NUMERICAL SIMULATION OF A GAS BUBBLE COLLAPSE USING THE SPH-ALE METHOD

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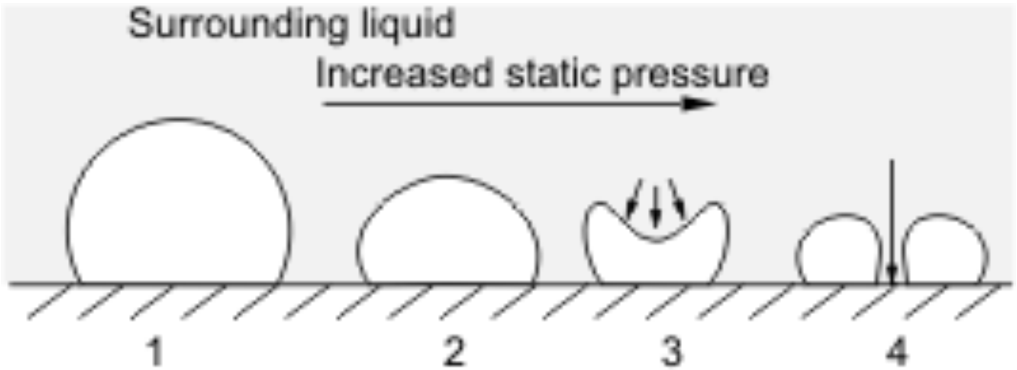
Numerical Modeling of Liquid-Vapor Interfaces in Fluid Flows
13.12.2016, Paris, France

Introduction: Cavitation erosion



https://en.wikipedia.org/wiki/Cavitation

lgsonic.com 2016

Governing equations

EULER EQUATIONS

$$\frac{\partial \mathbf{\Phi}}{\partial t} + \nabla \cdot (\mathbf{F}_C) = \mathbf{S}_T.$$

$$\mathbf{\Phi} = \begin{pmatrix} \rho \\ \rho \mathbf{v} \\ \rho E \end{pmatrix} \qquad \mathbf{F}_C = \begin{pmatrix} \rho \mathbf{v} \\ \rho \mathbf{v} \otimes \mathbf{v} + p\mathbf{I} \\ \mathbf{v}(\rho E + p) \end{pmatrix}$$

STIFFENED GAS EOS

$$e = e(\rho, p) = \frac{p + \gamma p_\infty}{\rho(\gamma - 1)}$$

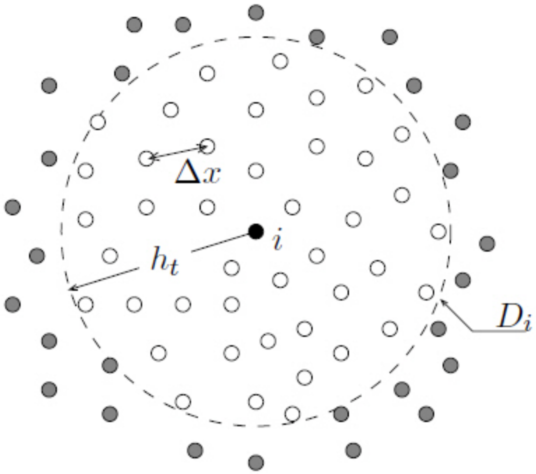
- ✓ Two-phase simulation (liquid - gas)
- ✓ Euler system
- ✓ Stiffened gas EOS and ideal gas EOS
- ○ No mass transfer

Numerical method

Smoothed Particle Hydrodynamics (SPH)

SPH

- ✓ A meshless method



SPH-ALE

- ✓ Riemann solver -> calculation of fluxes

$$\begin{cases} \dfrac{d(x_i)}{dt} = \mathbf{v}_0(\mathbf{x}_i, t) \\ \dfrac{d(\omega_i)}{dt} = \omega_i \sum_{j \in D_i} \omega_j (\mathbf{v}_0(\mathbf{x}_j, t) - \mathbf{v}_0(\mathbf{x}_i, t)) \cdot \nabla_i W_{ij} \\ \dfrac{d(\omega_i \mathbf{\Phi}_i)}{dt} + \omega_i \sum_{j \in D_i} \omega_j 2\mathbf{F}_{ij}(\mathbf{\Phi}, \mathbf{v}_0) \cdot \nabla_i W_{ij} = \omega_i \mathbf{Q}_i \end{cases}$$

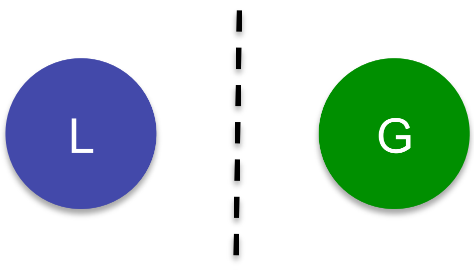
Pineda, SPHERIC 2016

- ✓ MUSCL method

Multiphase model

Multiphase model

✓ Strong variations of density at the interface without diffusion



V_0 = velocity from Riemann solver

Mass flux between the control volumes is blocked

Leduc, 2010

Correction of the particle motion

- Lagrangian motion is not adequate for compressible flows with Ma close to 1

✧ To correct the particle velocity in every time step in order to reduce the errors due to an unacceptable particle distribution

$$\forall i \in \Omega : \quad p_{ci} = \beta \rho_i c_c^2,$$

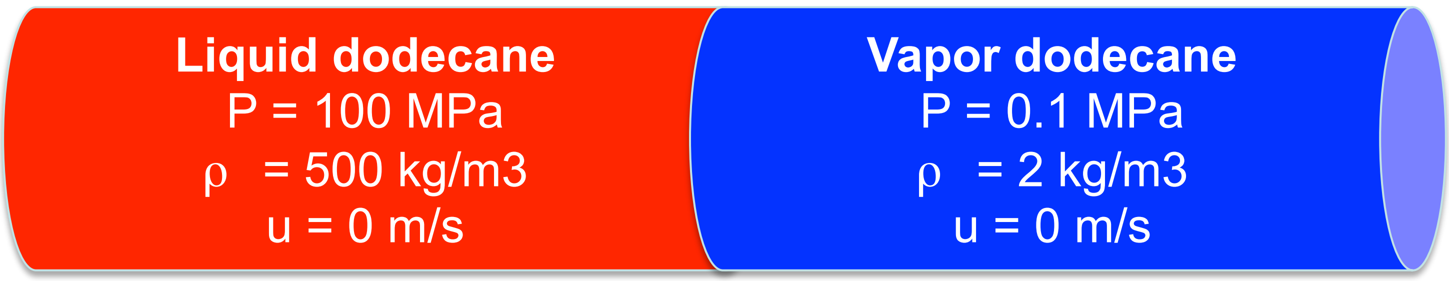
$$\left(\frac{d\mathbf{v}_0(\mathbf{x}_i)}{dt} \right)_c = \sum_{j \in D_i} \omega_j \left[\frac{p_{ci}}{\rho_i} + \frac{c_c}{2} \left(\mathbf{v}_0(\mathbf{x}_i) - \mathbf{v}_0(\mathbf{x}_j) \right) \cdot \mathbf{n}_{ij} \right] \nabla_i W_{ij}$$

