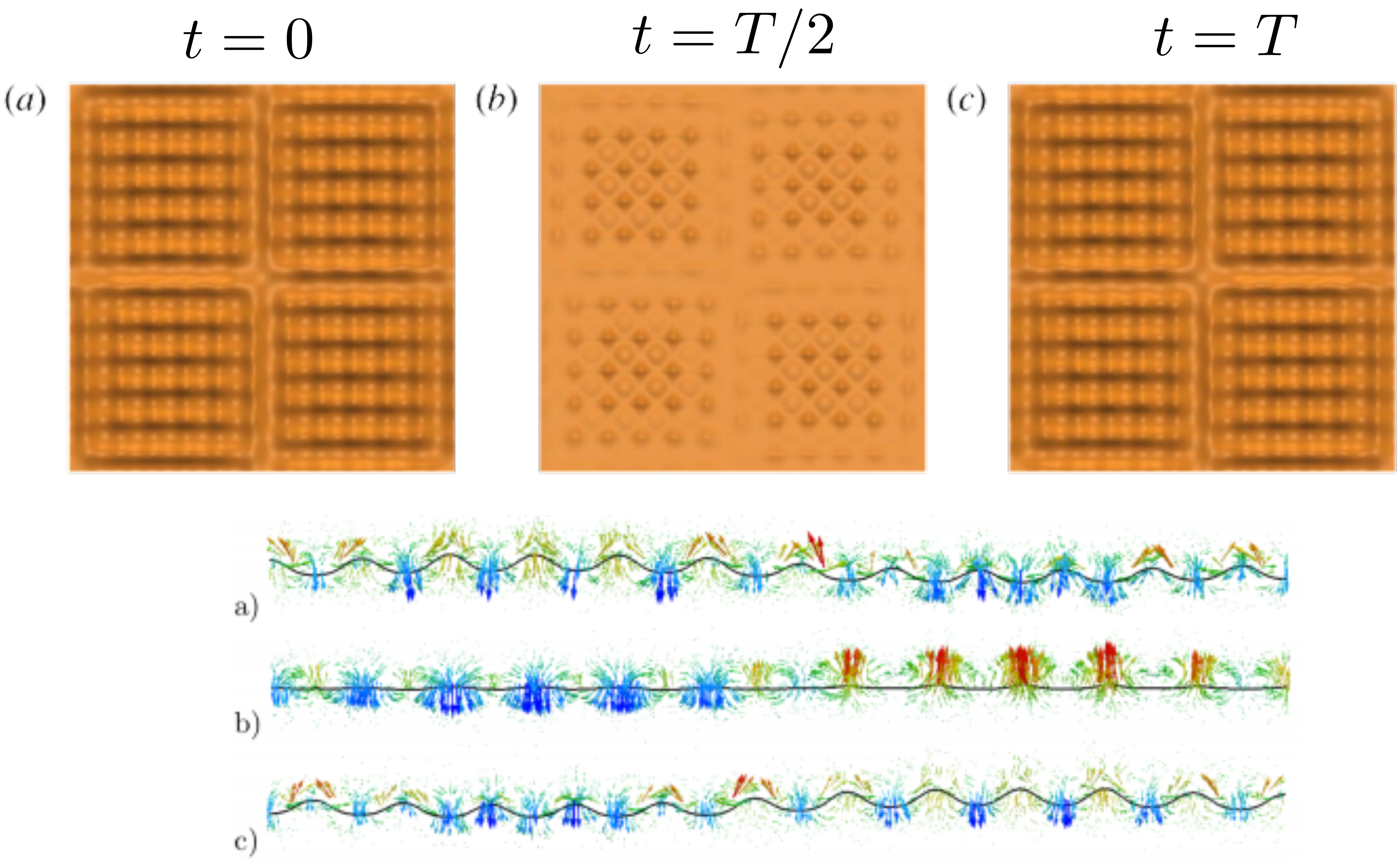






Faraday Super-squares

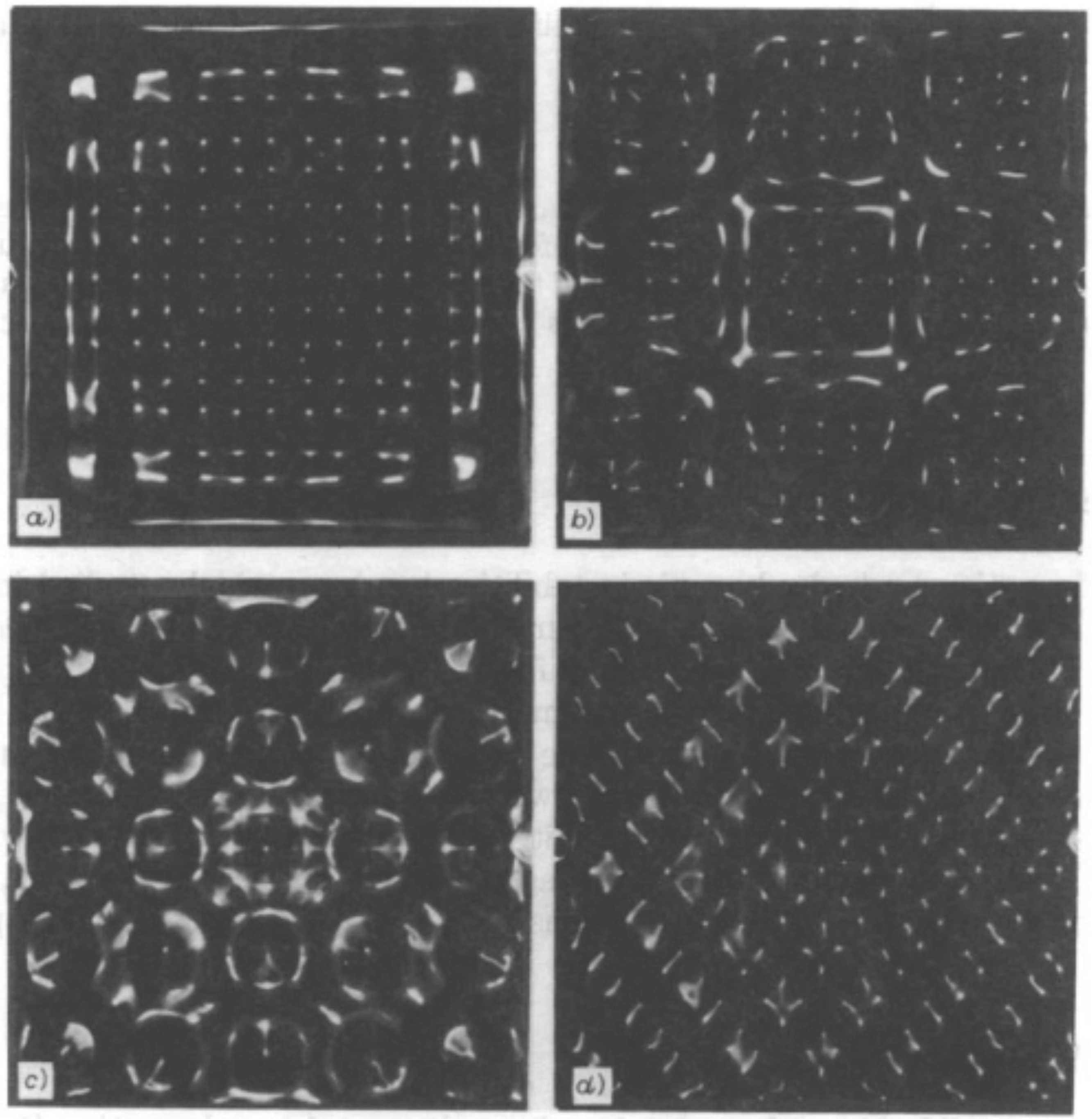
Simulation: L. Kahouadji



EUROPHYSICS LETTERS
Europhys. Lett., **6** (3), pp. 221-226 (1988)

Pattern Selection in Faraday Instability.

