High fidelity anisotropic adaptive FEM: towards physical couplings occurring in turbulent boiling

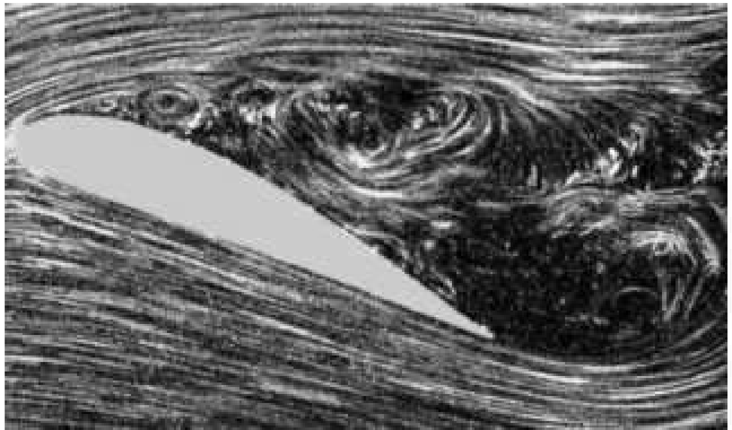
Elie Hachem*, Mehdi Khalloufi, Rudy Valette

CFL Research Group
(Computing and Fluids Lab)
MINES ParisTech, PSL Research University, CEMEF

elie.hachem@mines-paristech.fr

Context:

→ Looking for a robust Eulerian multiphase framework



gaz-solid flows

liquid-gaz flows

liquid-gaz-solid flows

Context:

→ Industrial application: **“Water Quench & Temper”**

- Multiphase flows
- Turbulent boiling
- Phase change
- Liquid-gaz-solid flows
- Water “agitators”
- Surface tension
- High thermal gradients
-…

Outline

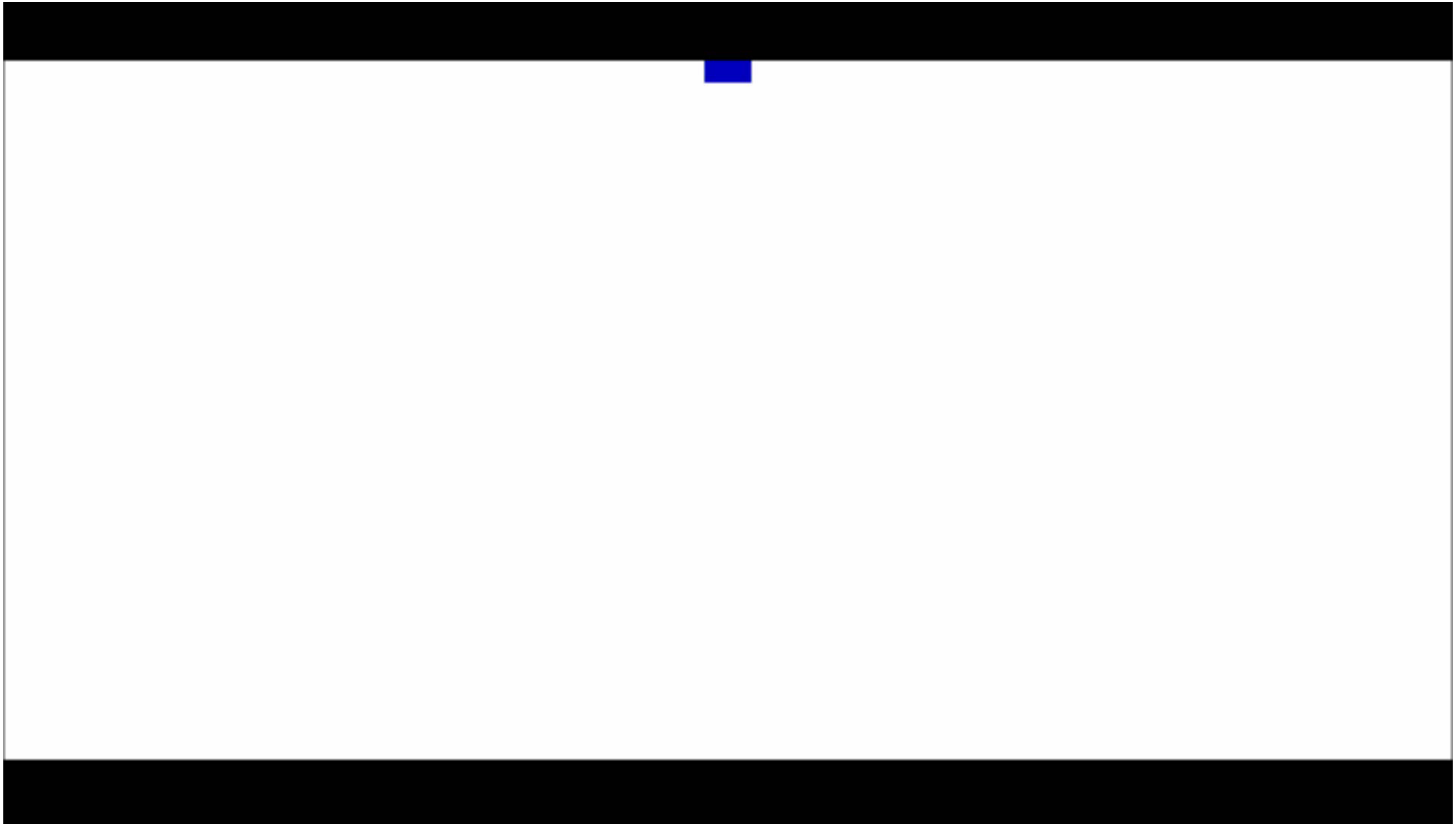
❑ Eulerian multiphase framework

- FEM solver suing Variational Multiscale Method
- A conservative Level-set method
- Implicit treatment of the surface tension
- Anisotropic mesh adaptation
- Extension towards a unified compressible-incompressible solver

- Heat transfer and phase change
- 2D and 3D validations

High fidelity multiphase framework: illustration

Unsteady NS, Anisotropic mesh adaptation, regularisation, parallel computing



FEM Flow solver:

The strong form of the incompressible Navier Stokes equations reads:

$$\begin{cases} \rho(\delta_t \mathbf{v} + \mathbf{v} \cdot \boldsymbol{\nabla} \mathbf{v}) - \boldsymbol{\nabla} \cdot \boldsymbol{\sigma} = \mathrm{f} & \text{in } \Omega \times [0,T] \\ \boldsymbol{\nabla} \cdot \mathbf{v} = 0 & \text{in } \Omega \times [0,T] \end{cases}$$

where ρ is the density, η is the viscosity, and the Cauchy stress tensor for a Newtonian fluid is given by:

$$\boldsymbol{\sigma} = 2\eta \, \boldsymbol{\varepsilon}(\mathbf{v}) - p \, \mathbf{I}_{\mathrm{d}}$$

[A. Masud, R.A. Khurram, 2006]
[T.J.R. Hugues et al.,1998]
[V. Gravemeier, W.A. Wall, E. Ramm, 2004]
[L.P. Franca, A. Nesliturk, 2001]
[R. Codina, 2002]

The Galerkin discrete problem consists therefore in solving the following mixed problem:

Find a pair $\mathbf{v}_h : [0,T] \to V_h$ and $p_h : (0,T] \to P_h$, such that: $\forall \, (\mathbf{w}_h, q_h) \in V_{h,0} \times P_h$

$$\begin{cases} (\rho \delta_t \mathbf{v}_h, \mathbf{w}_h)_\Omega + (\rho \mathbf{v}_h \cdot \boldsymbol{\nabla} \mathbf{v}_h, \mathbf{w}_h)_\Omega \\ \qquad + (2\eta \boldsymbol{\varepsilon}(\mathbf{v}_h) : \boldsymbol{\varepsilon}(\mathbf{w}_h))_\Omega - (p_h, \boldsymbol{\nabla} \cdot \mathbf{w}_h)_\Omega = (\mathbf{f}, \mathbf{w}_h)_\Omega \\ (\boldsymbol{\nabla} \cdot \mathbf{v}_h, q_h)_\Omega = 0 \end{cases}$$

…the stability of this formulation depends on appropriate compatibility restrictions on the choice of the finite element spaces

…oscillations due to convection dominated flows

Flow solver:

VMS: Variational MultiScale

- VMS methods consider large scales which are defined by projection into appropriate spaces
- Models both velocity and pressure unresolved scales
- Similarity with the implicit version of LES
- Allows high density and viscosity ratios

Variational Multiscale formulation: $\mathbf{v} = \mathbf{v}_h + \mathbf{v}'$ $\quad p = p_h + p'$

– the use of equal order continuous interpolations
– preventing from oscillations due to convection dominated flows

$$\text{find } (\mathbf{v}_h + \mathbf{v}', p_h + p') \in V_h \oplus V' \times P_h \oplus P' \text{ such that}$$

$$(\rho\delta_t(\mathbf{v}_h + \mathbf{v}'), \mathbf{w}_h + \mathbf{w}') + (\rho(\mathbf{v}_h + \mathbf{v}') \cdot \nabla(\mathbf{v}_h + \mathbf{v}'), \mathbf{w}_h + \mathbf{w}') - (p_h + p', \nabla \cdot (\mathbf{w}_h + \mathbf{w}'))$$
$$+2(\eta\varepsilon(\mathbf{v}_h + \mathbf{v}'), \varepsilon(\mathbf{w}_h + \mathbf{w}')) = \langle \mathbf{f}, \mathbf{w}_h + \mathbf{w}' \rangle$$
$$(q_h + q', \nabla \cdot (\mathbf{v}_h + \mathbf{v}')) = 0$$

$$\text{for all } (\mathbf{w}_h + \mathbf{w}', q_h + q') \in V_{h,0} \oplus V_0' \times P_{h,0} \oplus P_0'.$$

Flow solver:

The implemented version:

- Static subscales
- Approximation of the nonlinear term using only the large-scale part
- Well adapted to anisotropic mesh adaptation

The subscales are approximated within each element K by:

$$\mathbf{v}' = \alpha_v \Pi'_v(\mathcal{R}_v), \quad p' = \alpha_p \Pi'_p(\mathcal{R}_p),$$

Where Rc and Rp are the finite element residuals:

$$\mathcal{R}_v = \mathbf{f} - \rho\delta_t \mathbf{v}_h - \rho \mathbf{v}_h \cdot \nabla \mathbf{v}_h - \nabla p_h + \nabla \cdot (2\eta\varepsilon(\mathbf{v}_h))$$
$$\mathcal{R}_p = -\nabla \cdot \mathbf{v}_h$$

Flow solver:

Variational MultiScale method:

– approximate the fine scale within each element K
– inserting the expressions of the subscales in the coarse scale equations
– fully implicit resolution

$$
\begin{aligned}
&(\rho\delta_t \mathbf{v}_h, \mathbf{w}_h) + (\rho \mathbf{v}_h \cdot \nabla \mathbf{v}_h, \mathbf{w}_h) - (p_h, \nabla \cdot \mathbf{w}_h) + 2(\eta\varepsilon(\mathbf{v}_h), \varepsilon(\mathbf{w}_h)) \\
&\quad + \sum_K \alpha_v(\rho\delta_t \mathbf{v}_h + \rho \mathbf{v}_h \cdot \nabla \mathbf{v}_h + \nabla p_h - \nabla \cdot (2\eta\varepsilon(\mathbf{v}_h)), \rho \mathbf{v}_h \cdot \nabla \mathbf{w}_h + \nabla \cdot (2\eta\varepsilon(\mathbf{w}_h)))_K \\
&\quad + \sum_K \alpha_p(\nabla \cdot \mathbf{v}_h, \nabla \cdot \mathbf{w}_h) \\
&\quad = \langle \mathbf{f}, \mathbf{w}_h \rangle + \sum_K \alpha_v(\mathbf{f}, \rho \mathbf{v}_h \cdot \nabla \mathbf{w}_h + 2\eta \nabla \cdot \varepsilon(\mathbf{w}_h))_K
\end{aligned}
$$

$$
\begin{aligned}
&(q_h, \nabla \cdot \mathbf{v}_h) + \sum_K \alpha_v(\rho\delta_t \mathbf{v}_h + \rho \mathbf{v}_h \cdot \nabla \mathbf{v}_h + \nabla p_h - \nabla \cdot (2\eta\varepsilon(\mathbf{v}_h)), \nabla q_h)_K \\
&\quad = \sum_K \alpha_v(\mathbf{f}, \nabla q_h)_K
\end{aligned}
$$

and the stabilization parameters:

$$
\alpha_v = \left[\left(\frac{c_1 \eta}{\rho h^2} \right)^2 + \left(\frac{c_2 \|\mathbf{v}_h\|_K}{h} \right)^2 \right]^{-1/2} \qquad \alpha_p = \left[\left(\frac{\eta}{\rho} \right)^2 + \left(\frac{c_2 \|\mathbf{v}_h\|_K h}{c_1} \right)^2 \right]^{1/2}
$$

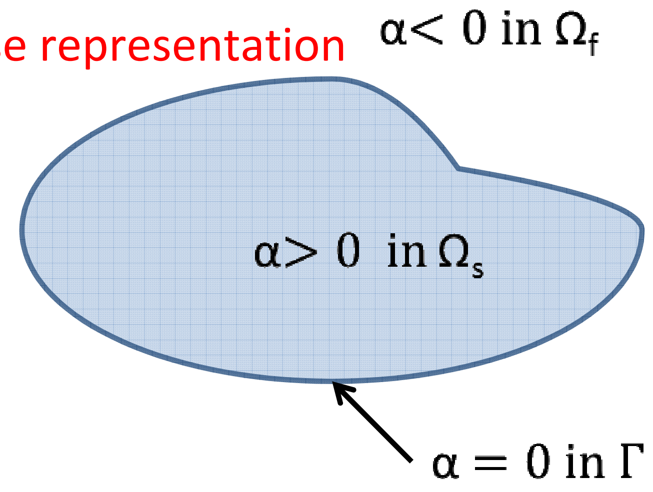
2-Level set

Basic definition

$$\alpha(X) = \begin{cases} -\mathrm{dist}(X,\Gamma) \text{ if } X \in \Omega_f \\ 0 \text{ if } X \in \Gamma \\ \mathrm{dist}(X,\Gamma) \text{ if } X \in \Omega_s \end{cases}$$

$$\|\nabla \alpha\| = 1$$

Phase representation



(1) Transport equation $\frac{\partial \alpha}{\partial t} + u \cdot \nabla \alpha = 0$

(2) Hamilton-Jacobi problem $\frac{\partial \alpha}{\partial \tau} + s(\alpha)(\|\nabla \alpha\| - 1) = 0$

Two possibilities:

Solving (1) and (2) separately

Fedkiw 2009, Osher 2000, Sussman 2005

Embedding (2) in (1) : auto reinitialisation

Ville et.al. 2011, Bonito et al 2015

Conservative Level Set method:

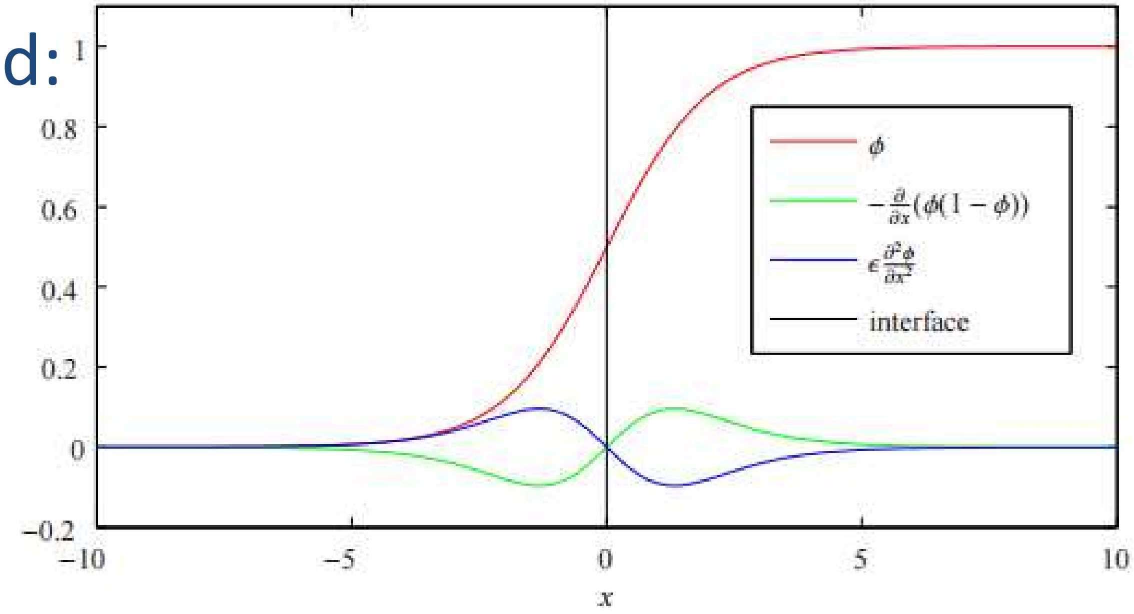
Conservative Level set method:

(1) Filtering $\phi(\alpha) = \frac{1}{2}\left(1 + \tanh\left(\frac{\alpha}{2\varepsilon}\right)\right)$

(2) Convection $\frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = 0$

(3) Reinitialization

$$\frac{\partial \phi}{\partial \tau} + \nabla \cdot \left(\phi(1-\phi)\mathrm{n} - \varepsilon((\nabla \phi \cdot \mathrm{n})\mathrm{n}\right) = 0$$



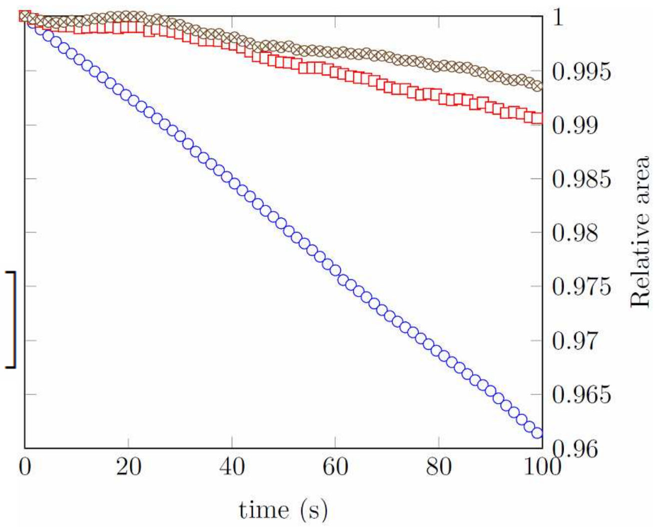
Walker, Müller 2014

E. Olsson, G. Kreiss, A conservative level set method for two phase flow,
***Journal of Computational Physics*, Volume 210, Issue 1, 2005, Pages 225-246**

(2) + (3) $\frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = \nabla \cdot \left(\varepsilon(\nabla \phi \cdot \mathrm{n})\mathrm{n} - \phi(1-\phi)\mathrm{n}\right)$

↓

Stabilized conservative FEM $\frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = \nabla \cdot \left[\chi\left(\varepsilon \nabla \phi - \phi(1-\phi)\frac{\nabla \phi}{|\nabla \phi|}\right)\right]$



X : local Index

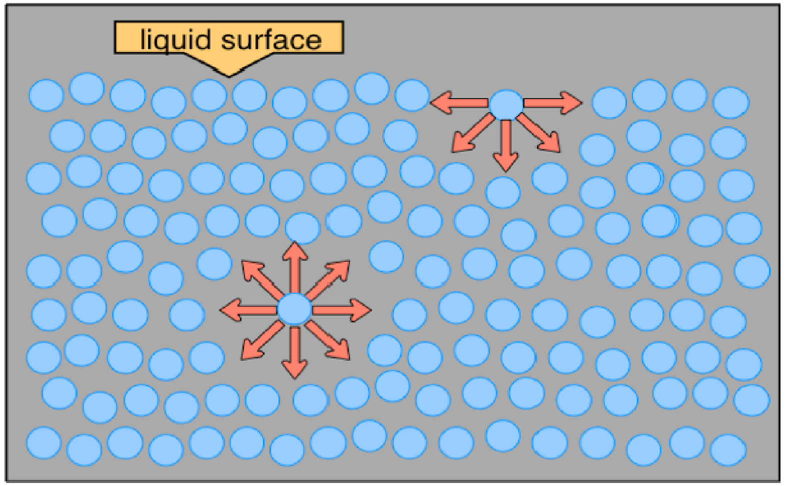
3- Surface tension

Surface tension using CSF

$$f_{\mathrm{ST}} = -\gamma\kappa\delta(\Gamma)\mathrm{n}$$

$$\mathrm{n} = \nabla\alpha/|\nabla\alpha| \qquad \kappa = -\nabla\cdot\mathrm{n}$$

γ is the surface tension coefficient



http://butane.chem.uiuc.edu/

Usually implemented as a source term in NS equations

$$\begin{aligned} \rho(\partial_t u + u\cdot\nabla u) - \nabla\cdot(2\mu\varepsilon(u)) + \nabla p &= f + f_{ST} \\ \nabla\cdot u &= 0 \end{aligned}$$

Numerical analysis shows that: time-step restriction $\Delta t < (\Delta x)^{\frac{3}{2}}\sqrt{\dfrac{\bar{\rho}}{2\pi\gamma}}$

Mesh adaptation: h→ 0 so the restriction becomes stronger !!!