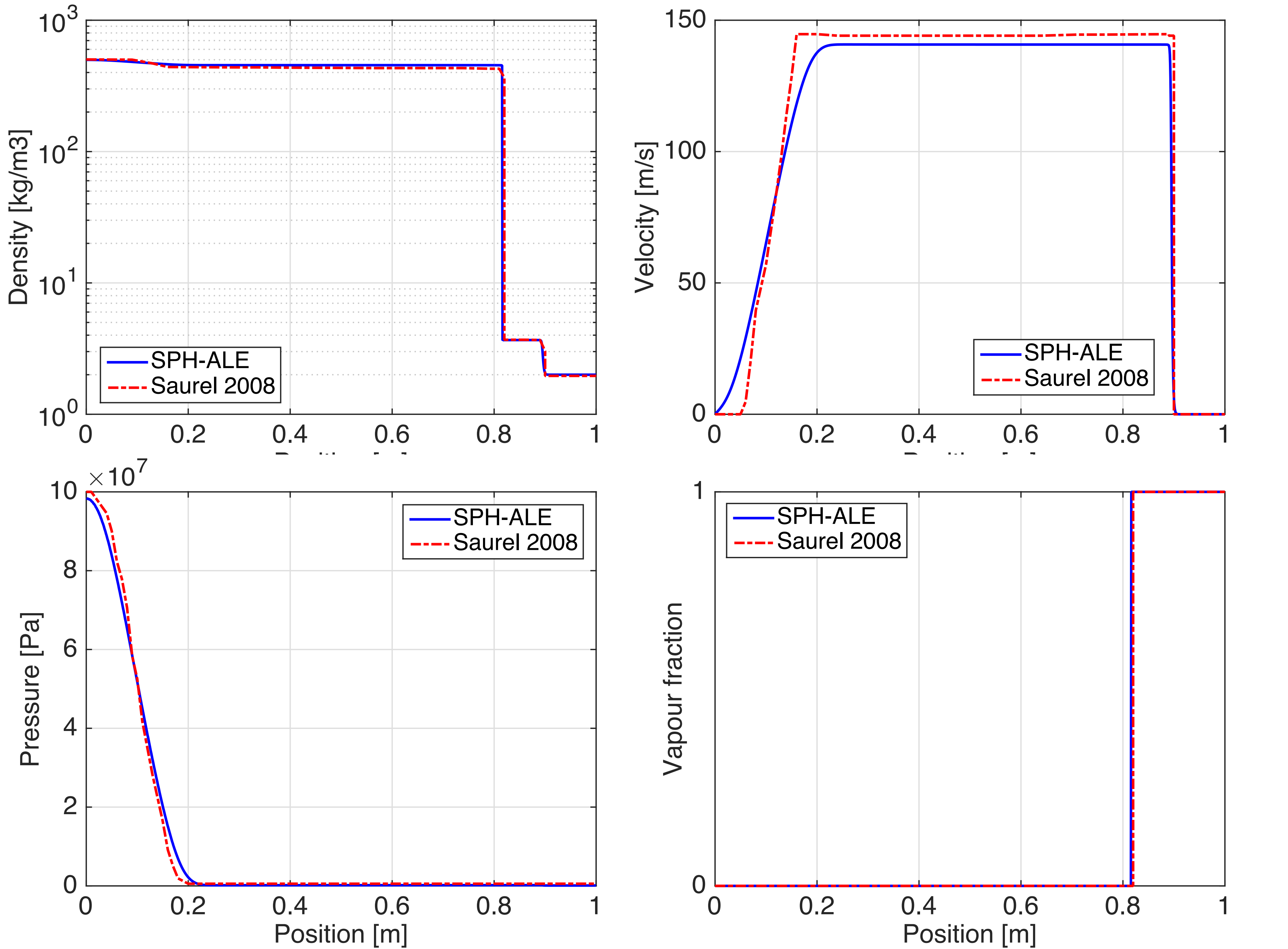
Neuhauser, 2014

Validation: Non-isentropic model

Multiphasic shock tube

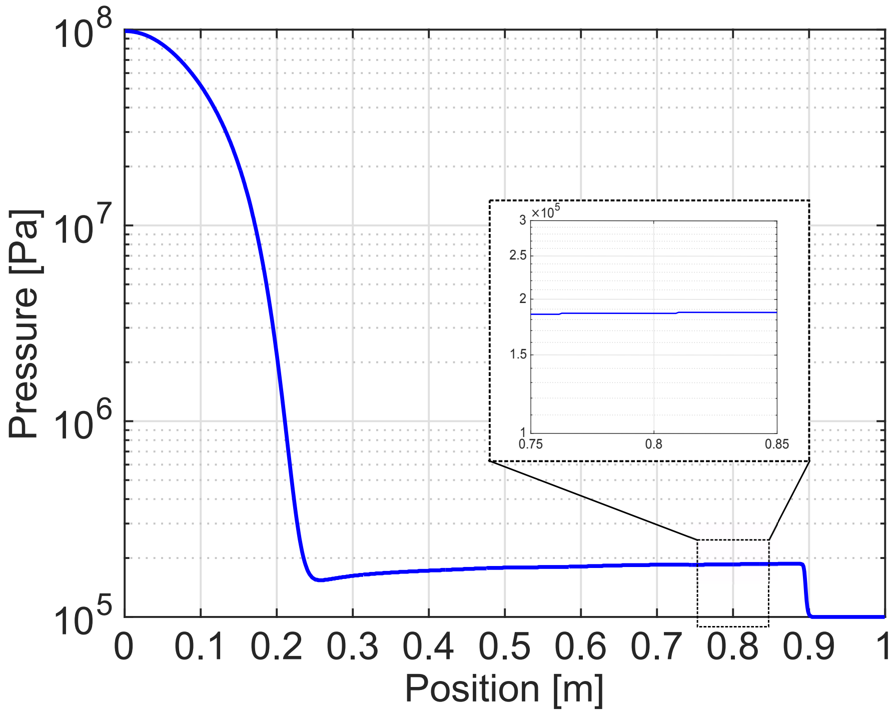
Saurel, et al J. Fluid Mech. (2008) vol. 607, pp. 313-350