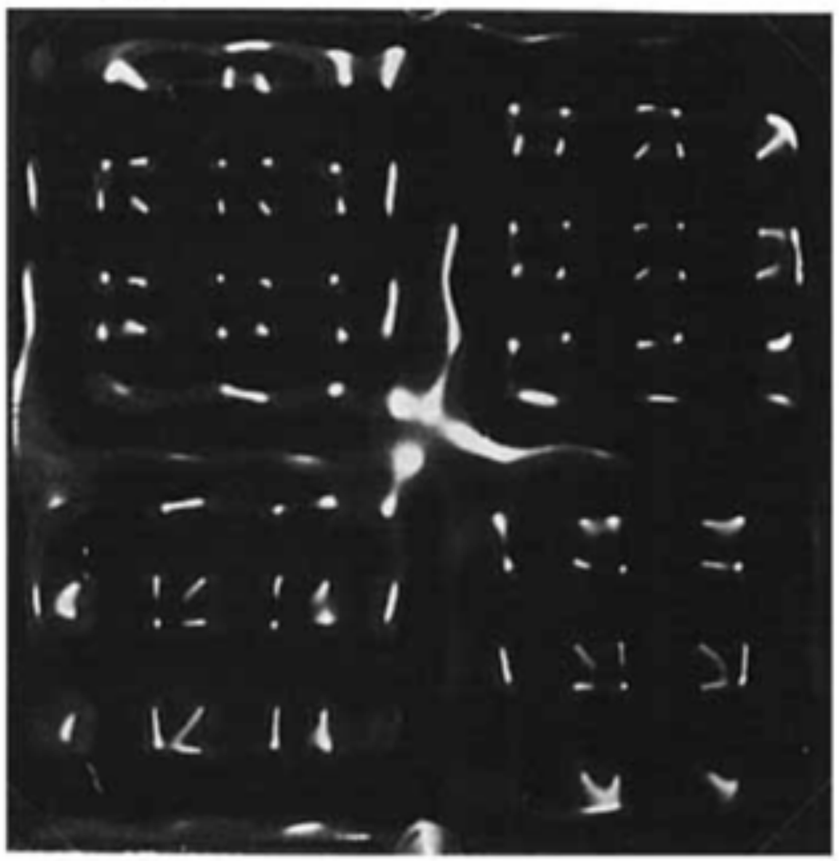
S. DOUADY and S. FAUVE

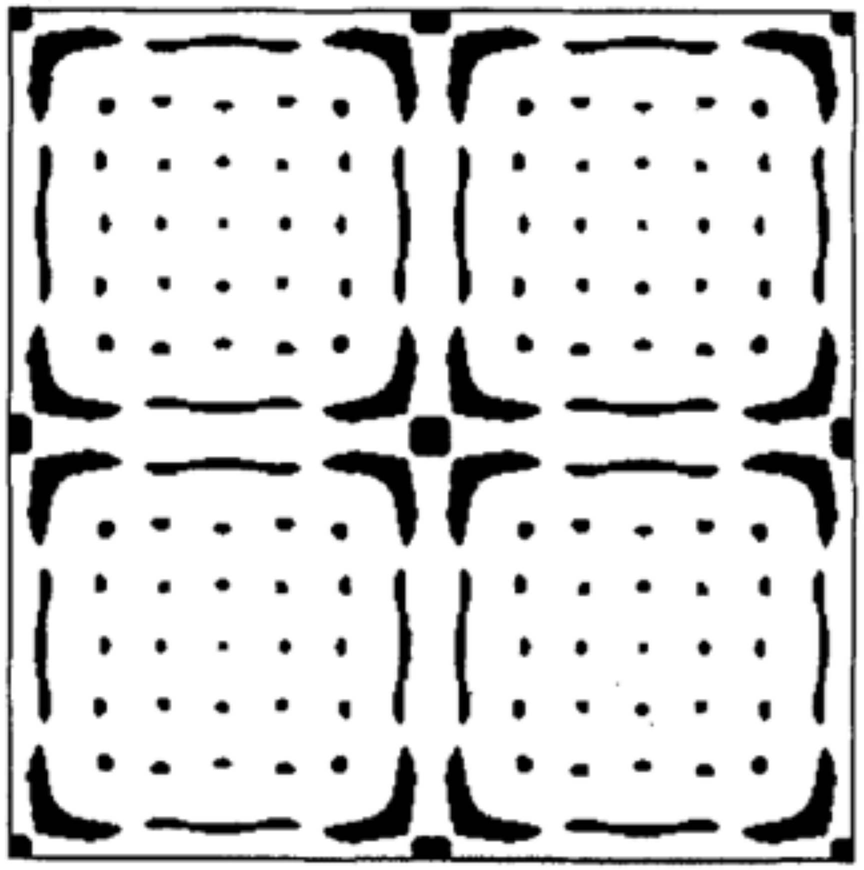


J. Fluid Mech. (1990), vol. 221, pp. 383–409
Printed in Great Britain

Experimental study of the Faraday instability

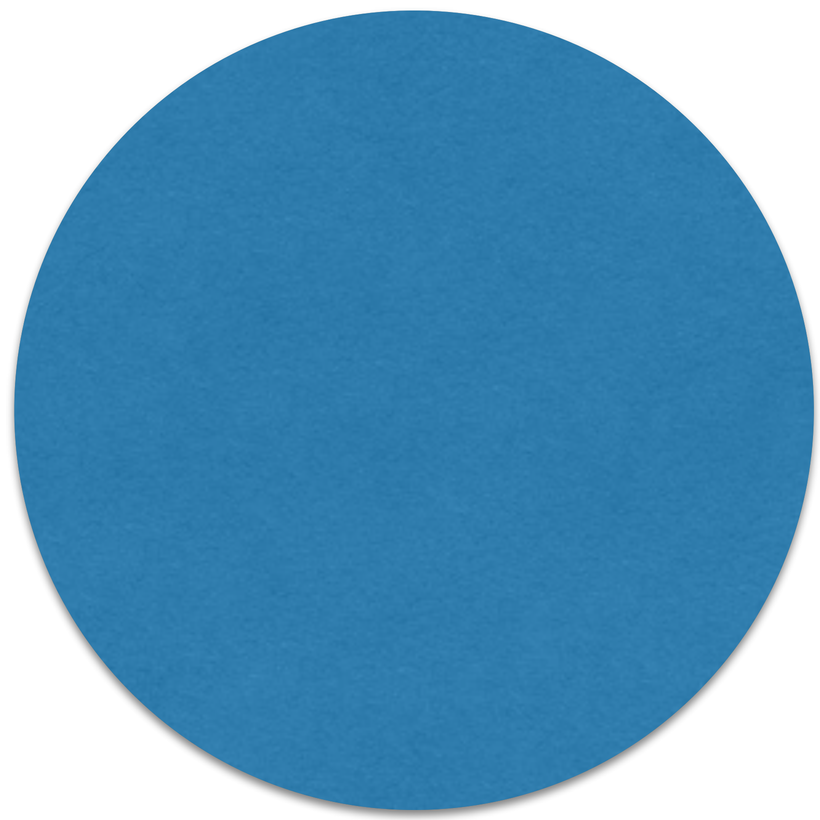
By S. DOUADY





Spherical Faraday Instability

Ali-higo Ebo Adou



