Locally conservative approximation of (conservative) systems written in non conservation form: application to Lagrangian hydrodynamics and multifluid problems

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Since the celebrated Lax Wendroff theorem, it is known that the right way of discretising systems of hyperbolic equations written in conservation form is to use a flux formulation. However, in many occasions, the relevant formulation, from an engineering point of view, is not to consider this conservative formulation but one non conservative form. For example, with standard notations, a one fluid model writes

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho\mathbf{u}\\ E\end{pmatrix} + \text{ div }\begin{pmatrix}\rho\mathbf{u}\\ \rho\mathbf{u}\otimes\mathbf{u}+p\text{Id}\\ (E+p)\mathbf{u}\end{pmatrix} = 0, \tag{1}$$

but the interesting quantities are the mass, velocity and pressure, which evolution is described by:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho\mathbf{u}\\ p\end{pmatrix} + \begin{pmatrix}\text{div }\rho\mathbf{u}\\ \text{div }(\rho\mathbf{u}\otimes\mathbf{u}+p\text{Id})\\ \mathbf{u}\cdot\nabla p+\rho c^2 \text{ div }\mathbf{u}\end{pmatrix} = 0 \tag{2}$$

Unfortunately this form is not suitable to approximation. In the case of a multi-fluid system, the same problem occurs.

In this talk, we will describe a method to overcome this issue. It does not use any flux formulation *per se*, but can be shown to provide the right solutions. In order to illustrate the method, we will consider several examples in Eulerian and Lagrangian hydrodynamics

We will first start from the Residual Distribution (RD) (re-)interpretation of the Dobrev et al. scheme [1] for the numerical solution of the Euler equations in Lagrangian form. The first ingredient of the original scheme is the staggered grid formulation which uses continuous node-based finite element approximations for the kinematic variables and cell-centered discontinuous finite elements for the thermodynamic parameters. The second ingredient of the Dobrev et al. scheme is an artificial viscosity technique applied in order to make possible the computation of strong discontinuities. Using a reformulation in term of RD scheme, we can show that the scheme is indeed locally conservative while the formulation is *stricto sensu* non conservative. Using this, we can generalise the construction and develop locally conservative artificial viscosity free schemes. To demonstrate the robustness of the proposed RD scheme, we solve several one-dimensional shock tube problems from rather mild to very strong ones: we go from the classical Sod problem, to TNT explosions (with JWL EOS) via the Collela-Woodward blast wave problem.

In a second part, we show how to extend this method to the Eulerian framework and give applications on single fluid and multiphase problems via the five equation model.

References

[1] V. Dobrev, T. Kolev, and R. Rieben. High order curvilinear finite element methods for Lagrangian hydrodynamics. *SIAM J. Sci. Comput*, 34:B606–B641, 2012.