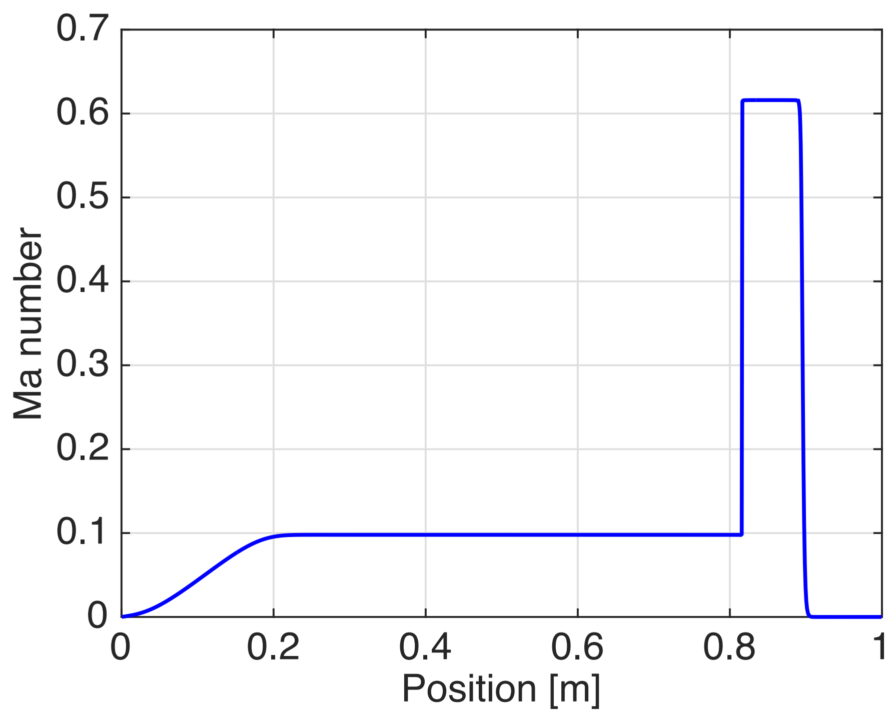




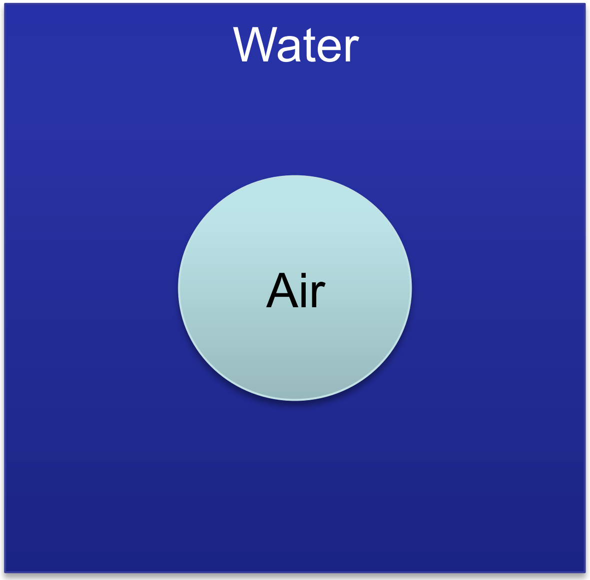
Density [kg/m3]
SPH-ALE
Saurel 2008
Velocity [m/s]
SPH-ALE
Saurel 2008
Pressure [Pa]
×10^7
SPH-ALE
Saurel 2008
Vapour fraction
SPH-ALE
Saurel 2008
Position [m]
Position [m]

Validation: Non-isentropic model
Multiphasic shock tube





Two phase simulations in SPH-ALE code

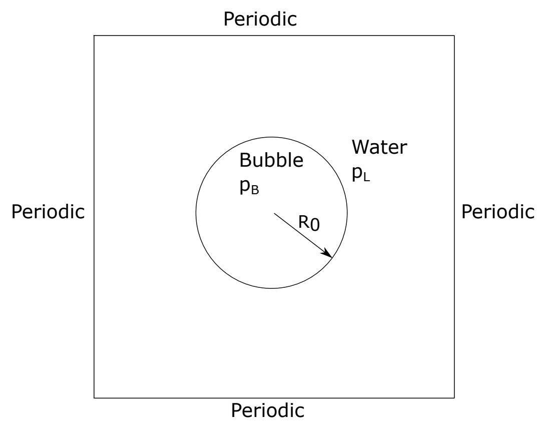


- ✓ Two-phase simulation (liquid - gas)
- ✓ Euler system
- ✓ Stiffened gas EOS and ideal gas EOS
- ○ No mass transfer