Oscillating radial force $\mathbf{G}(r,t) = (g - a\cos(\omega t))\mathbf{r}$

Planar

Spherical

Surface height

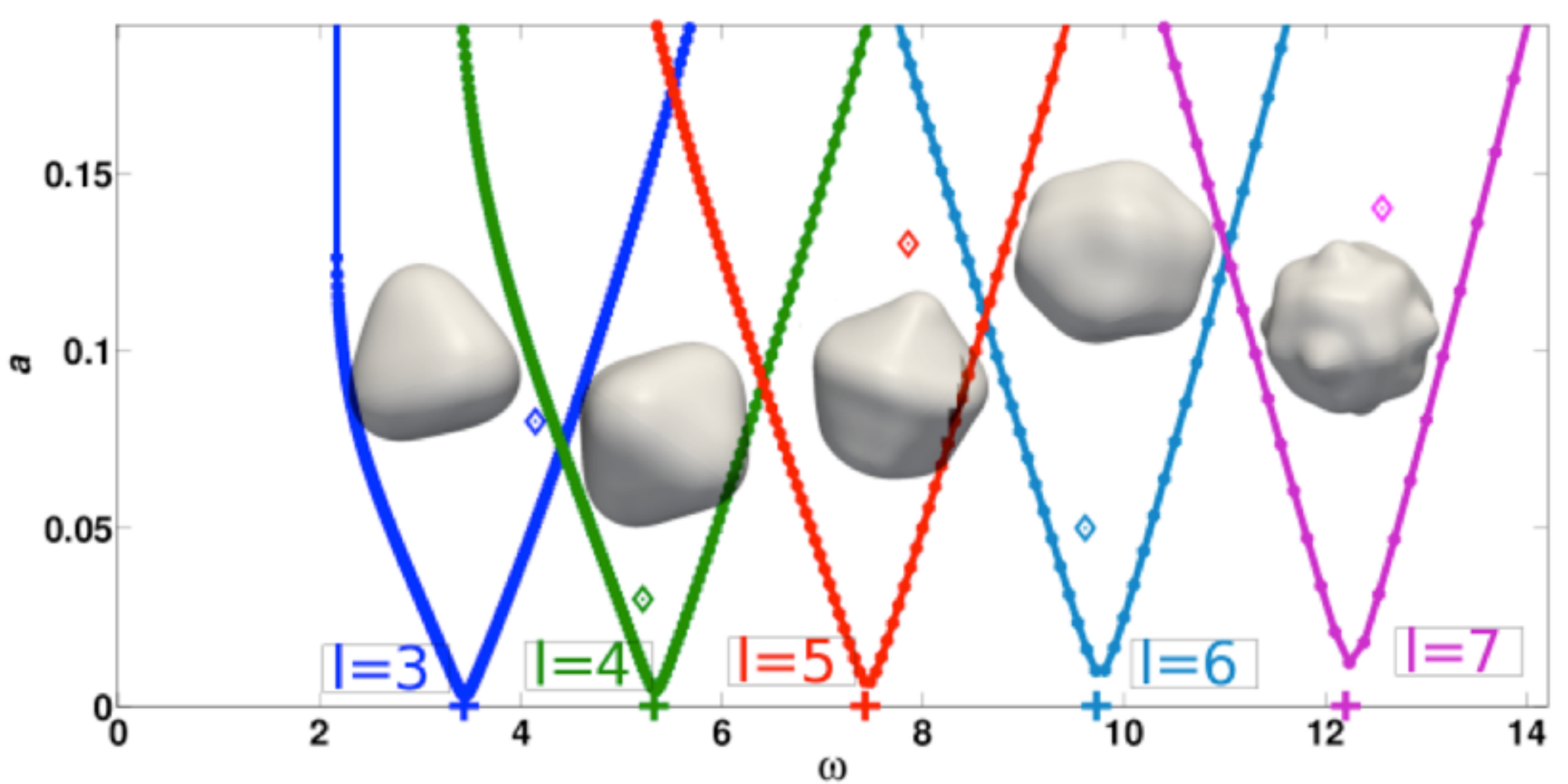
$$\zeta(\mathbf{x}) = \sum_{\mathbf{k}} \zeta_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{x})$$

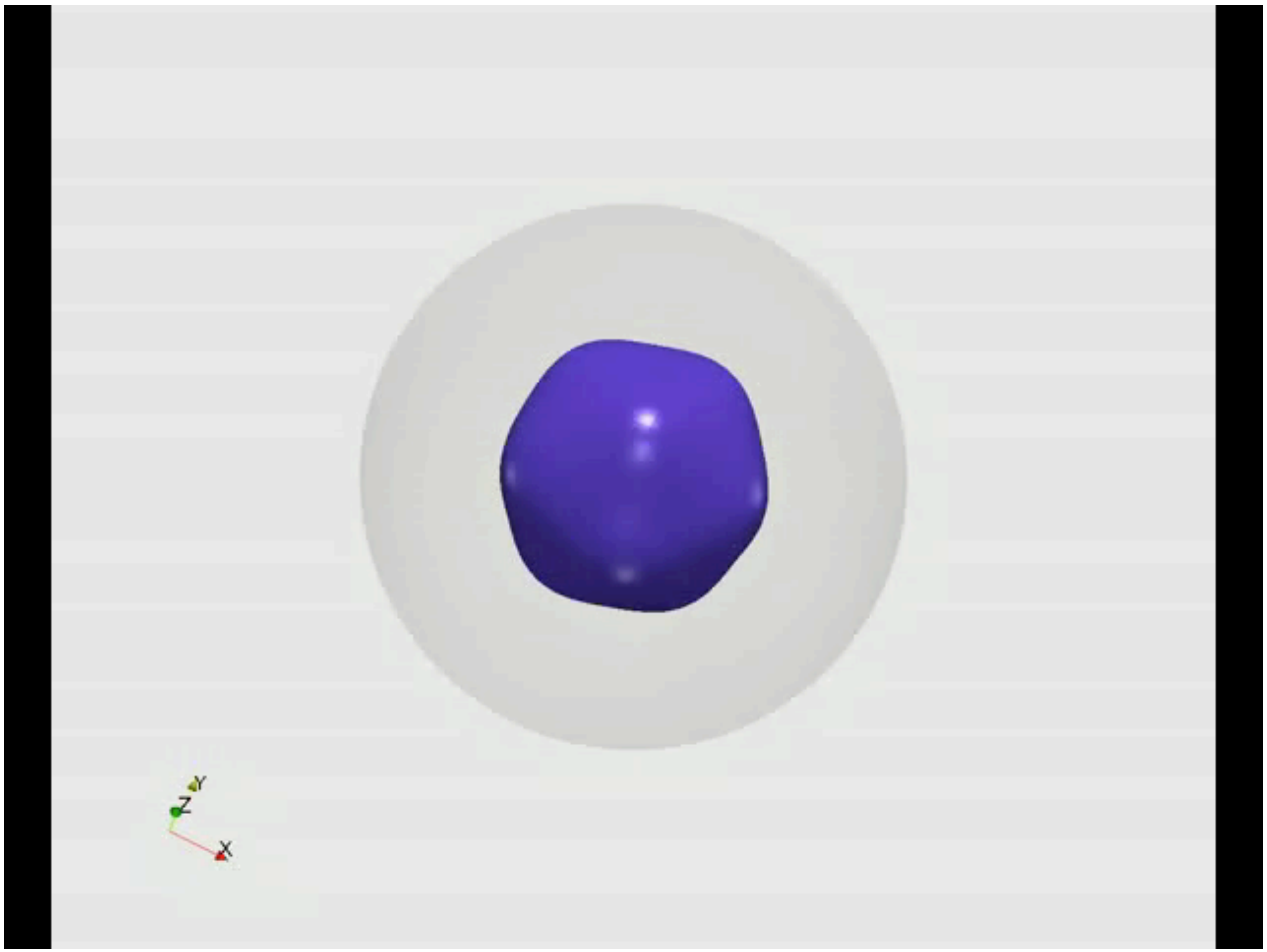
$$\zeta(\theta, \phi) = R + \sum_{\ell,m} \zeta_{\ell,m} Y_{\ell,m}(\theta, \phi)$$