3- Surface tension

Going back to the theorem of differential geometry

$$\Delta_s I_\Gamma = \nabla_s \cdot \nabla_s I_\Gamma = -\kappa \mathrm{n}$$

Implicit time integration of the interface position

$$\mathrm{I}_\Gamma^{n+1} = \mathrm{I}_\Gamma^{n} + \mathrm{u}^{n+1}\Delta t$$

Applying the theorem to the interface position

$$\begin{aligned}
\Delta_s I_\Gamma^{n+1} &= \Delta_s I_\Gamma^{n} + \Delta t \Delta_s \mathrm{u}^{n+1} \\
-(\kappa \mathrm{n})^{n+1} &= -(\kappa \mathrm{n})^{n} + \Delta t(\Delta_s \mathrm{u}^{n+1}) \\
-\gamma(\kappa \mathrm{n})^{n+1} &= -\gamma \kappa \mathrm{n} + \gamma \Delta t(\Delta_s \mathrm{u}^{n+1})
\end{aligned}$$

Decomposition of the surface Laplacian into a standard Laplacian

$$\Delta_s = \nabla_s^2 = \nabla^2 - \frac{\partial^2}{\partial \mathrm{n}^2} - \kappa \frac{\partial}{\partial \mathrm{n}} \qquad \text{with} \qquad \frac{\partial u}{\partial \mathrm{n}} = \nabla u \cdot \mathrm{n}$$

Semi-implicit surface tension

$$f_{\mathrm{ST}} = \boxed{-\gamma \kappa \delta(\alpha) \mathrm{n}} - \underline{\gamma \delta(\alpha) \Delta t \left(\frac{\partial^2 \mathrm{u}}{\partial \mathrm{n}^2} + \kappa \frac{\partial \mathrm{u}}{\partial \mathrm{n}} - \nabla^2 \mathrm{u}^{n+1} \right)}$$

3- Modifying NS equations

$$f_{\mathrm{ST}} = -\gamma\kappa\delta(\alpha)\mathrm{n} - \gamma\delta(\alpha)\Delta t\left(\frac{\partial^2 \mathrm{u}}{\partial \mathrm{n}^2} + \kappa\frac{\partial \mathrm{u}}{\partial \mathrm{n}} - \nabla^2 \mathrm{u}^{n+1}\right)$$

Variational formulation

- $$\rho(\partial_t u_h, v_h)_\Omega + \rho(u_h^i . \nabla u_h, v_h)_\Omega - \sum_{K\in\mathcal{T}_h} (\tau_K \mathcal{R}_M, \rho u_h \nabla v_h)_K + (2\mu\varepsilon(u_h) : \varepsilon(v_h))_\Omega$$
  $$-(p_h, \nabla\cdot v_h)_\Omega + \gamma\delta(\alpha)\Delta t\,(\nabla u_h : \nabla v_h)_\Omega + \sum_{K\in\mathcal{T}_h}(\tau_C \mathcal{R}_C, \nabla\cdot v_h)_K =$$
  $$\left(f - \boxed{\gamma\delta(\alpha)\kappa\mathrm{n}} - \gamma\delta(\alpha)\Delta t\left(\partial_{nn}u_h^i + \kappa\partial_n u_h^i\right), v_h\right)_\Omega$$
- $$(\nabla u_h, q_h)_\Omega - \sum_{K\in\mathcal{T}_h}(\tau_K \mathcal{R}_M, \nabla q_h)_K = 0$$

Residuals

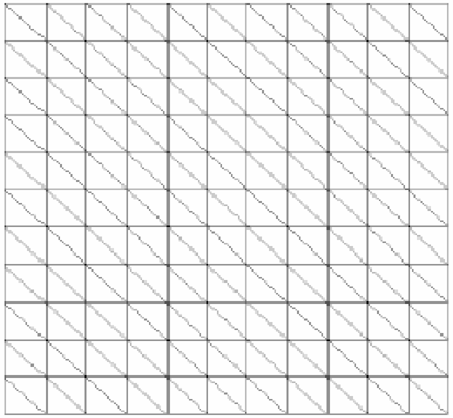
$$\mathcal{R}_M = f - \boxed{\gamma\delta(\alpha)\kappa\mathrm{n}} - \gamma\delta(\alpha)\Delta t\left(\partial_{nn}u_h^i + \kappa\partial_n u_h^i\right) - \rho\partial_t u_h - \rho u_h^i\cdot\nabla u_h - \nabla p_h$$
$$\mathcal{R}_C = -\nabla\cdot u_h$$

M. Khalloufi, Y. Mesri, R. Valette, E. Hachem, High fidelity anisotropic adaptive variational multiscale method for multiphase flows with surface tension, **Computer Methods in Applied Mechanics and Engineering**, Vol. 307, pp.44-67, **2016**

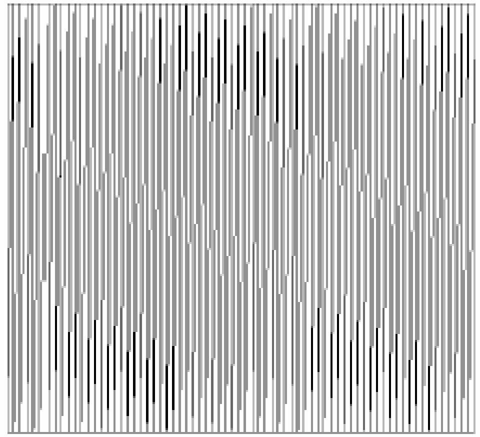
4- Anisotropic mesh adaptation

We are concern to capture automatically:

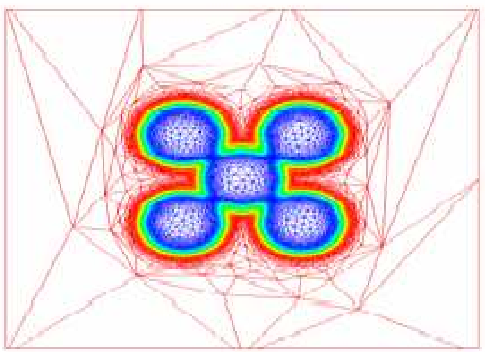
(i) boundary layers (ii) inner layers (iii) flow detachments

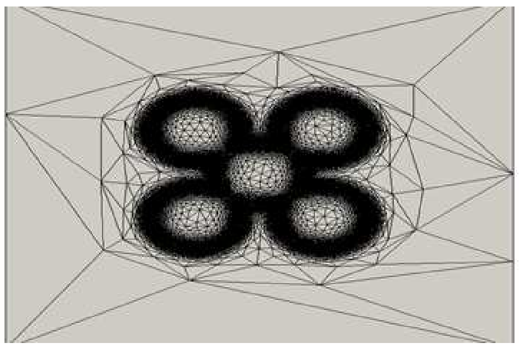


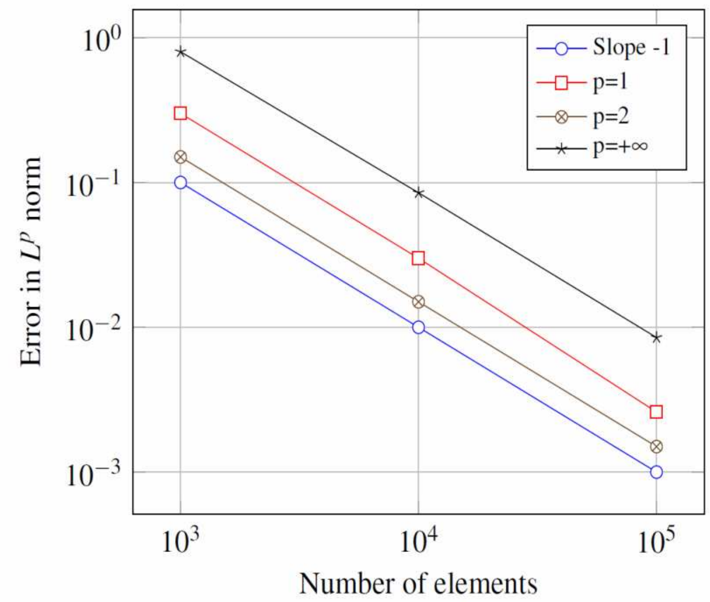
$$+\ M = \begin{pmatrix} \frac{1}{0.01^2} & \\ & \frac{1}{1^2} \end{pmatrix} +$$ imrpoved remesher =



Metric (directions, size)



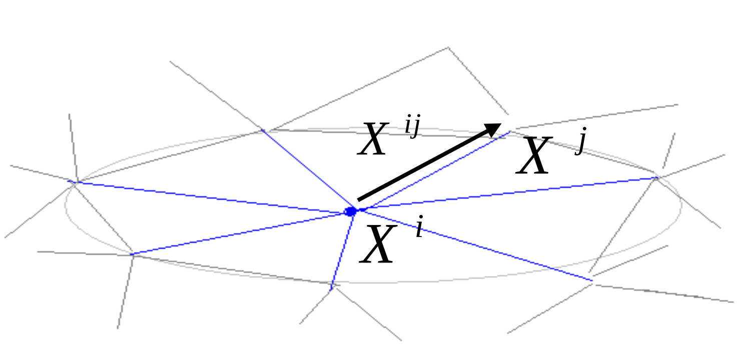




4- Anisotropic mesh adaptation

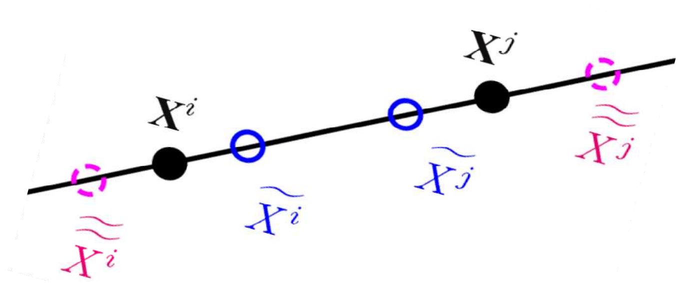
Candidate for directions:

$$\mathbb{X}^i = \frac{d}{|\Gamma(i)|} \sum_{j \in \Gamma(i)} \mathbf{X}^{ij} \otimes \mathbf{X}^{ij}$$



Control the size of each edge X^{ij} using a stretching factor:

$$\widetilde{\mathbf{X}^{ij}} = s_{ij} \mathbf{X}^{ij}$$



Drive these stretching factors using an edge based error estimator:

$$s_{ij} = \left(\frac{e}{e(N)}\right)^{-\frac{1}{2}} = \left(\frac{\sum_i n^i(1)}{N}\right)^{\frac{2}{d}} e_{ij}^{-1/2}$$

$$\widetilde{\mathbb{M}}^i = \frac{|\Gamma(i)|}{d} \left(\widetilde{\mathbb{X}}^i\right)^{-1}$$