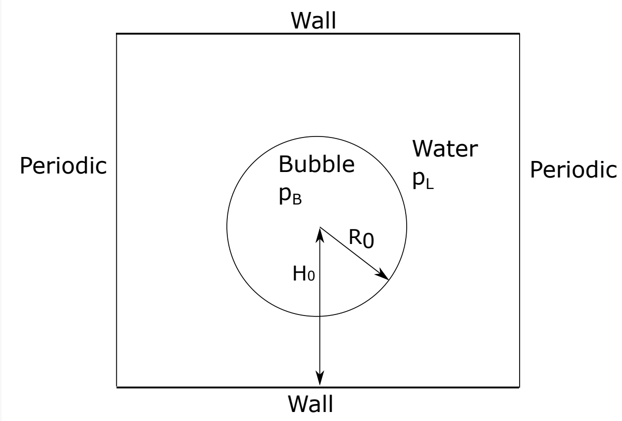
Stiffened gas parameters

	P_0 [Pa]	γ	Cv
Liquid	1e9	2.35	1816
Gas	0	1.43	1040

Bubble dynamic: free field

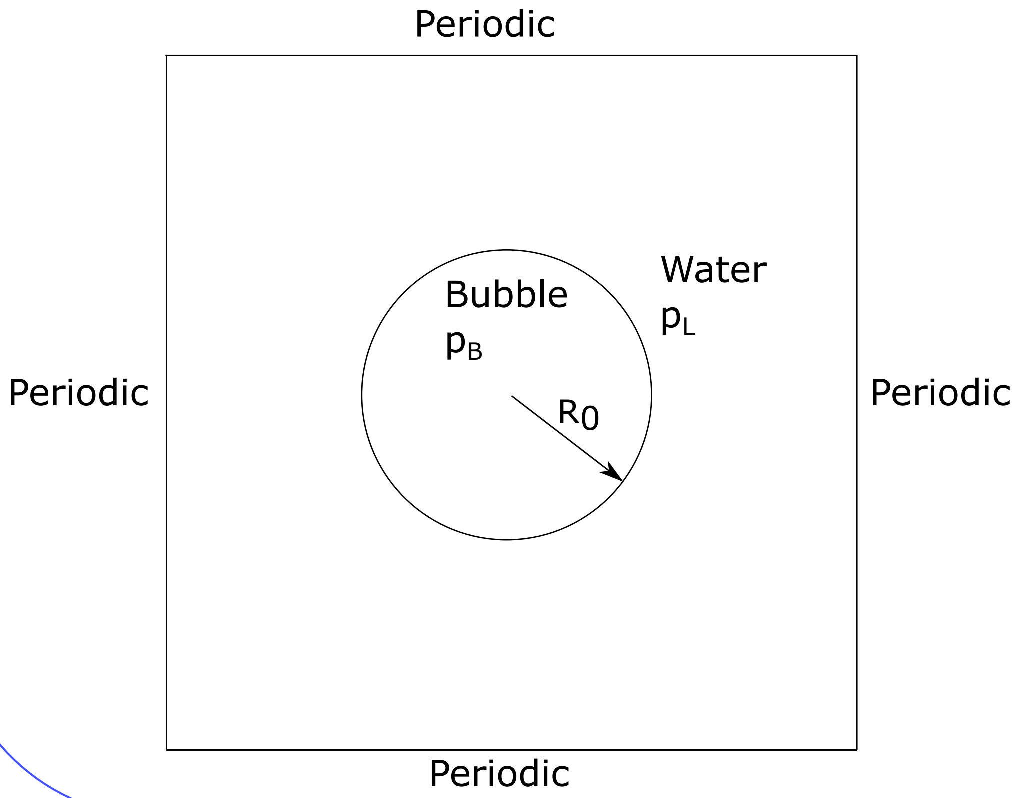


Bubble dynamic: near a wall



Bubble dynamic: free field

$P_L/P_B = 100$



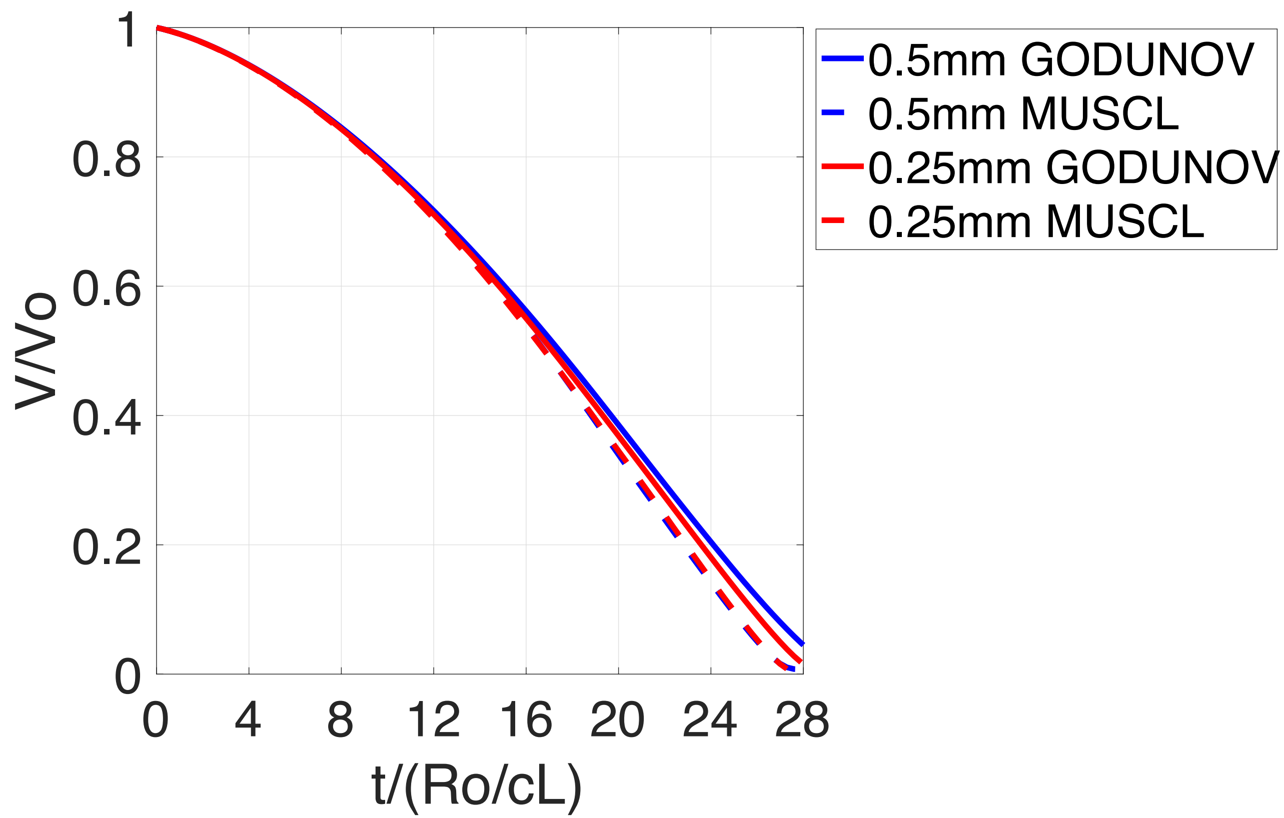
2D simulations

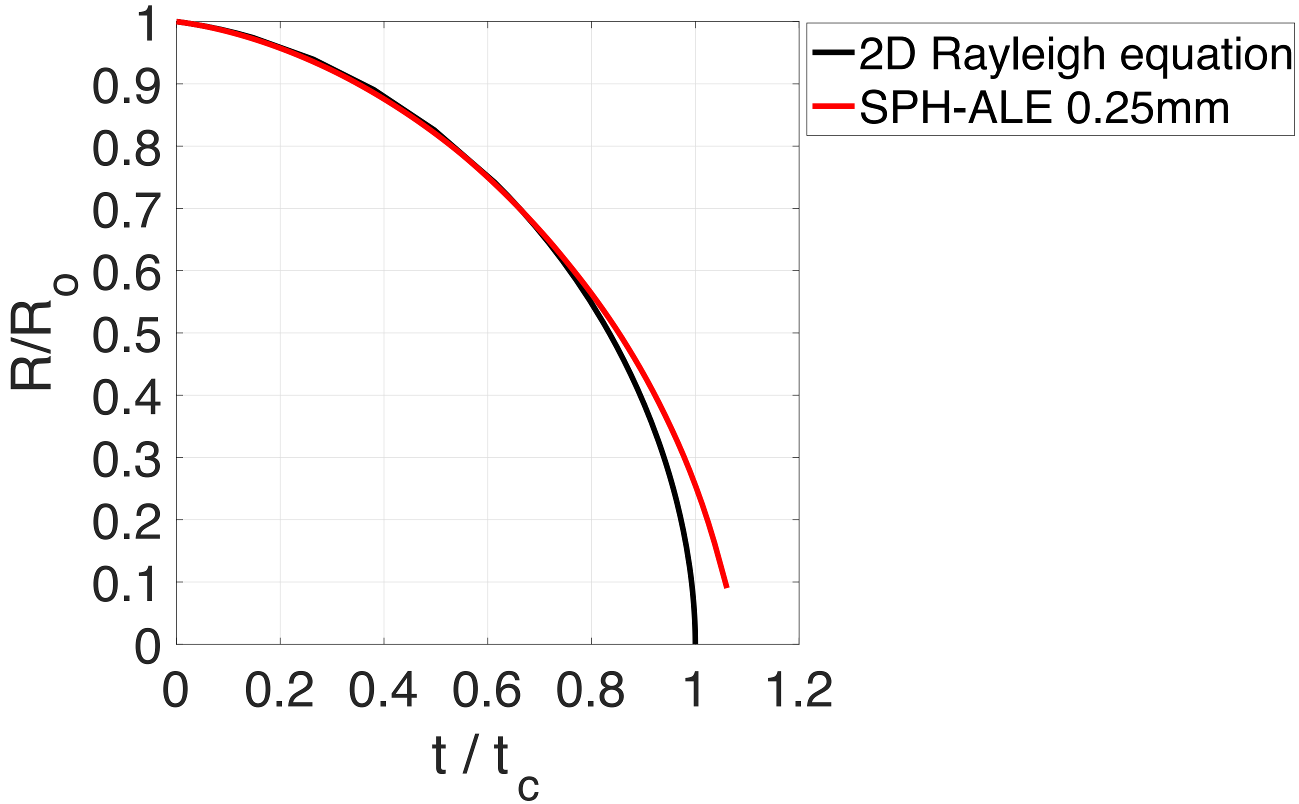
Collapse free field

Non-dimensional variables

Characteristic distance : Ro
Characteristic velocity : c_L

$t_{ND} = t/(Ro/c_L)$
$u_{ND} = u/c_L$
$p_{ND} = p/(\rho_L c_L^2)$





$$\left(R\ddot{R}+\dot{R}^2\right)\log\left(\frac{R}{R_\infty}\right)+\frac{1}{2}\dot{R}^2=\frac{p}{\rho}$$

R_∞: distance at which the velocity in the fluid has dropped to zero (geometrical limits of calculation domain)