Inviscid dispersion relation

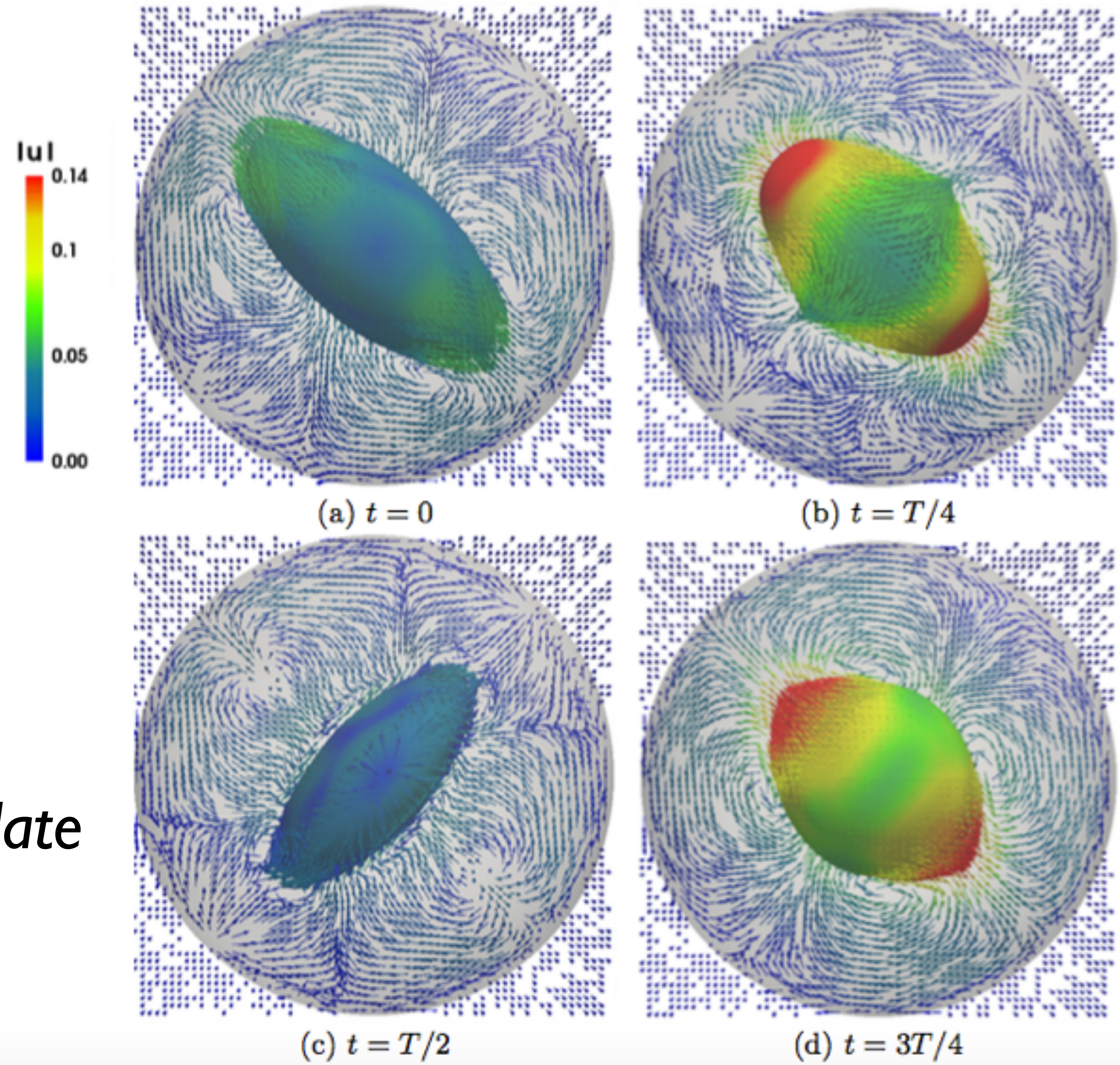
$$\omega^2 = gk + \frac{\sigma}{\rho} k^3$$

$$\omega^2 = g\frac{\ell}{R} + \frac{\sigma}{\rho} \frac{\ell(\ell-1)(\ell+2)}{R^3}$$