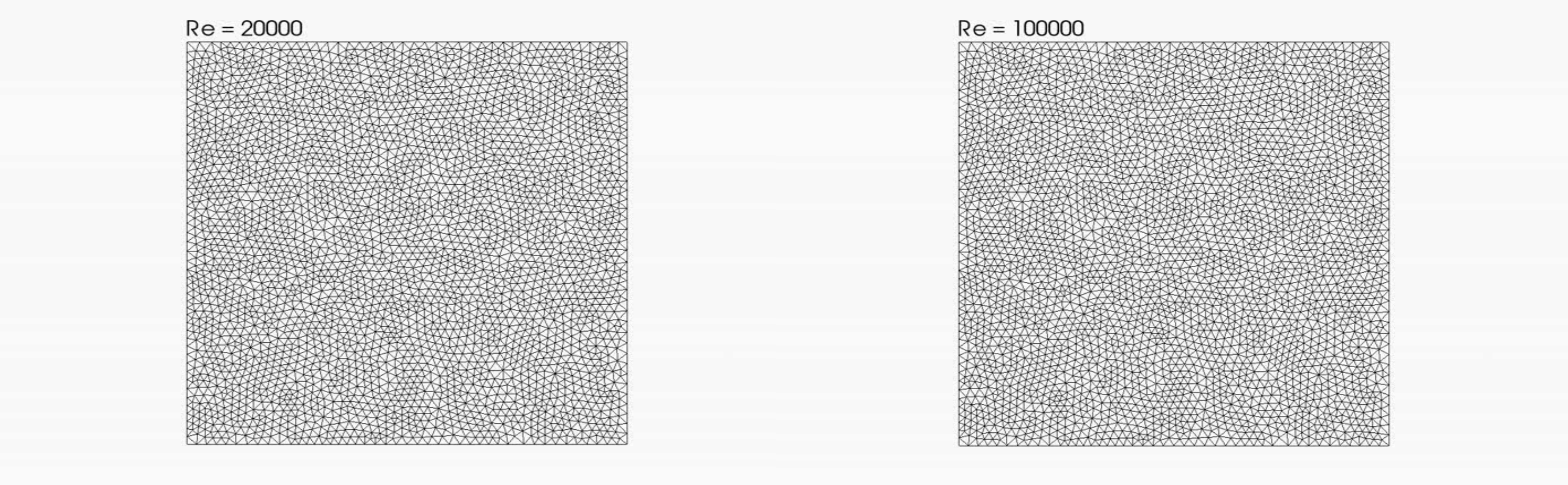
T. Coupez and E. Hachem, Solution of High-Reynolds Incompressible Flow with Stabilized Finite Element and Adaptive Anisotropic Meshing, Computer Methods in Applied Mechanics and Engineering, Vol. 267, pp. 65-85, 2013

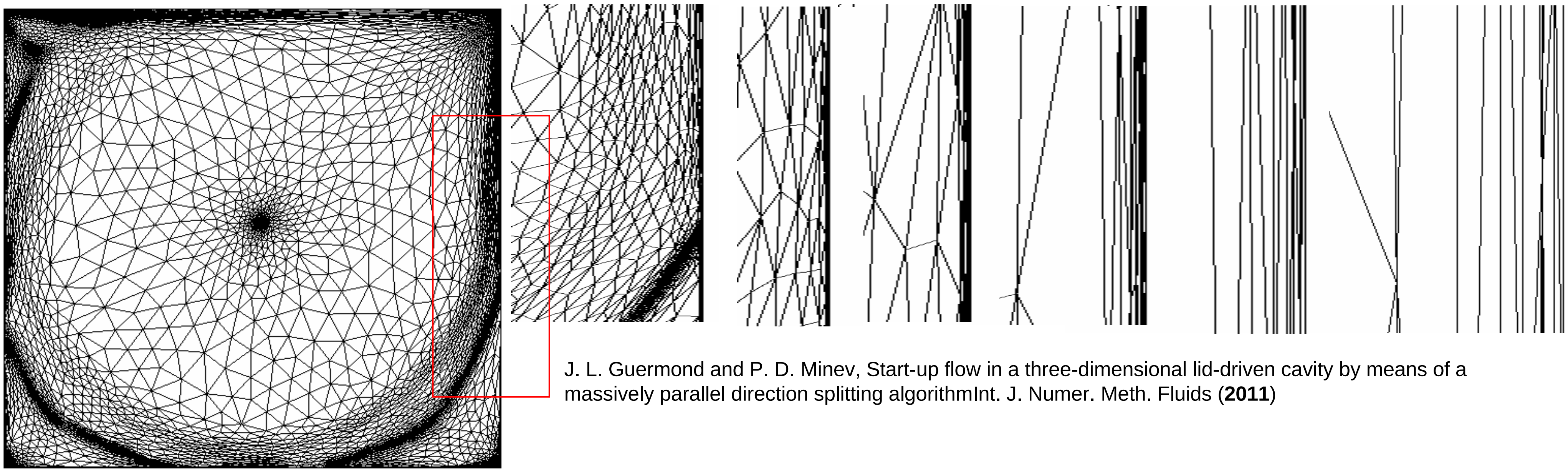
L. Billon, Y. Mesri, E. Hachem, Anisotropic boundary layer mesh generation for immersed complex geometries, Engineering with Computers, pp. 1-12, 2016

Illustration 1: complex geometry

- ❑Unlock new features in complex geometries

Illustration 2: NS with dynamic anisotropic meshing





J. L. Guermond and P. D. Minev, Start-up flow in a three-dimensional lid-driven cavity by means of a massively parallel direction splitting algorithmInt. J. Numer. Meth. Fluids (**2011**)

Illustration 3: Dam breaking (water column with surface tension)

- ❑ Independent from the problem at hand

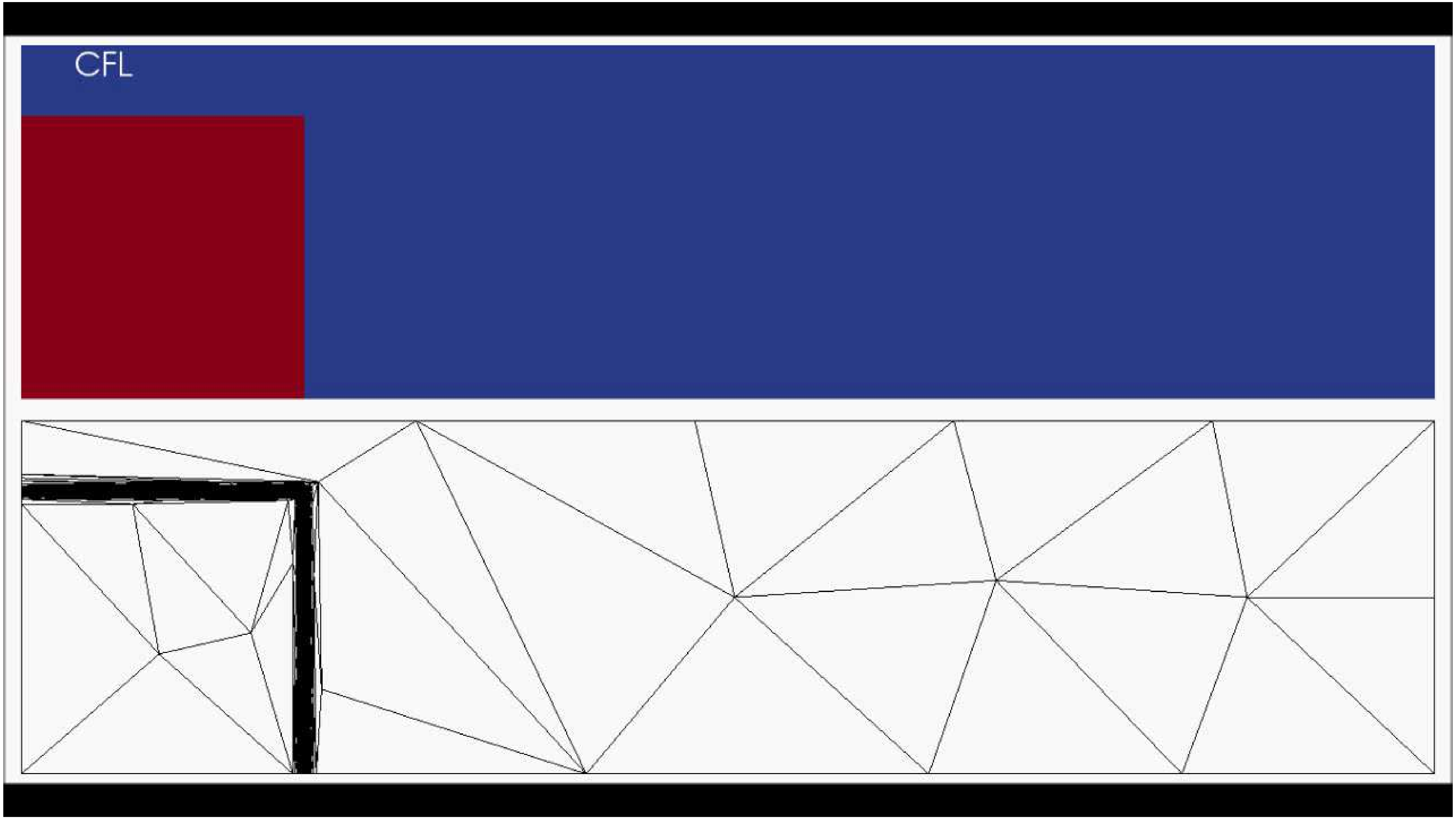
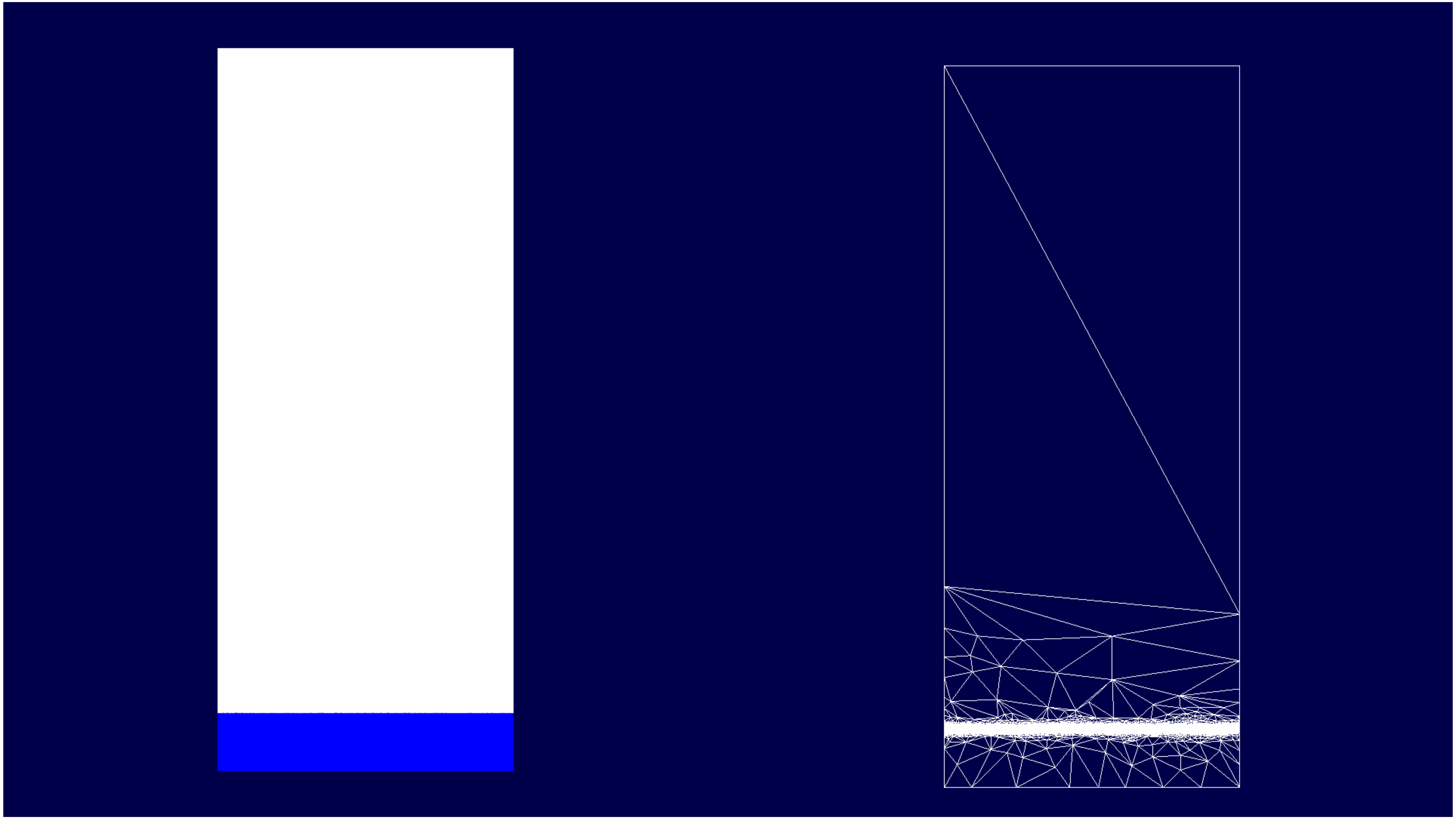