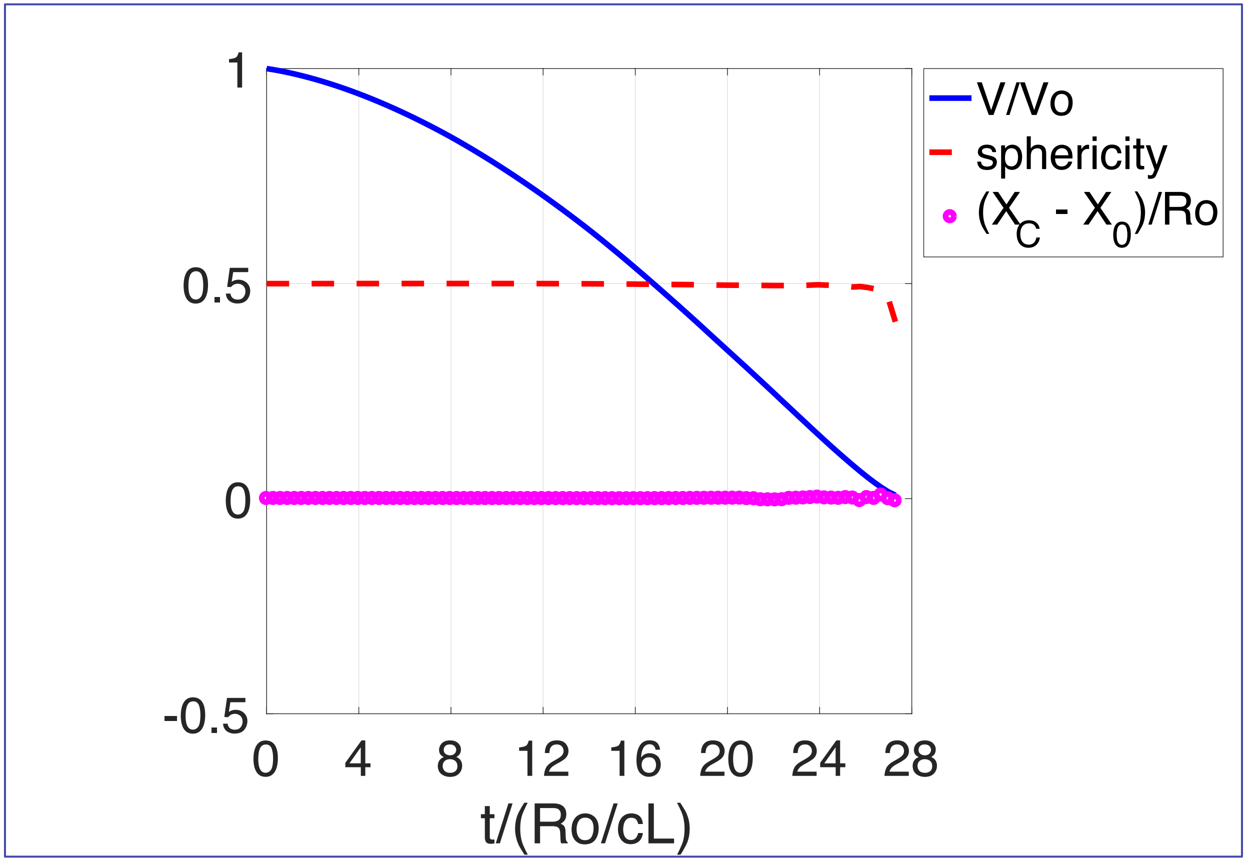
p: pressure far from the bubble

ρ: liquid density

Integrating with the following initial conditions t = 0 s

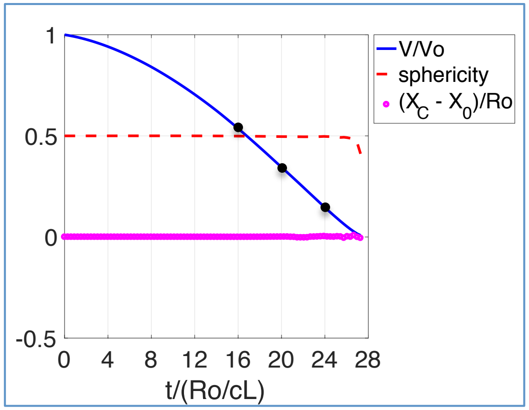
R = Ro
$\dot{R}$ = u* (velocity from Riemman solver)

Collapse free field



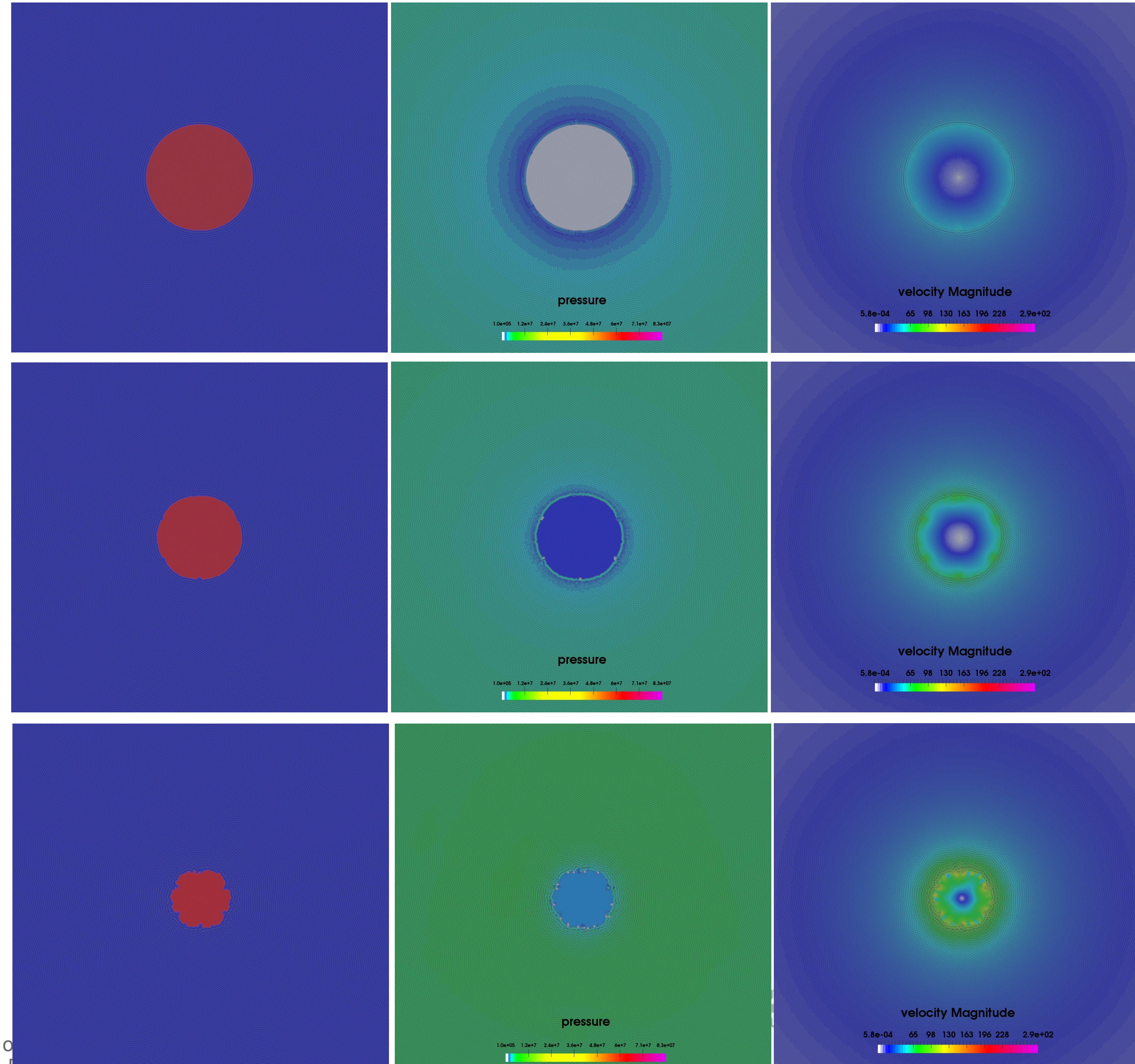
$$sphericity = \frac{radius\ calculated}{2\ (average\ of\ each\ radius)}$$