0.15
0.1
0.05
0
a
0
2
4
6
8
10
12
14
ω
l=3
l=4
l=5
l=6
l=7


Y
Z
X

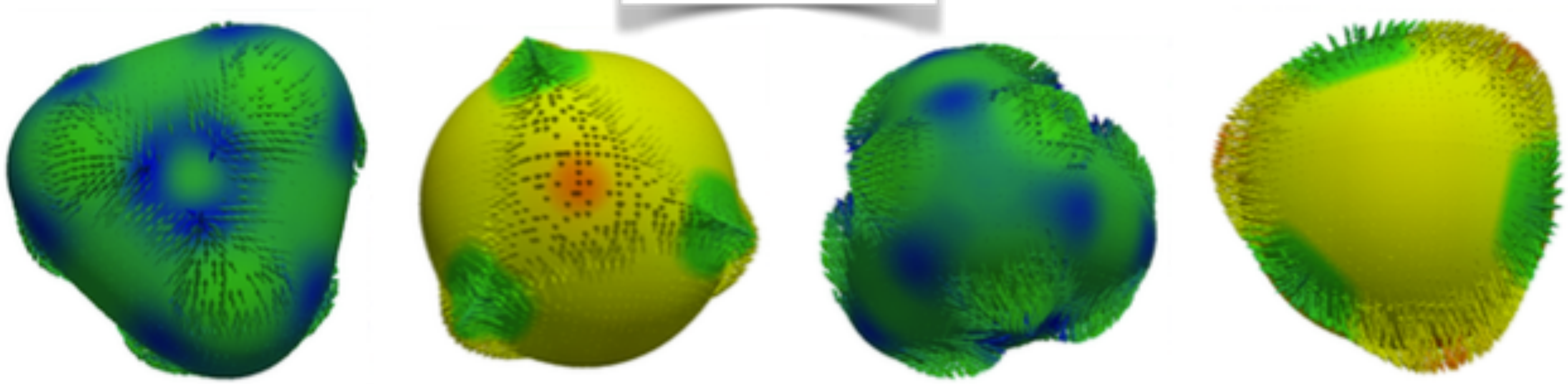
$\ell = 2$



prolate

oblate

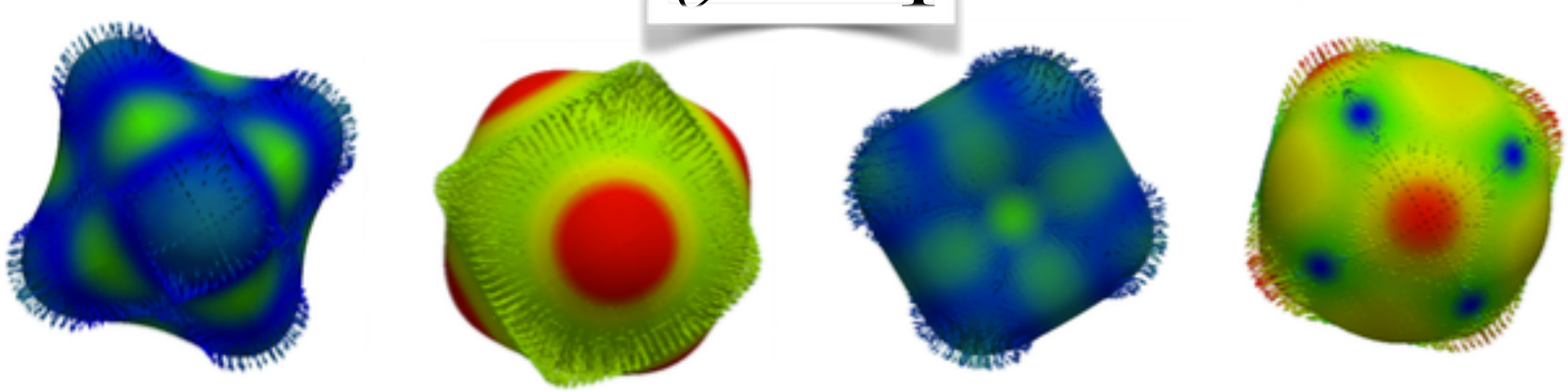
$\ell = 3$



(a) $t = 0$ (b) $t = T/2$ (c) $t = T$ (d) $t = 3T/2$

tetrahedron

$\ell = 4$

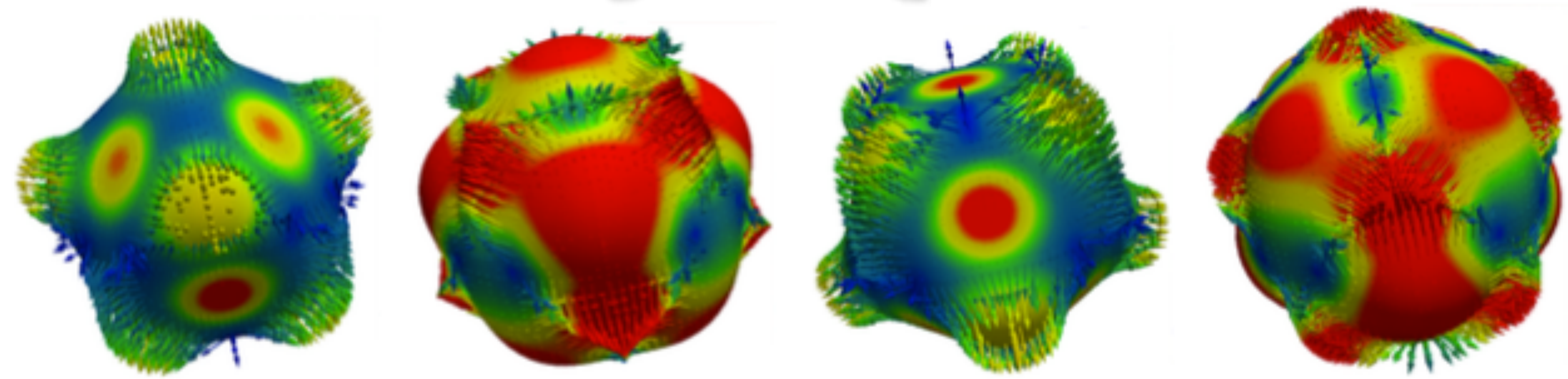


(a) $t = 0$ (b) $t = T/2$ (c) $t = T$ (d) $t = 3T/2$

octahedron ⟷ *cube*

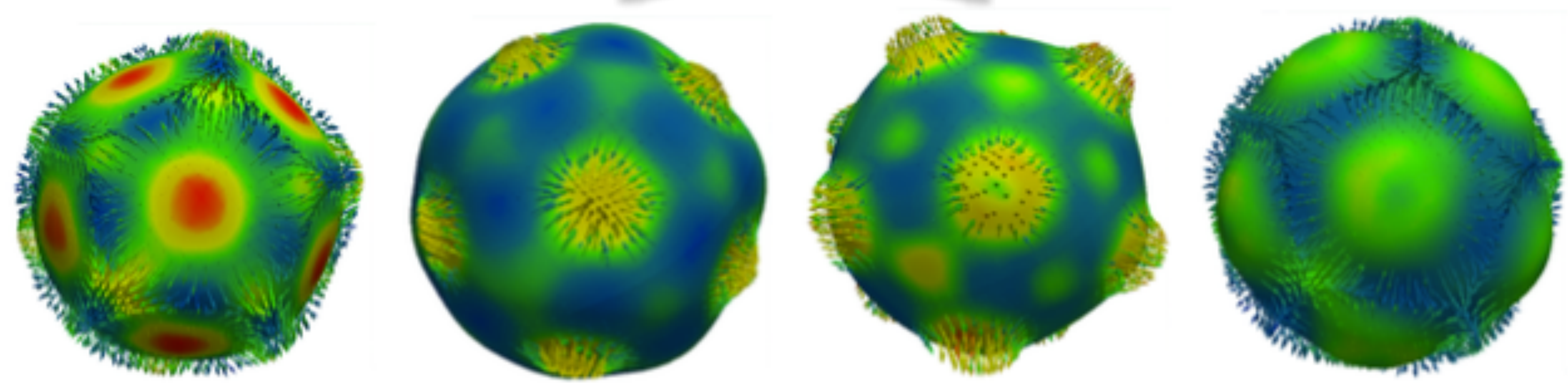
dual

$\ell = 5$



(a) $t = 0$ (b) $t = T/2$ (c) $t = T$ (d) $t = 3T/2$

$\ell = 6$

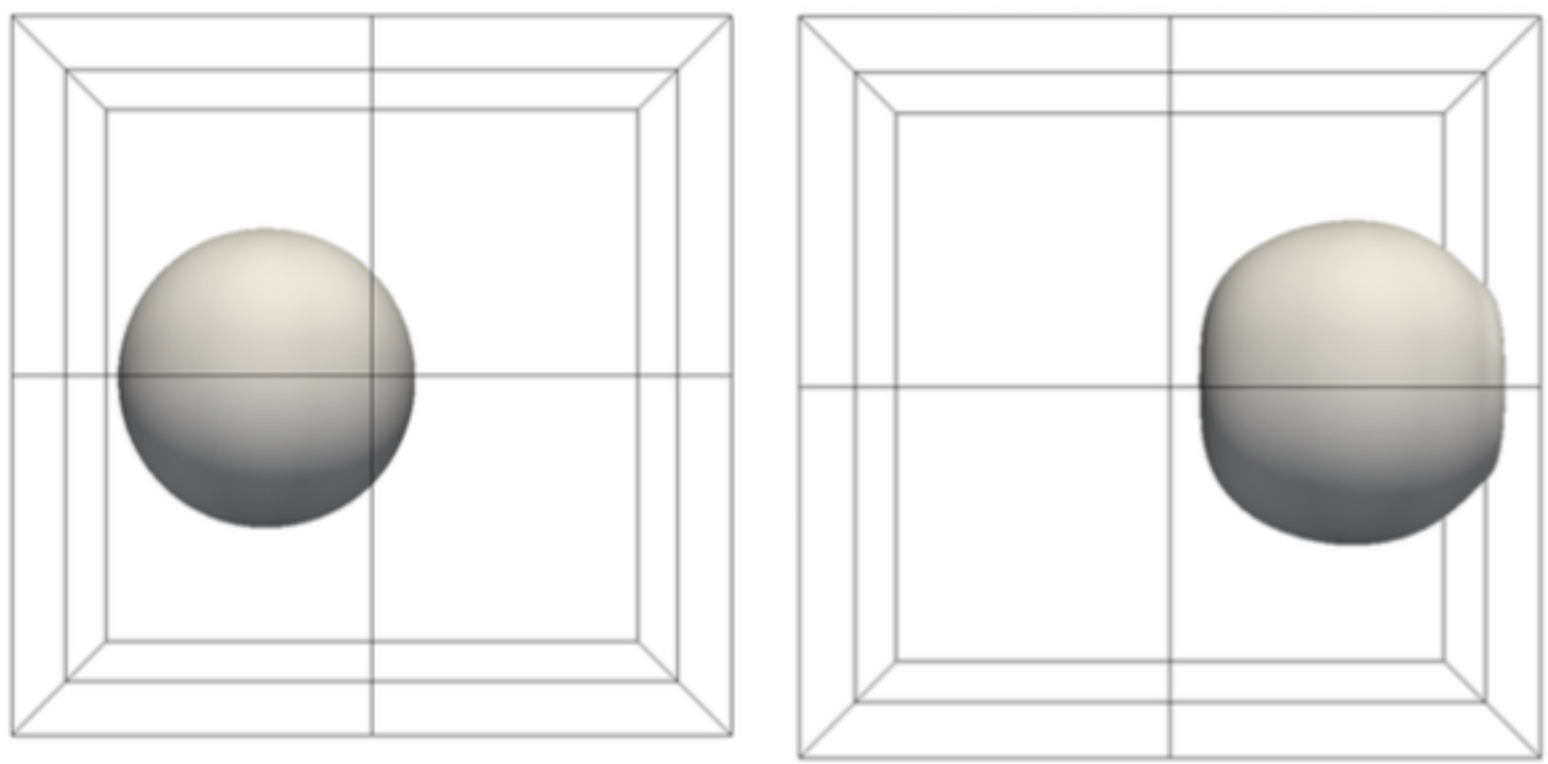


(a) $t = 0$ (b) $t = T/2$ (c) $t = T$ (d) $t = 3T/2$

dodecahedron (12 pentagons)