Illustration 4: Filling and dome formation

- Length redistribution under the constraint of a fixed number of nodes



5- Unified Compressible/Incompressible solver

Navier-Stokes equations

$$\rho(\partial_t u + u \cdot \nabla u) - \nabla \cdot \sigma = f \quad \text{in } \Omega \times [0,T]$$

$$\nabla \cdot u = 0 \quad \text{in } \Omega \times [0,T]$$

$$\frac{d\rho}{dt} = \left(\frac{\partial\rho}{\partial T}\right)_p \frac{dT}{dt} + \left(\frac{\partial\rho}{\partial p}\right)_T \frac{dp}{dt} \quad \Rightarrow \quad \chi_P = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)_T \quad \text{and} \quad \chi_T = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p$$

χ_T is the volume expansivity

χ_P is the isothermal compressibility coefficient

Conservation equation

$$\boxed{\nabla \cdot u + \chi_P \frac{\partial p}{\partial t} + \chi_P u \cdot \nabla p = \chi_T \frac{dT}{dt}}$$

M. Billaud, G. Gallice, B. Nkonga, A simple stabilized finite element method for solving two phase compressible–incompressible interface flows, *Computer Methods in Applied Mechanics and Engineering*, Volume 200, Issues 9–12, 2011, pp.1272-1290

5-Unified Compressible/Incompressible solver

$$\rho\frac{\partial u}{\partial t}+\rho u\cdot\nabla u-\nabla\cdot(2\mu\varepsilon(u))+\nabla p = f_v$$

$$\nabla\cdot u+\chi_P\frac{\partial p}{\partial t}+\chi_P u\cdot\nabla p = f_p$$

By splitting the velocity and the pressure into a coarse scale and a fine scale:
Variaztional Multiscale Stabilized Finite Element Method

$$\left(\frac{\rho\partial(u_h+\tilde{u})}{\partial t},v_h\right)+\left(\rho(u_h+\tilde{u})\cdot\nabla(u_h+\tilde{u}),v_h\right)-\left(p_h+\tilde{p},\nabla\cdot v_h\right)+\left(2\eta\varepsilon(u_h):\varepsilon(v_h)\right)=\left(f_v,v_h\right)\quad\forall v_h\in\mathcal{V}_h$$

$$\left(\nabla\cdot(u_h+\tilde{u}),q_h\right)+\chi_P\left(\frac{\partial(p_h+\tilde{p})}{\partial t},q_h\right)+\chi_P\left((u_h+\tilde{u}).\nabla(p_h+\tilde{p}),q_h\right)=\left(f_p,q_h\right)\quad\forall q_h\in Q_h$$

$$\tilde{u}=\sum_{K\in\Im_h}\tau_u\tilde{P}_u(R_u)$$

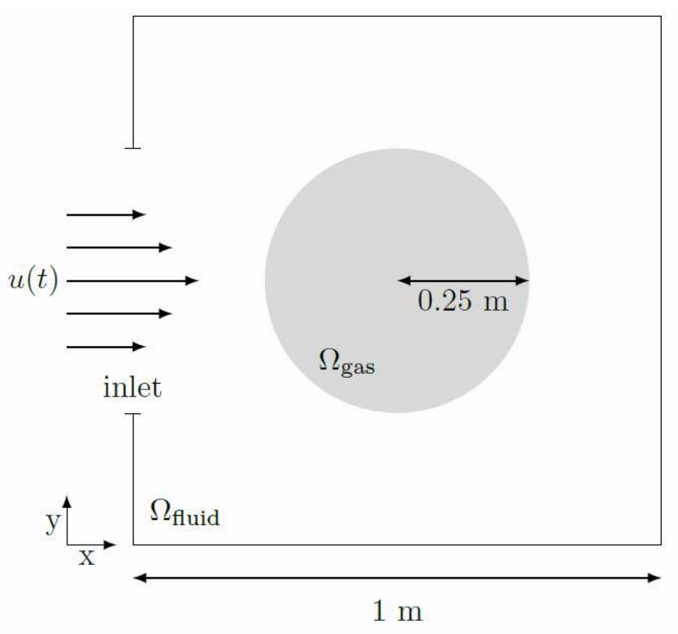
$$\tilde{p}=\sum_{K\in\Im_h}\tau_c\tilde{P}_c(R_c)$$

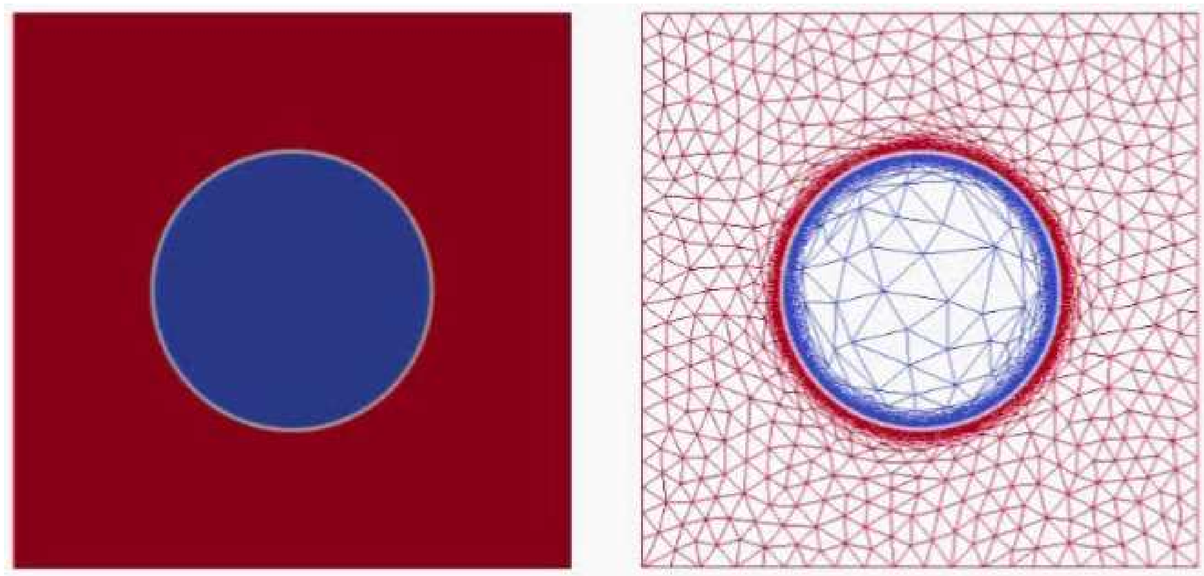
$$R_u=f_v-\rho\frac{\partial u_h}{\partial t}-\rho u_h^c\cdot\nabla u_h+\nabla\cdot(2\mu\varepsilon(u_h))-\nabla p_h$$

$$R_c=f_p-\nabla\cdot u_h-\chi_P\frac{\partial p_h}{\partial t}-\chi_P u_h^c\cdot\nabla p_h$$

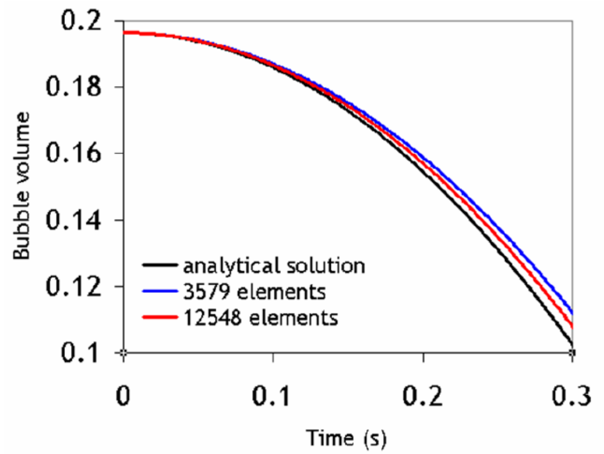
5-Unified Compressible/Incompressible solver