Collapse free field

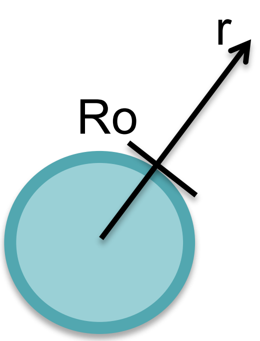


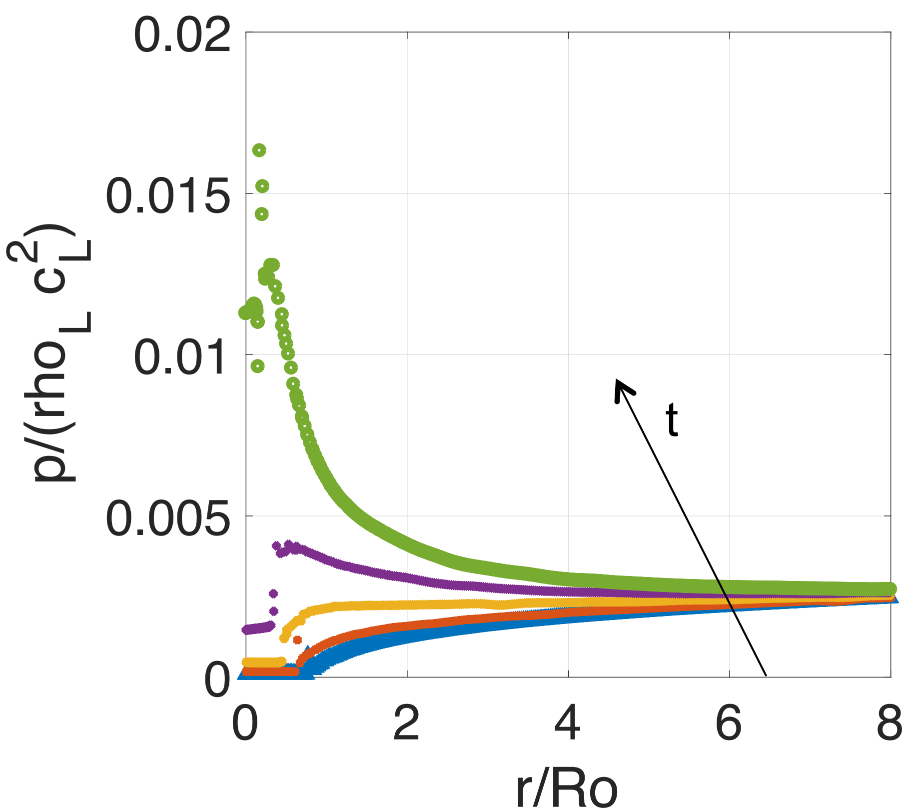
PRESSURE

VELOCITY



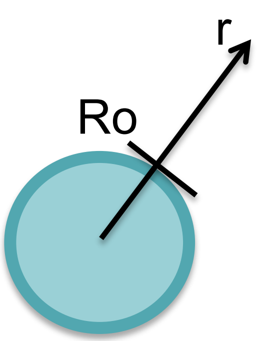
Collapse free field

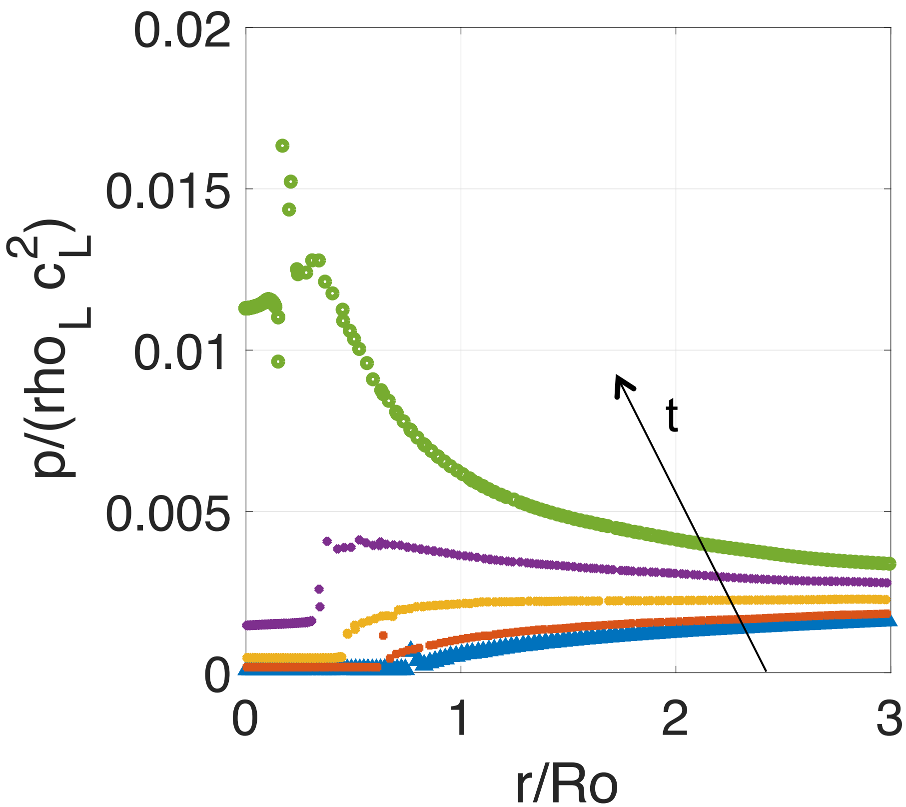




$t/(Ro\ cL^2)$:
15
18.75
22.5
24.75
26.25

Collapse free field



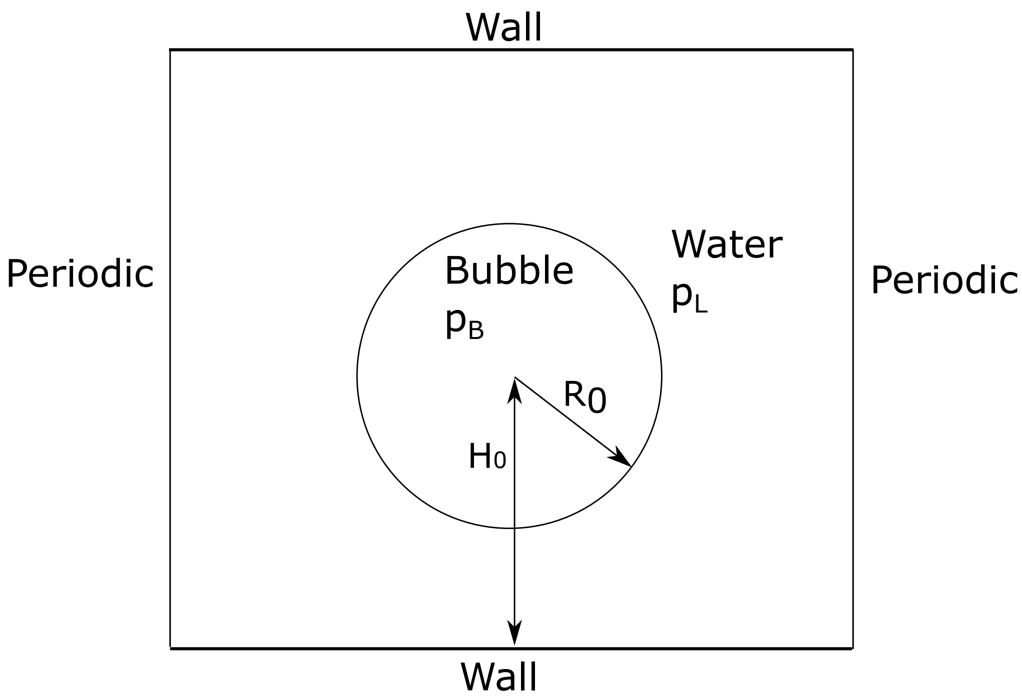


$t/(Ro\ cL^2)$:
15
18.75
22.5
24.75
26.25

Bubble dynamic: near a wall

Ho/Ro	->	1.10
		1.25
		2.00

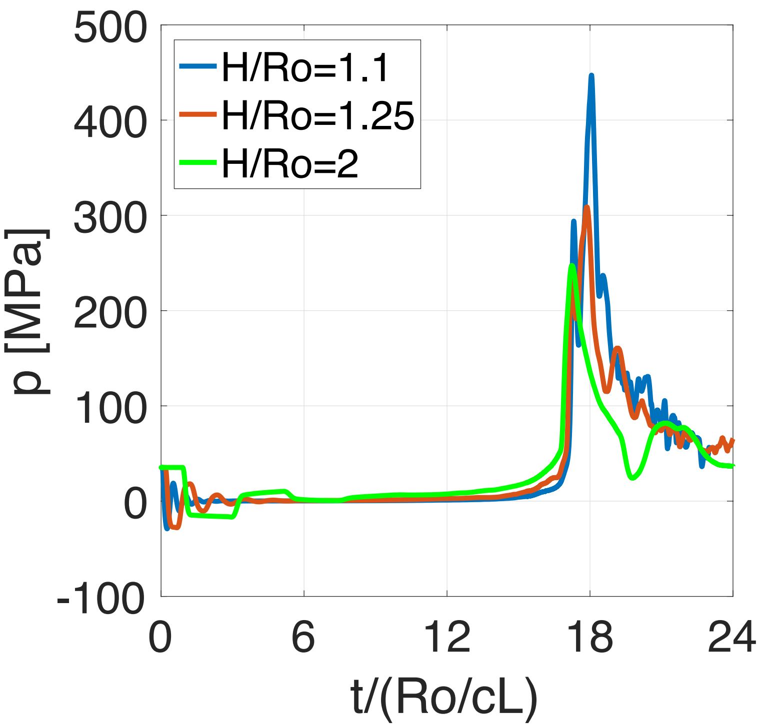
$P_L/P_B = 353$

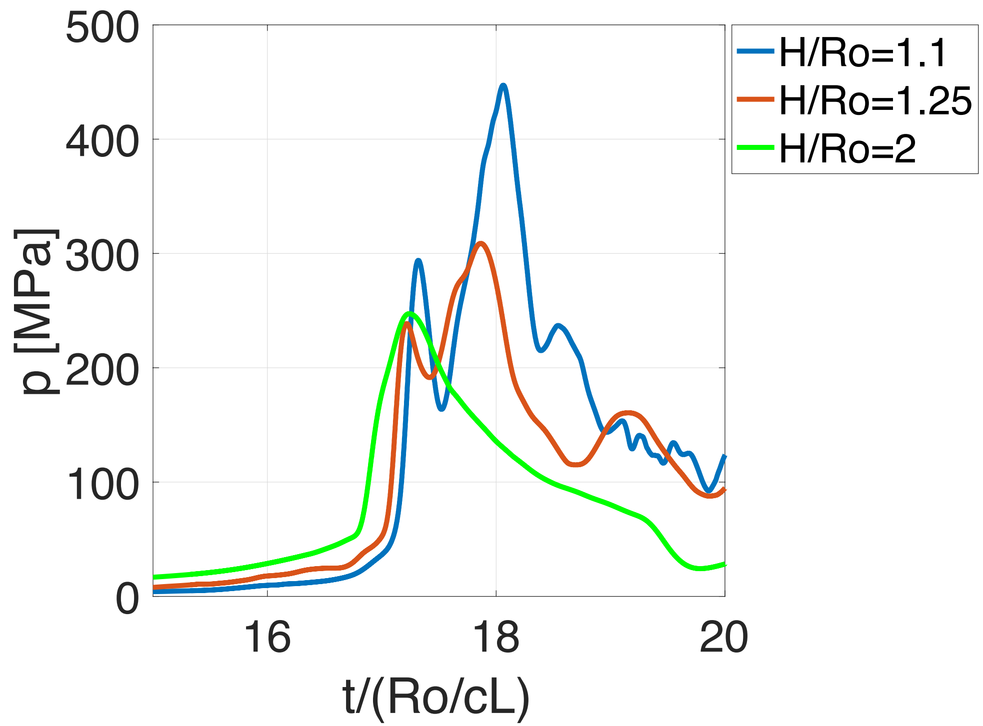