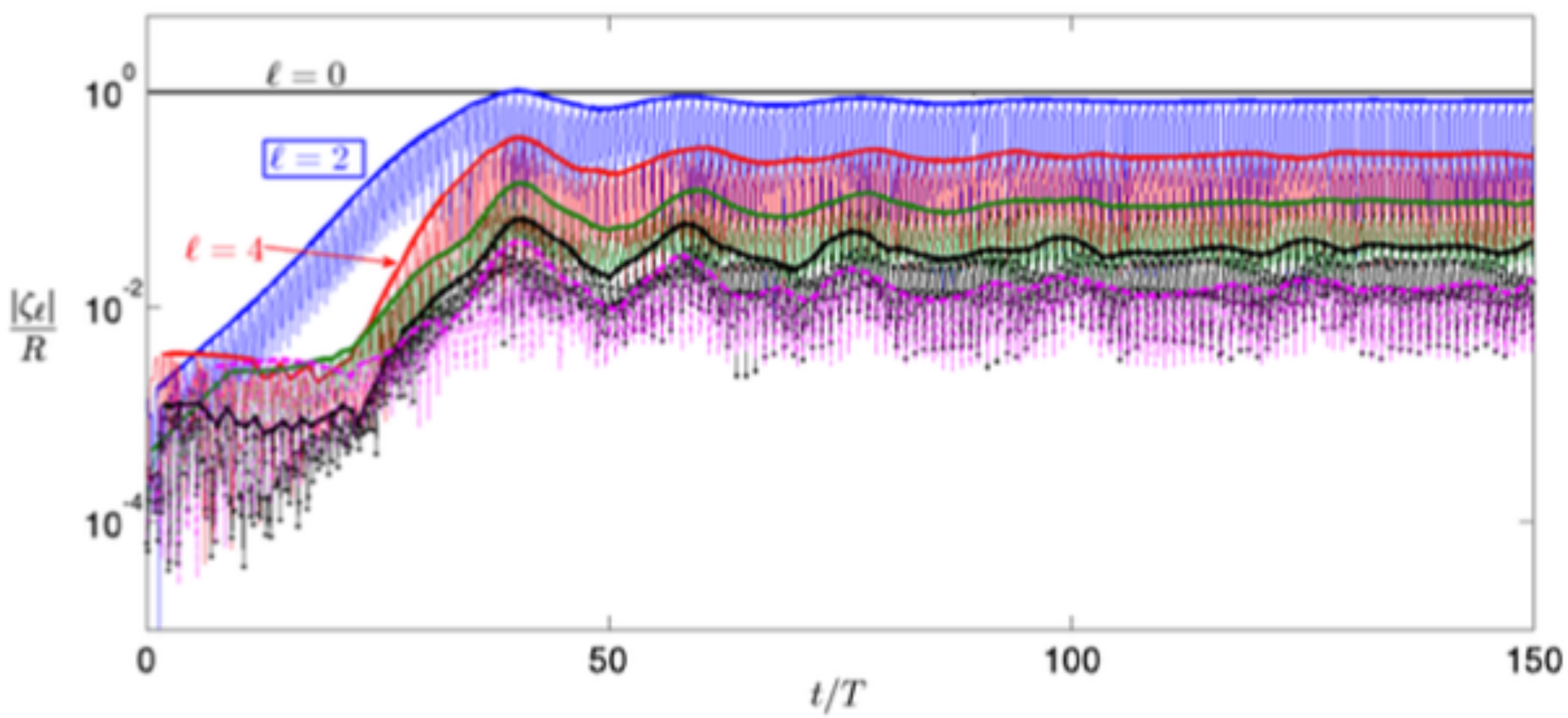
icosahedron (20 triangles)

$\ell = 1$



Spherical harmonic transforms

$\ell = 2$

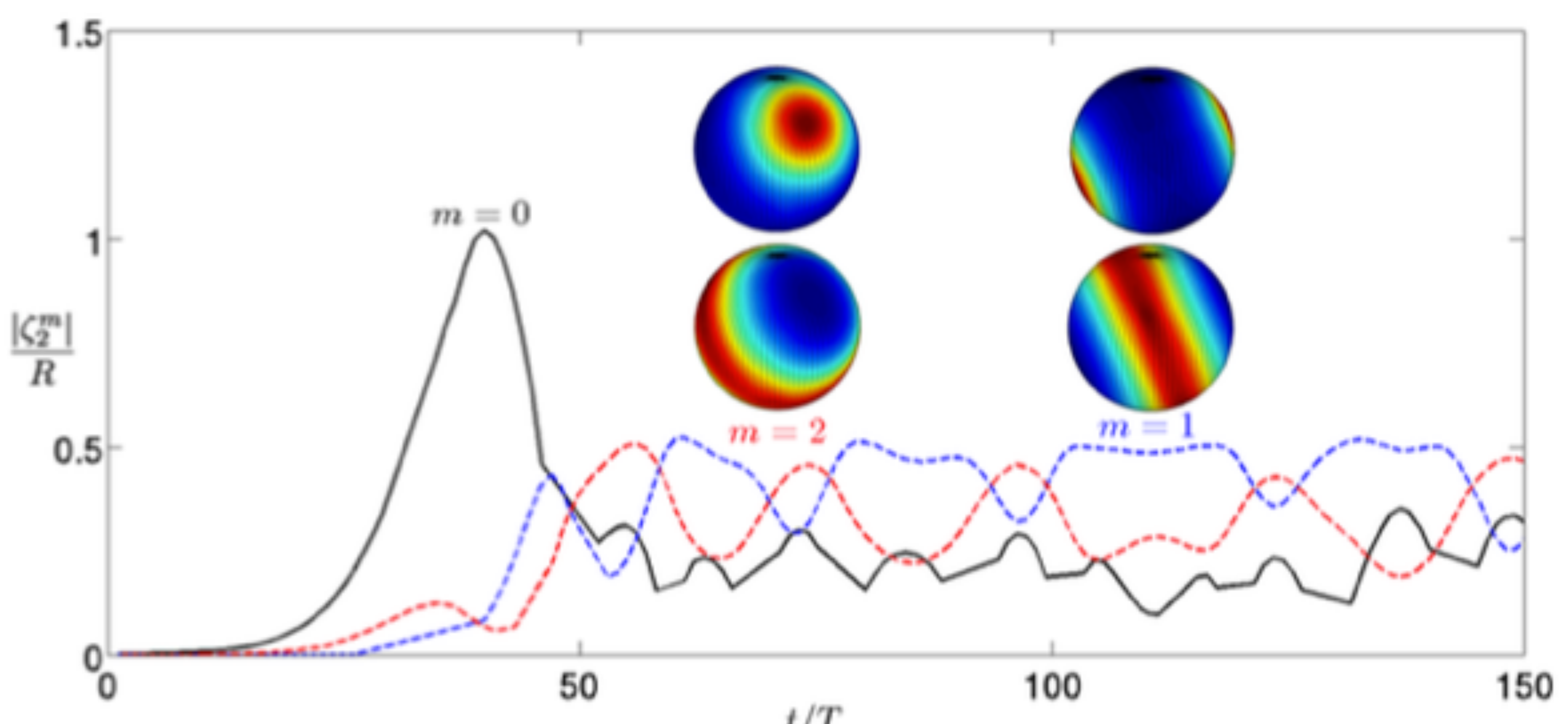


$\ell = 0, 2, 4, \dots$

volume conservation

higher harmonics

envelope



$\ell = 2$

$m = 0, 1, 2$

drift in orientation

similar drift for 3, 5, 6

Original computations: $64^3 = 2^3 \times 32^3$

Eight processors, each with 32^3 grid

Now repeated with: $256^3 = 8^3 \times 32^3$

512 processors, each with 32^3 grid

BLUE

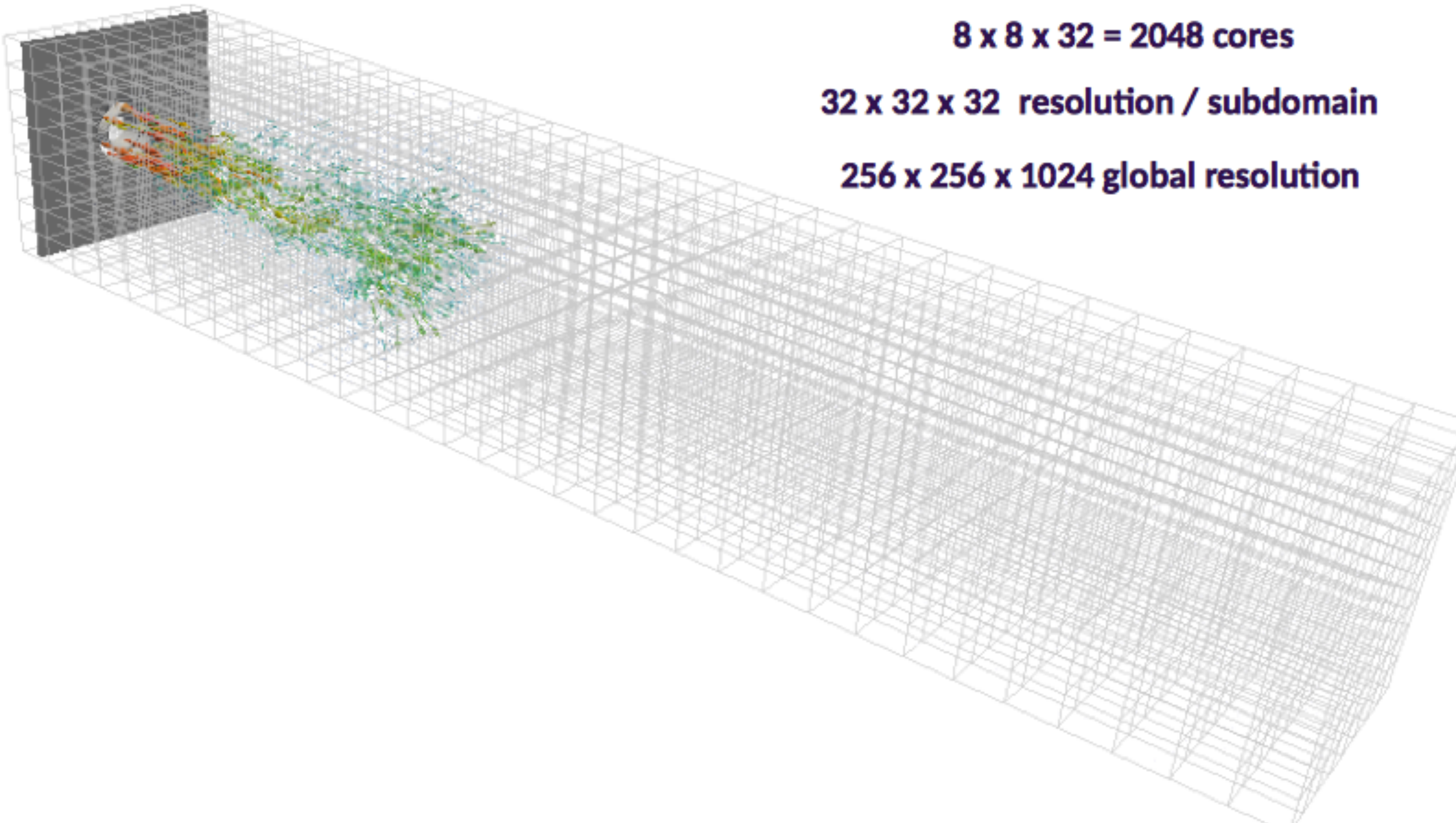
Damir Juric, Jalel Chergui & Seungwon Shin