Compression of a bubble – Challenging case

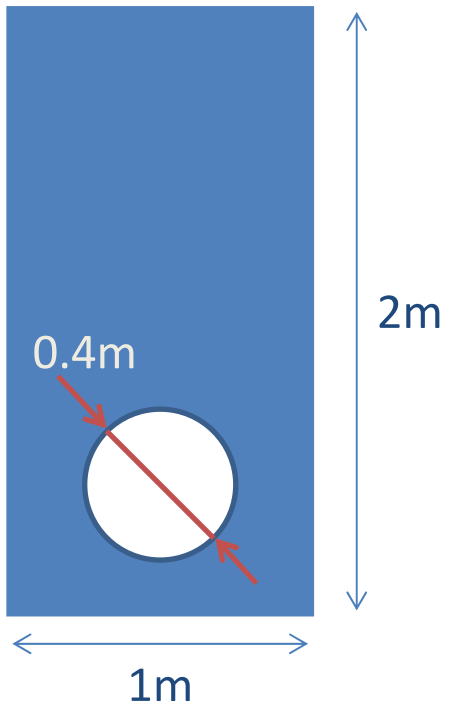




E. Hachem, M. Khalloufi, J. Bruchon, R. Valette, Y. Mesri, Unified adaptive Variational MultiScale method for two phase compressible-incompressible flows, Computer Methods in Applied Mechanics and Engineering, Vol. 308, pp. 238-255, 2016



2D and 3D validations



- Liquid
  Density: 10^4 kg/m^3
  Viscosity: 1 kg/(m.s)

- Vapor
  Density: 10^3kg/m^3
  Viscosity: 1 kg/(m.s)

Surface tension 0,5 kg/s^2
Gravity $g=-8x10^{-4}$m/s^2

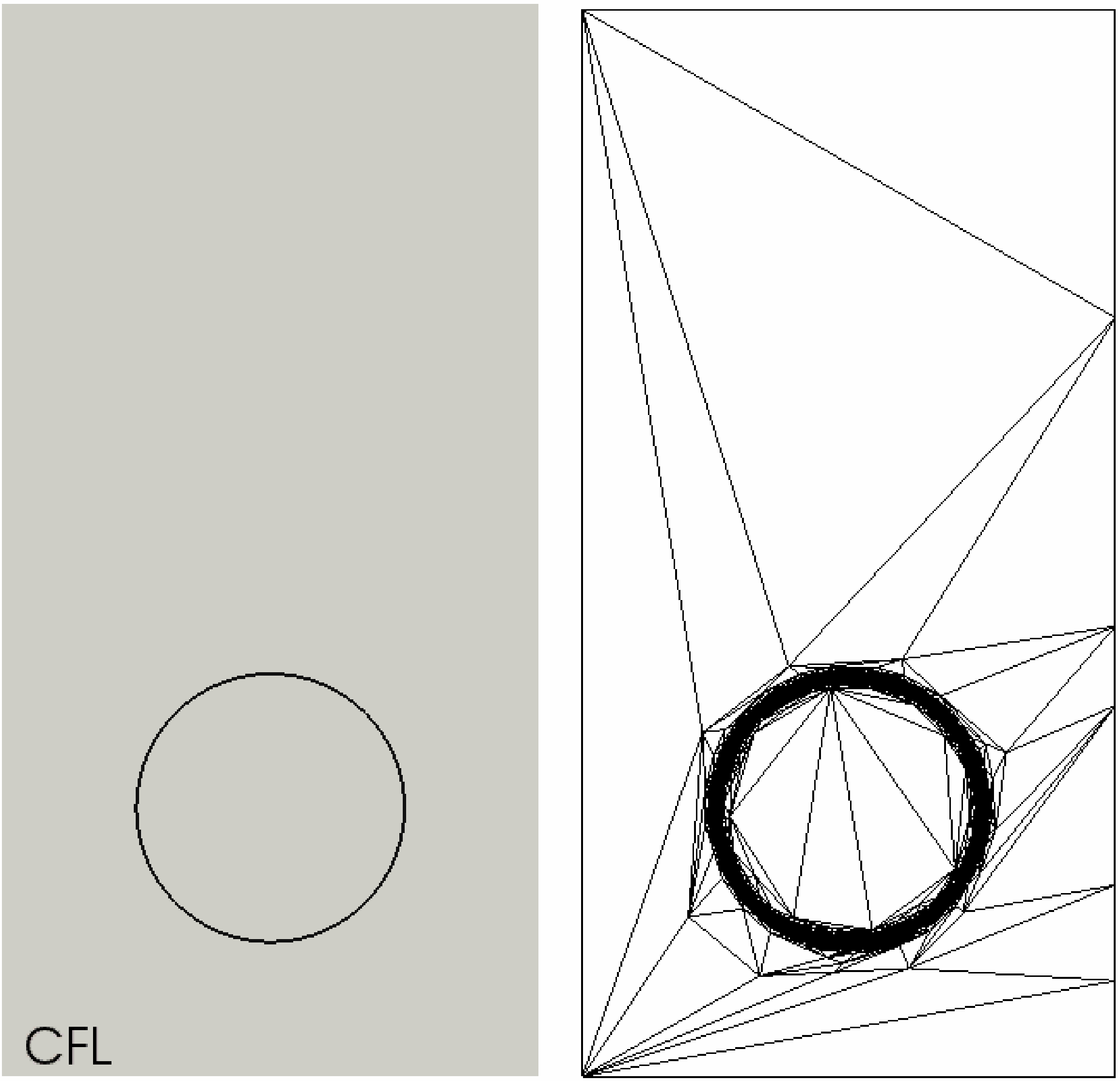
Mesh size h=1/80

Validation: Rising bubble with mesh adaptation with / without surface tension

(a) t=0.6s (b) t=1.2s (c) t=1.8s (d) t=2.2s

(e) t=2.4s (f) t=2.6s (g) t=2.8s (h) t=3.0s

Validation: Rising bubble with mesh adaptation



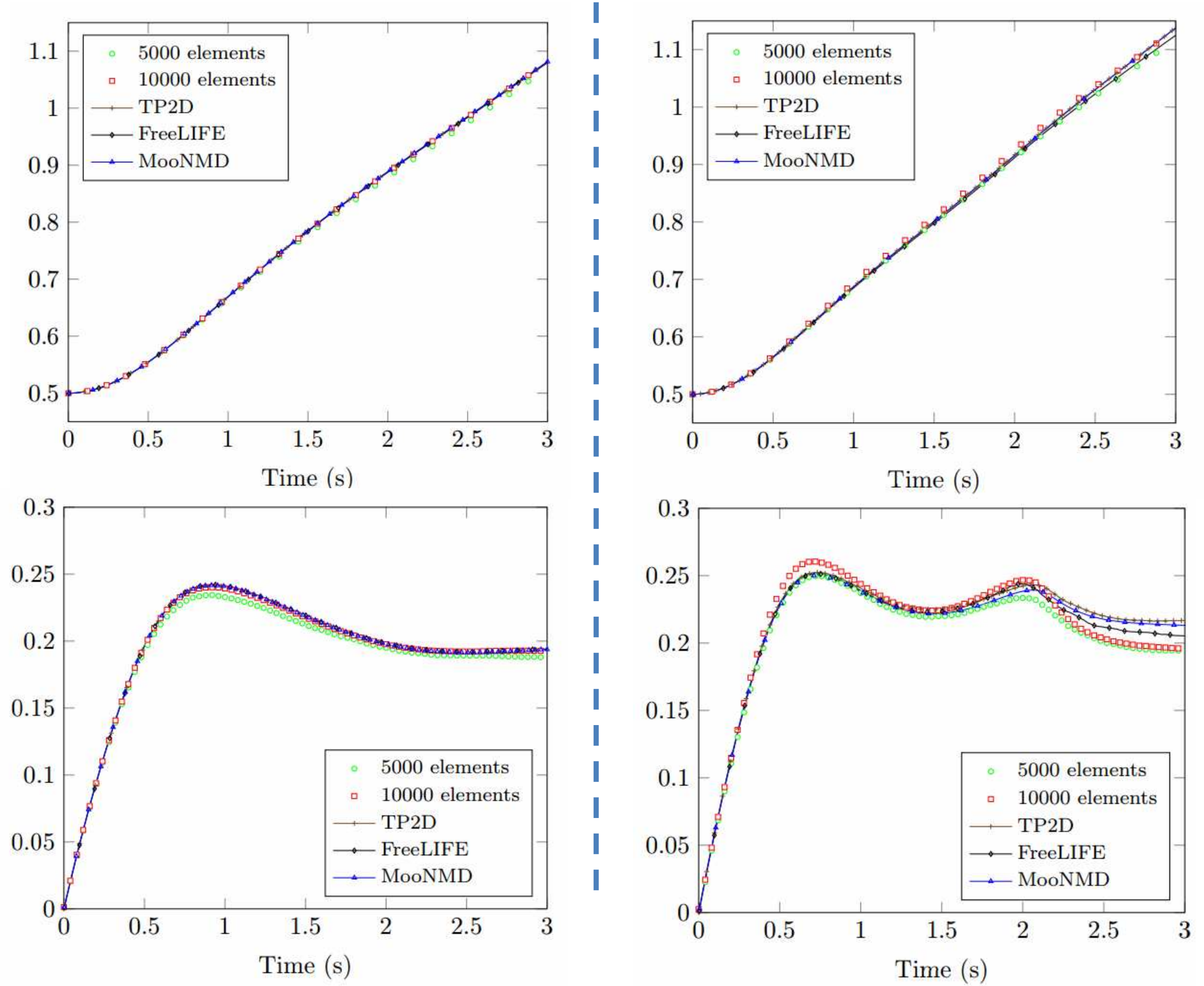
Validation: Rising bubble with mesh adaptation