Collapse near a wall

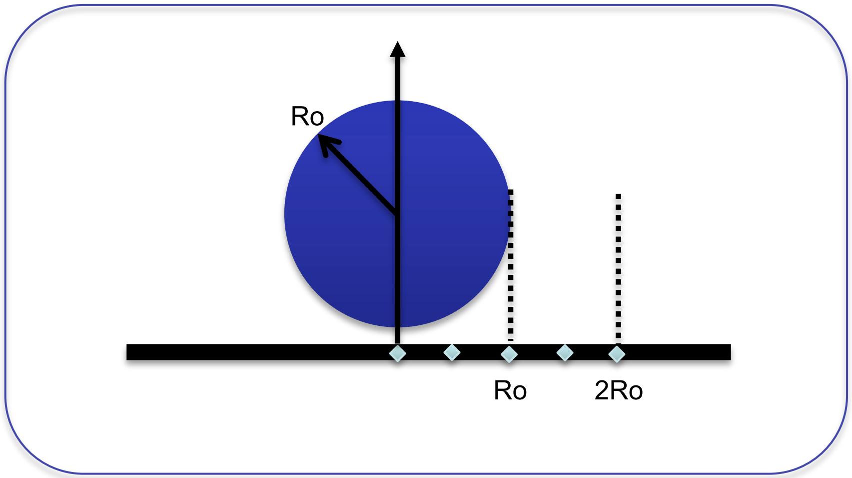
pressure at the center of the wall

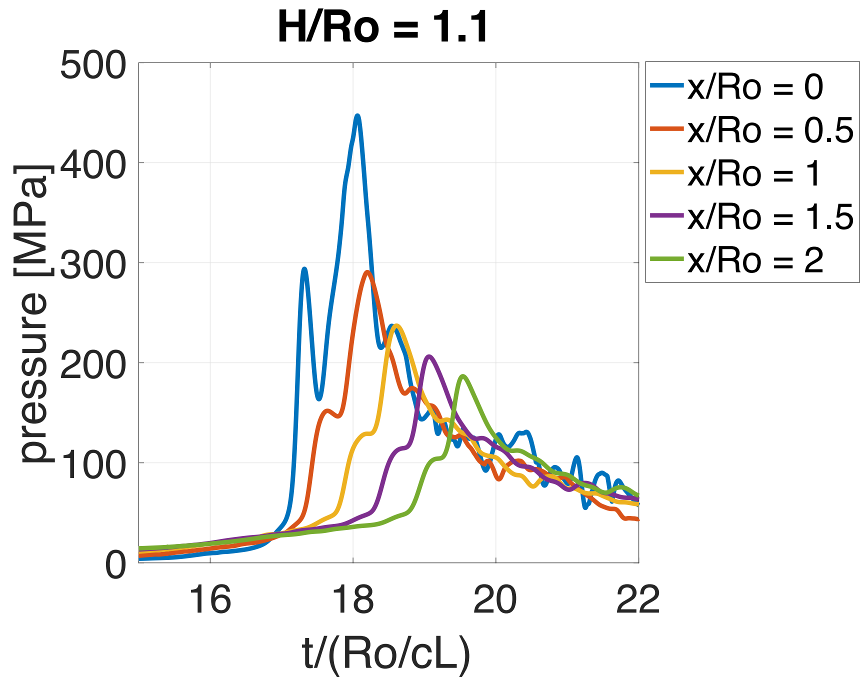
$p_L / p_B = 353$

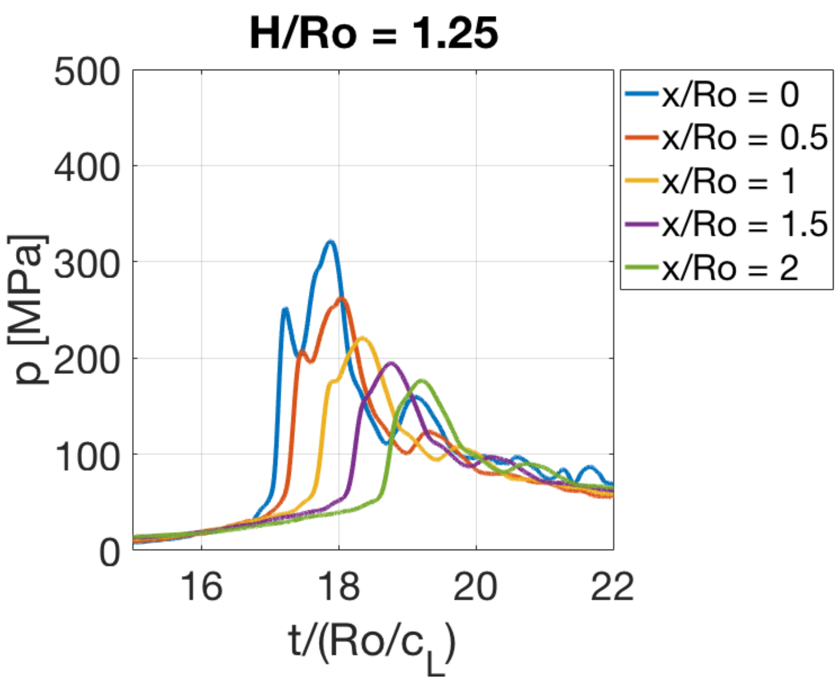


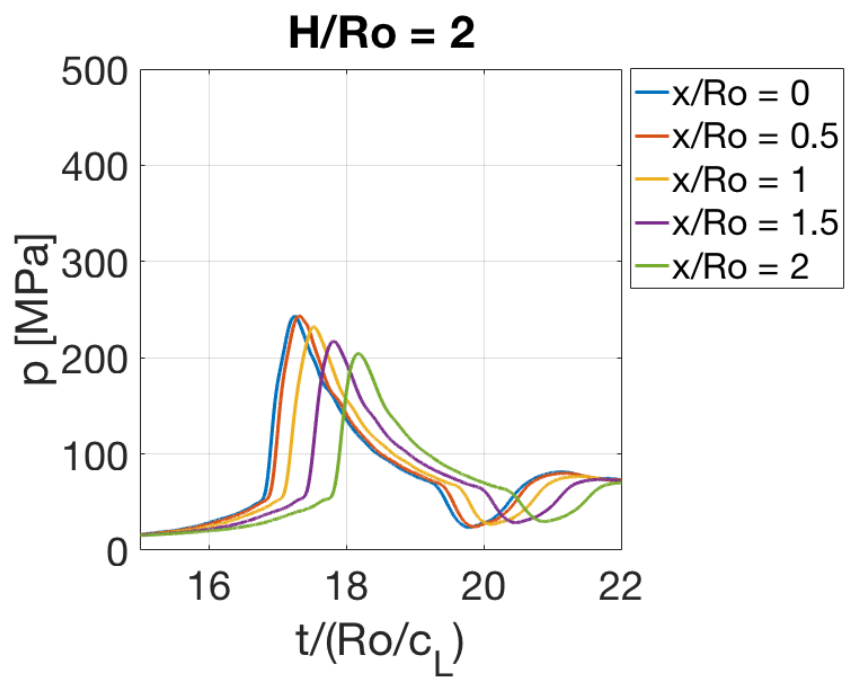


Collapse near a wall pressure $p_L / p_B = 353$



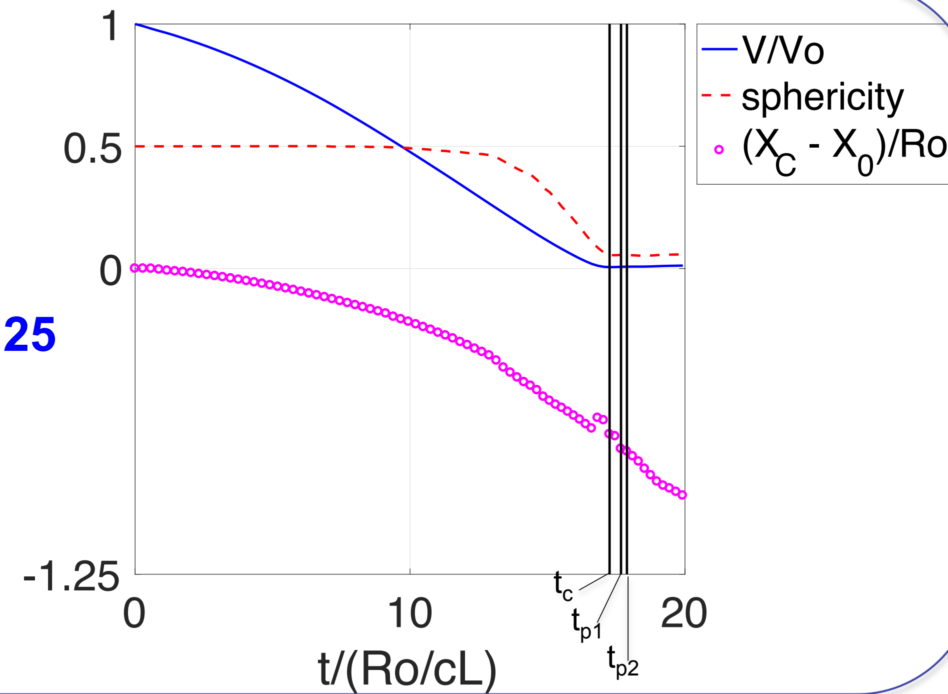




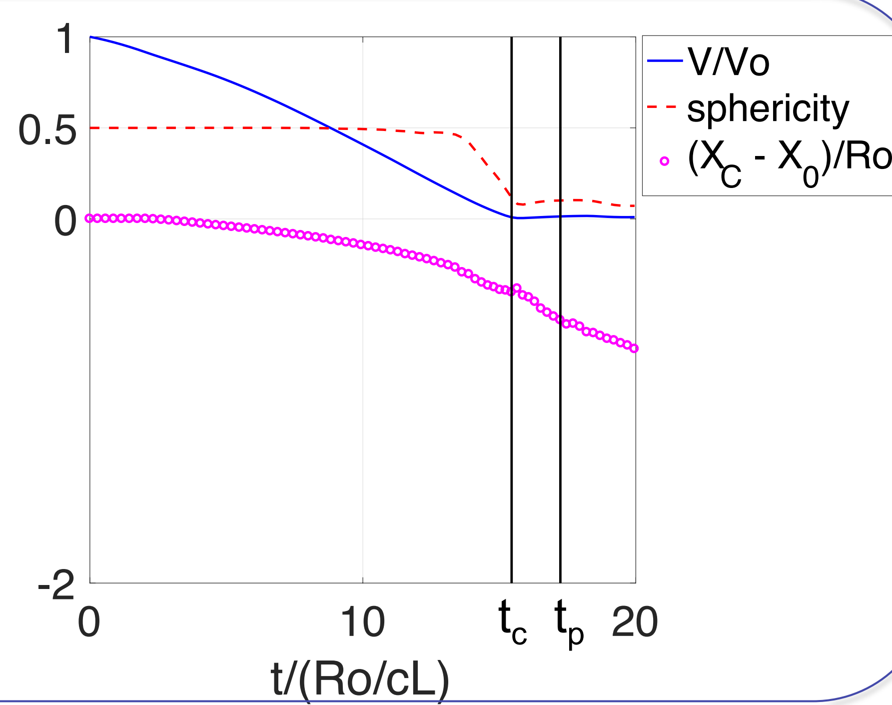


Collapse near a wall
p_L / p_B = 353

H/Ro = 1.25



H/Ro = 2



Conclusion

- Compressible SPH-ALE implemented for multiphase flow simulations.
- For the free-field collapse case:
  - Good agreement of collapse behavior with respect to analytical equation.
- For the case of the bubble collapse near a wall:
  - A re-entrant jet directed towards the surface is observed due to the non- symmetry initial configuration.
  - A precursor shock is observed due to the re-entrant jet in cases where H/Ro = 1.1 and 1.25.
  - An amplification of the pressure peak at the wall center is observed for case H/Ro = 1.1.
  - Collapse time bigger for bubbles closer to the wall.