Jet Spray/Atomization

8 x 8 x 32 = 2048 cores

32 x 32 x 32 resolution / subdomain

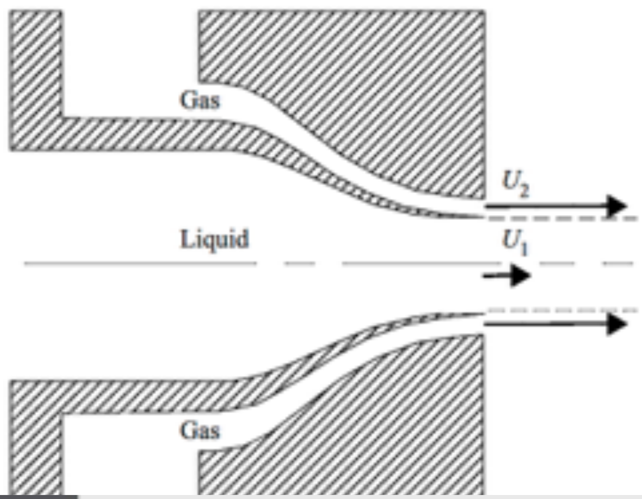
256 x 256 x 1024 global resolution

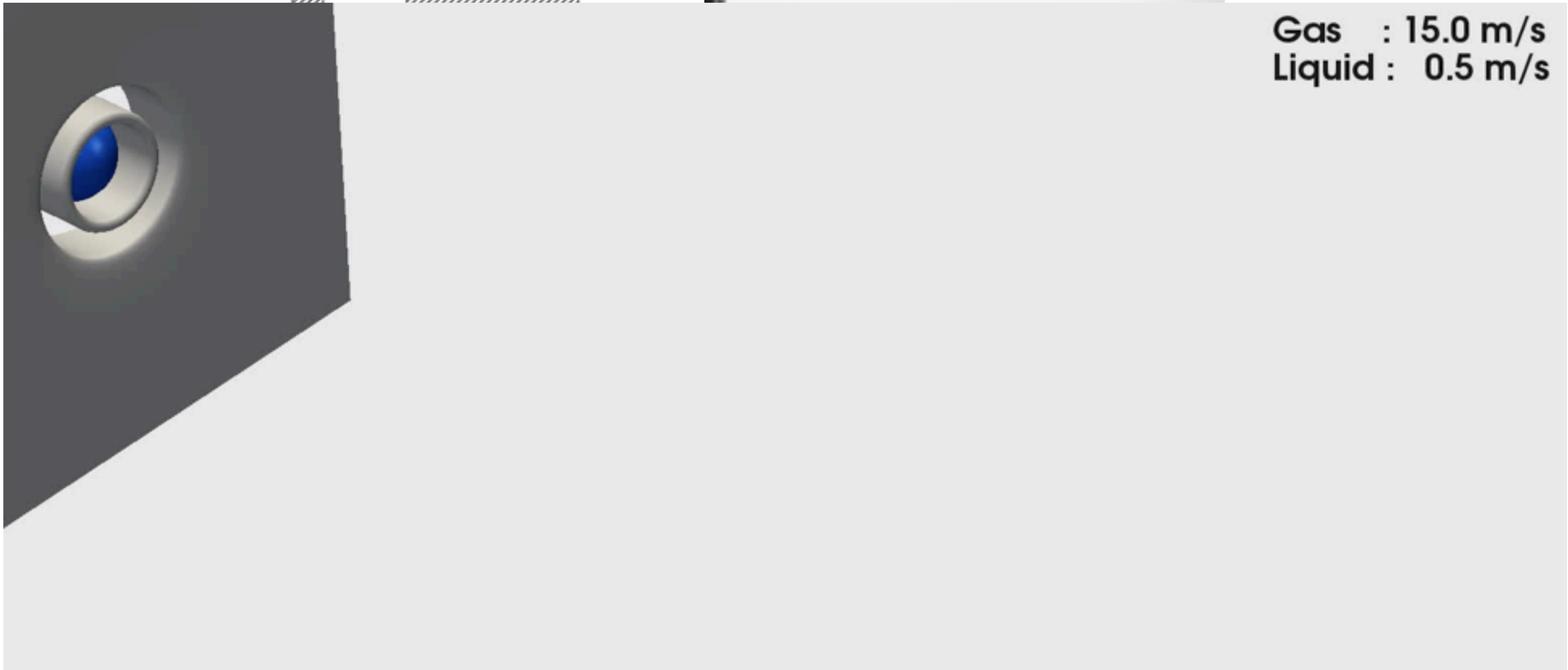


Imperial College London

Jet Spray/Atomization

U_l=0.5 m.s^{-1} / U_g=15 m.s^{-1}





High Performance Computing
Massive parallelism
Careful Memory Management

- Fortran 2003, modular, structures, derived types …
  no external libraries needed
- Parallelization - Domain Decomposition, MPI
- Tested on up to 131072 threads
  limited only by availability of computing resources
  262144 threads with increased hyperthreading

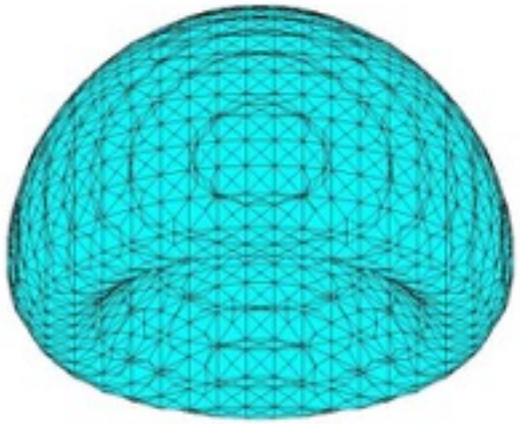
Eulerian Fields

- 3D
  - incompressible, multi-phase, multi-component
    - momentum, heat, mass transfer
- MAC/Projection method on staggered grid
- Velocity, temperature, species solution :
  - Parallel GMRes discontinuous coefficient Helmholtz solver
- Pressure solution :
  - Parallel Multigrid/GMRes discontinuous density Poisson solver
- Immersed solid objects
  - contact model
- Phase change
  - microlayer model
- Marangoni, Surfactant
- Non-Newtonian
- Coriolis, Centrifugal

Lagrangian Front Tracking for Accurate Advection

Parallel Immersed Boundary Method

- Curvature Free Surface tension
  Essentially no parasitic currents (Hybrid JCP 2005, Compact IJNMF 2009)
- Contact/wetting model (solid/liquid/gas)
  receding/advancing angles (JMST 2009)