Case1

Case2

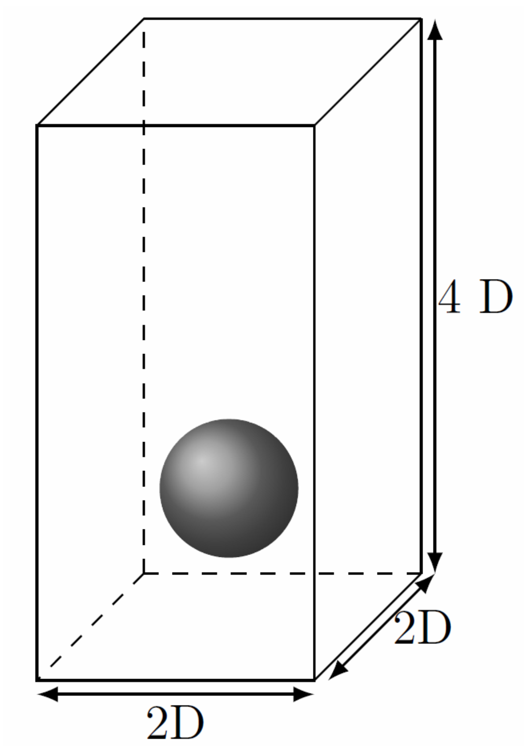
Position of the center of mass

Rising velocity



Validation: 3D Rising bubble with mesh adaptation

D=0,01m



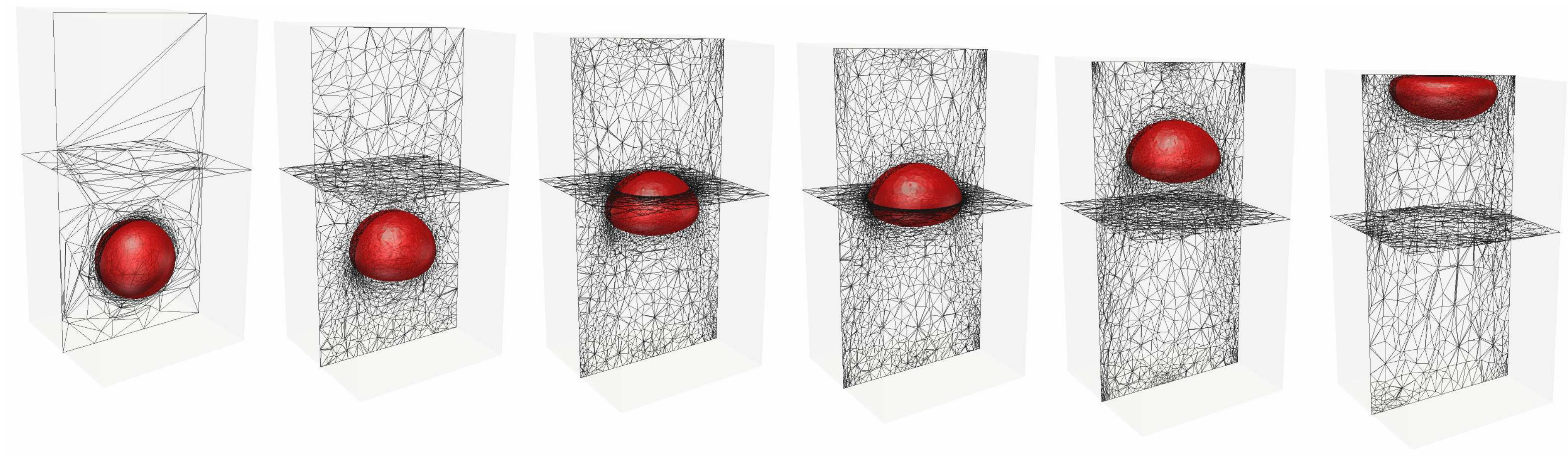
- Outer fluid
  Density 1000 kg/m^3
  Viscosity 0.35 kg/(m.s)

- Bubble
  Density 1.225kg/m^3
  Viscosity 0.00358 kg/(m.s)

Surface tension 0.11kg/s^2
Gravity g=9.81m/s^2

U. Rasthofer, F. Henke, W.A. Wall, V. Gravemeier, An extended residual-based variational multiscale method for two-phase flow including surface tension, Computer Methods in Applied Mechanics and Engineering, Volume 200, Issues 21–22, 1 May 2011, Pages 1866-1876

Validation: 3D Rising bubble with mesh adaptation



With surface tension

6- Heat transfer and Phase change

Energy conservation (neglecting viscosity and capillary forces)

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \cdot \nabla T \right) - \nabla \cdot (k \nabla T) = - \left(L + (c_p^v - c_p^l)(T - T_{\text{vap}}) \right) \dot{m} \delta |\nabla \phi| \frac{\rho^2}{\rho_v \rho_l}$$

Mass conservation

$$\nabla \cdot u = \dot{m} \left(\frac{1}{\rho_v} - \frac{1}{\rho_l} \right) |\nabla \alpha| \delta$$

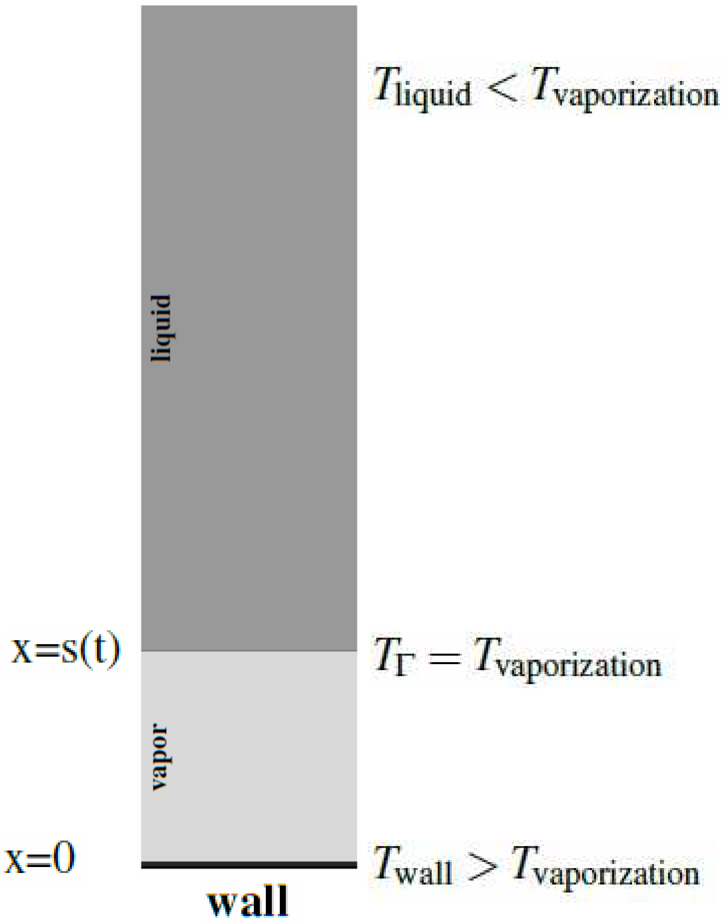
Interface evolution

$$\frac{\partial \phi}{\partial t} + \left[u - \frac{\rho}{\rho_l \rho_v} \dot{m} \frac{\nabla \phi}{|\nabla \phi|} \right] \cdot \nabla \phi = 0$$

Recall the density distribution

$$\rho = (\rho_v - \rho_l) H + \rho_l$$

6- Heat transfer and Phase change: Stefan problem



$$\boxed{\rho c_p \frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial x^2} = 0}$$

$$T(x,t) = T_{\text{wall}} + \frac{T_{\text{vaporization}} - T_{\text{wall}}}{\text{erf}\left(\frac{s}{2\sqrt{\alpha_v t}}\right)} \text{erf}\left(\frac{x}{2\sqrt{\alpha_v t}}\right)$$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt$$

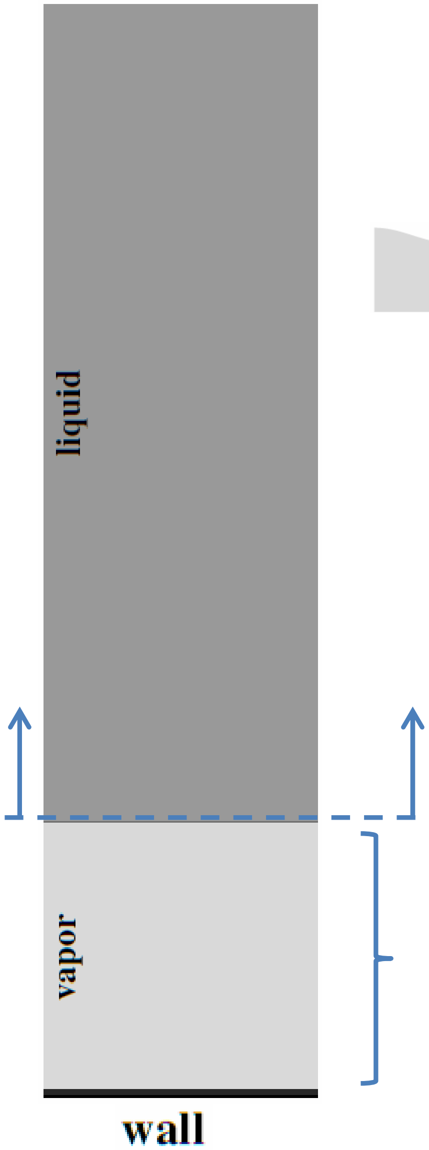
Stefan condition with constant density

$$\rho L \frac{ds}{dt} = (-k_v \nabla T_v + k_l \nabla T_l) \cdot \vec{e}_x$$

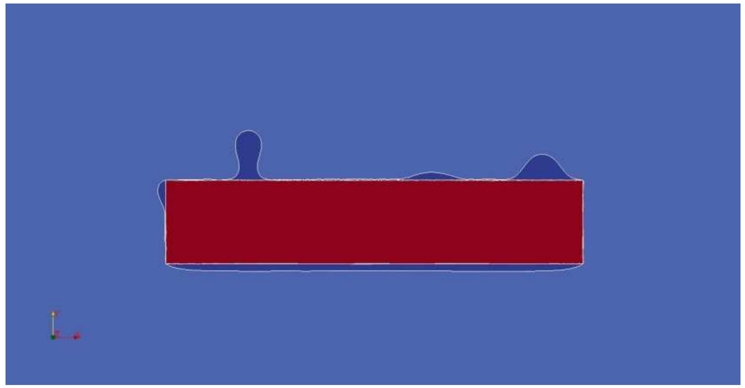
Stefan condition with variable density

$$-\rho_v L_{\text{vap}} \frac{\mathbf{ds}}{\mathbf{dt}} - \frac{1}{2}\rho_v \left(1 - \frac{\rho_v}{\rho_l}\right)^2 \left(\frac{\mathbf{ds}}{\mathbf{dt}}\right)^{\mathbf{3}} = (-k_l \nabla T_l + k_v \nabla T_v) \cdot \vec{e}_x$$

6- Heat transfer and Phase change: zoom



Rayleigh-Taylor instability



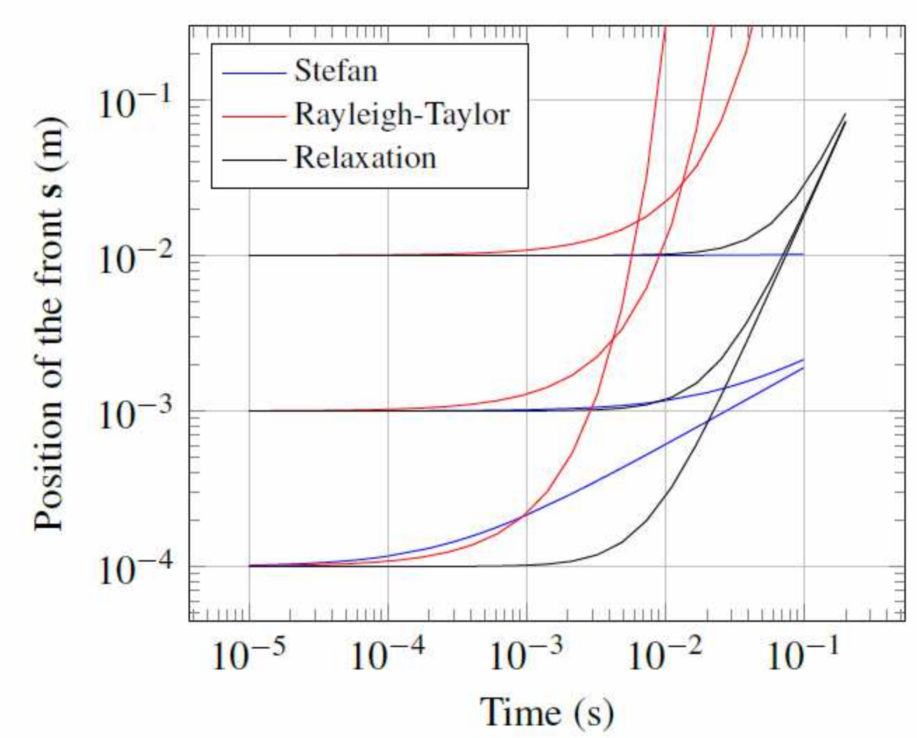
Relaxation

Phase change ⇒ Stefan problem

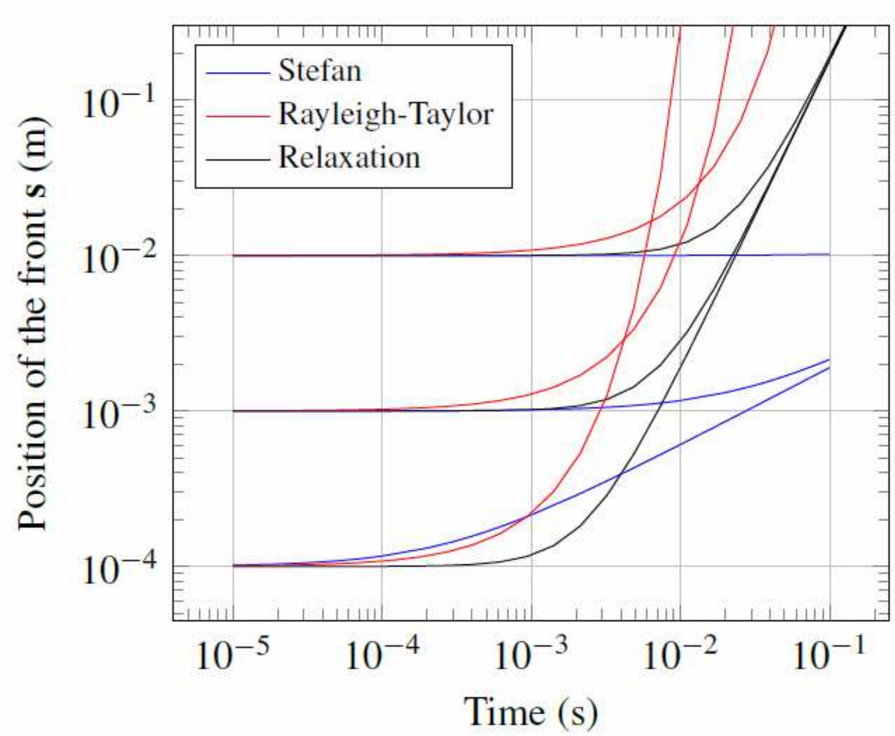
	ρ [kg/m^3]	μ [kg/(s·m)]	c_p [kJ/(kg·K)]	k [W/(m·K)]	L_{vap} [kJ/kg]
Vapor (100°C)	0.6	1.2×10^{-5}	2.027	0.0248	
Water (100°C)	960	2.83×10^{-4}	4.215	0.682	2264.76

6- Heat transfer and Phase change:

For a given initial thickness of the film



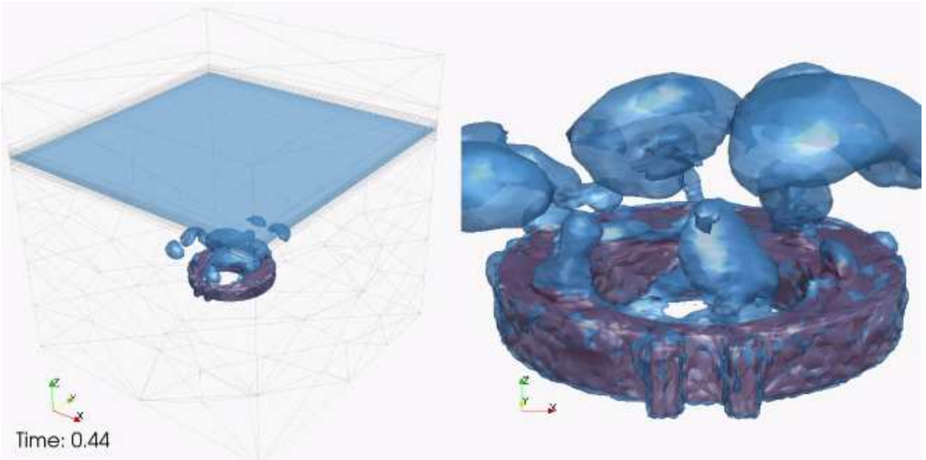
Water level : 1m above the vapor film P=1.1bar

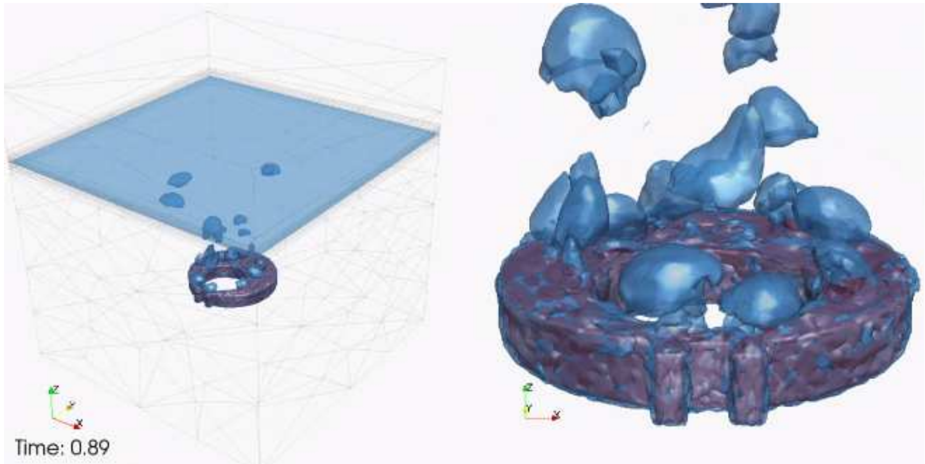


Water level : 0.1m above the vapor film P=1bar

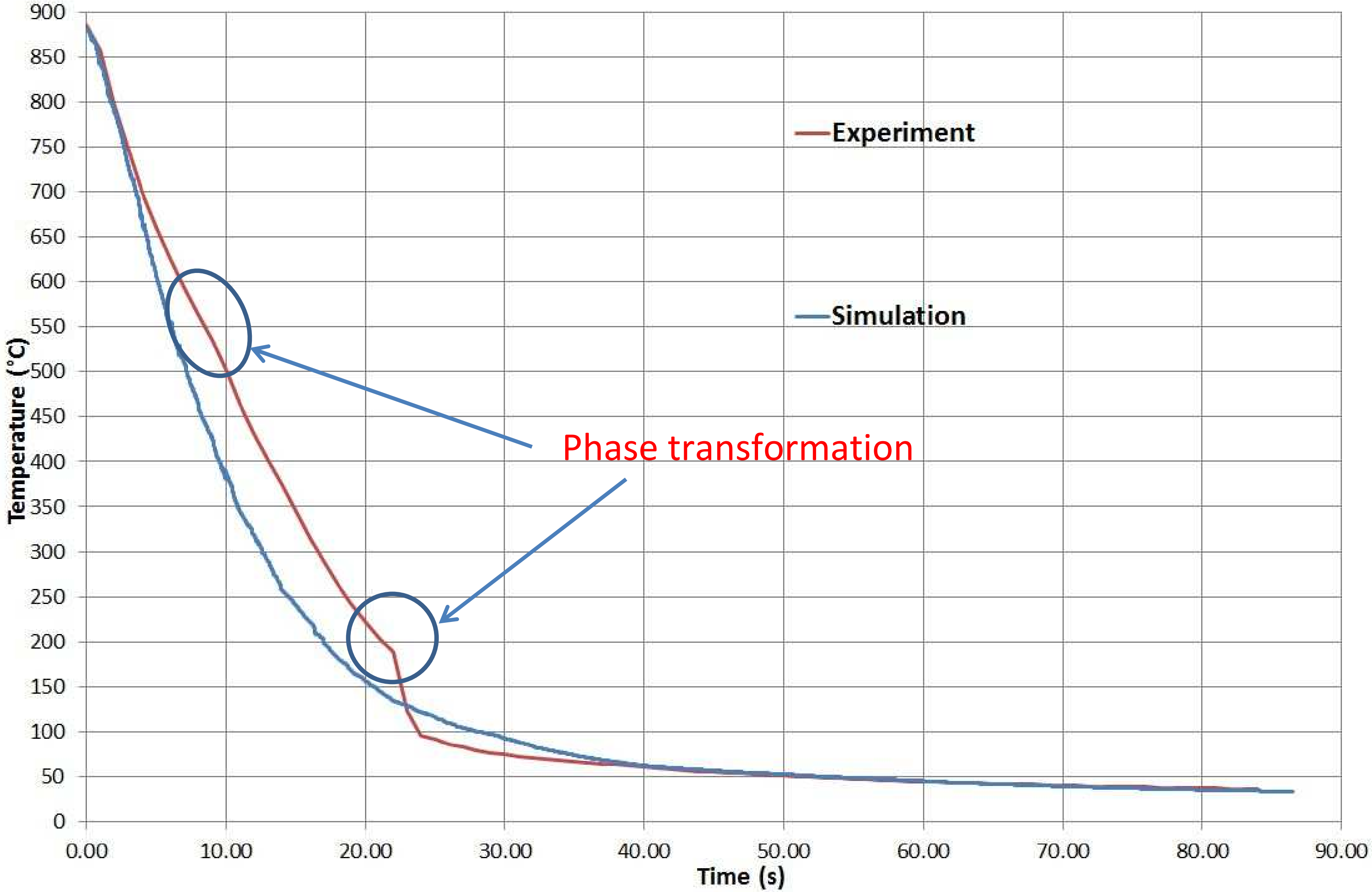
6- liquid – gaz – solid flow (2D)

6- Industrial Parts





6- Industrial Parts



Evolution of the temperature at the center of the sample

Conclusion & Perspectives

- An Eulerian framework for multiphase flows
- Combined with a high fidelity anisotropic mesh adaptation
- Variational Multiscale FE Method for turbulent multiphase flows
- Implementation of an implicit surface tension and a conservative LS
- Towards turbulent boiling and phase change

- Conservative interpolation after meshing (phd in progress)
- Nonliear subscales and higher order discretisation (in progress)

- Phase change with compressible features
- Phase transformation
- Wetting and contact angle