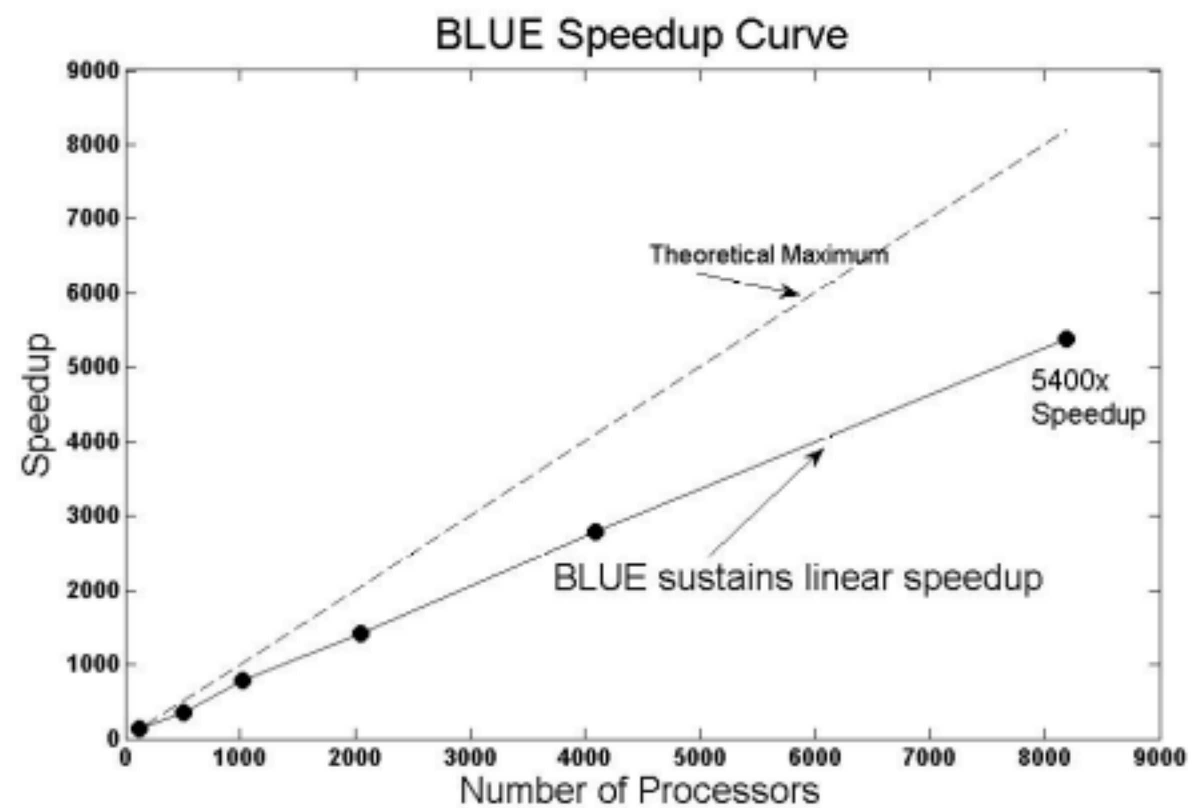


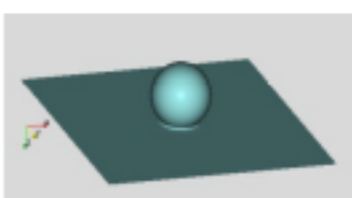
Level Set for coalescence/rupture

- High Order Level Contour Reconstruction (JMST, IJNMF 2009)
- Local Front Reconstruction Method/Adaptive (JCP 2011)

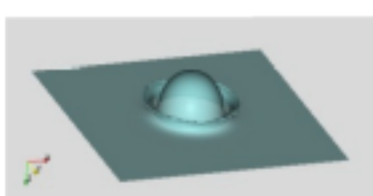
BLUE

Drop Splash – Parallel Performance



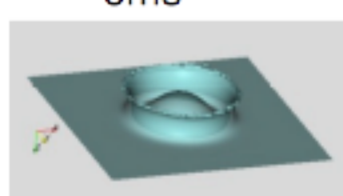


2.3ms

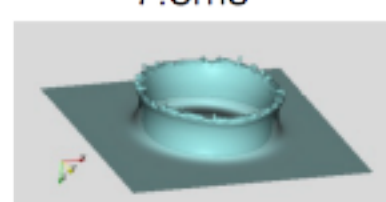


3.4ms

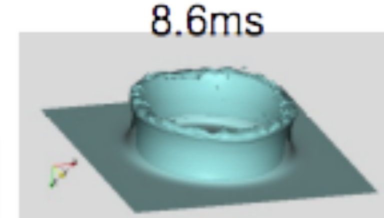
6ms

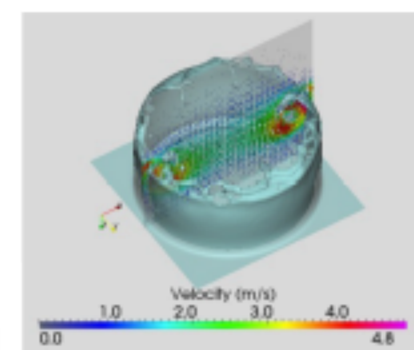


7.8ms



8.6ms



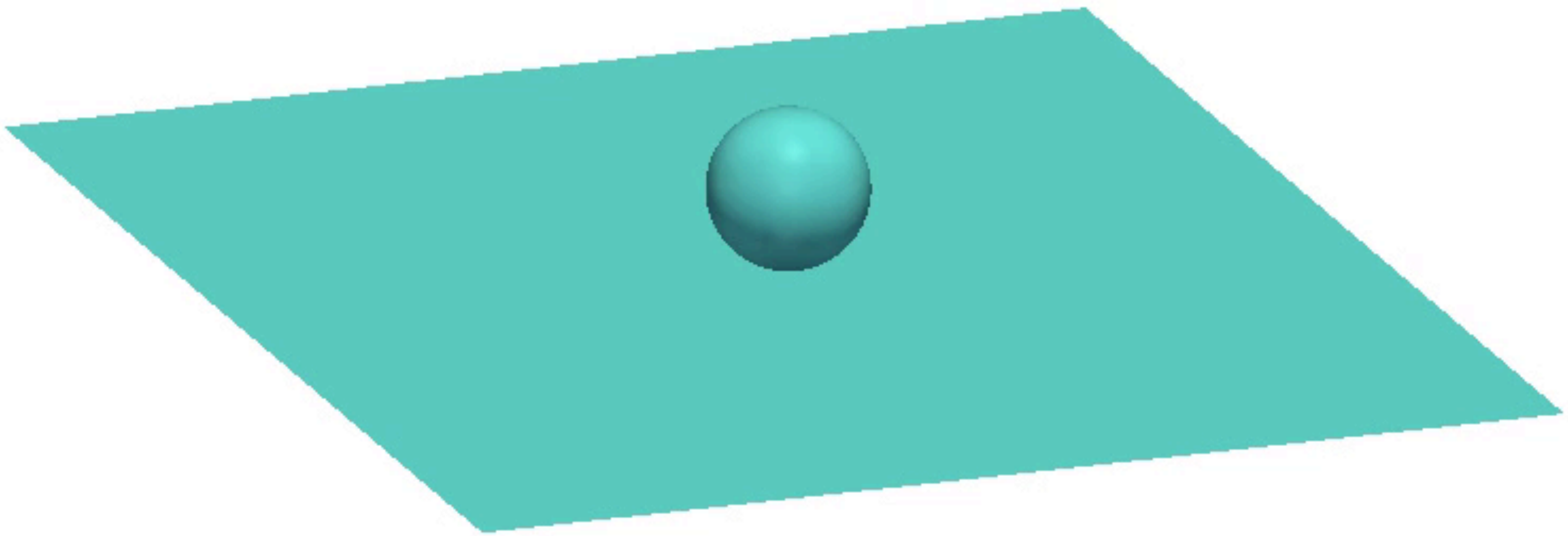


17.7ms

Water drop in air, r=4.53 mm, v_o=2.193 m/s, water film h=1mm.
$(3.6cm)^3$ closed domain, 512^3 Cartesian mesh with 16x16x16=4096 subdomains (processor cores) each subdomain at a resolution of 32^3.

Drop Splash on a Thin Film

Air/Water D=7mm h=0.7mm U=2.13m/s



Thank you