Precision Gravity

Gravitational waves using Feynman diagrams

Raj Patil

MPI for Gravitational Physics (AEI) and Humboldt University

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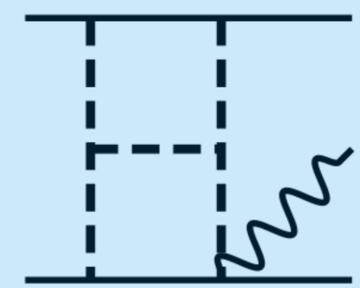
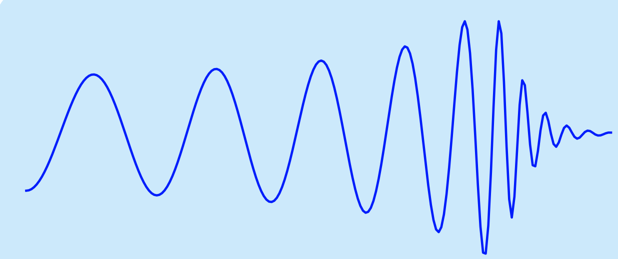
Amplitudes et Gravitation sur l'Yvette,
IHES/IPhT joint seminar

In collaboration with Manoj Mandal, Pierpaolo Mastrolia, Hector Silva, and Jan Steinhoff

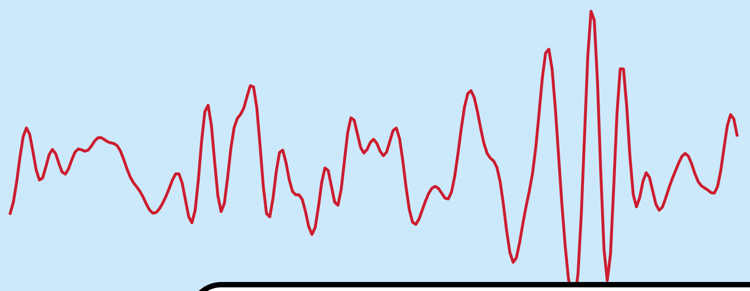


Muti-loop Feynman diagrams



High precision waveform

Why precision gravity?



Stra

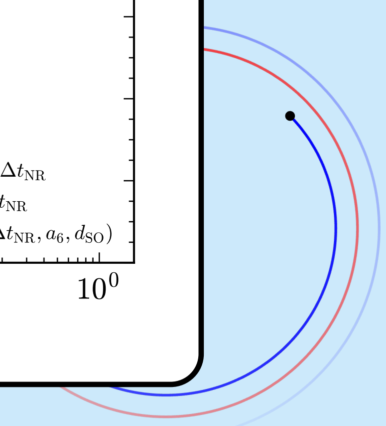
Precise and ac
parameter esti

Primary black hole	
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

GW150914

$$\dot{r} = \frac{dH}{dp_r} \qquad \dot{p}_r = -\frac{dH}{dr} + F_r$$

$$\dot{\ } \quad \frac{dH}{\ } \qquad = -\frac{dH}{d\phi} + F_\phi$$



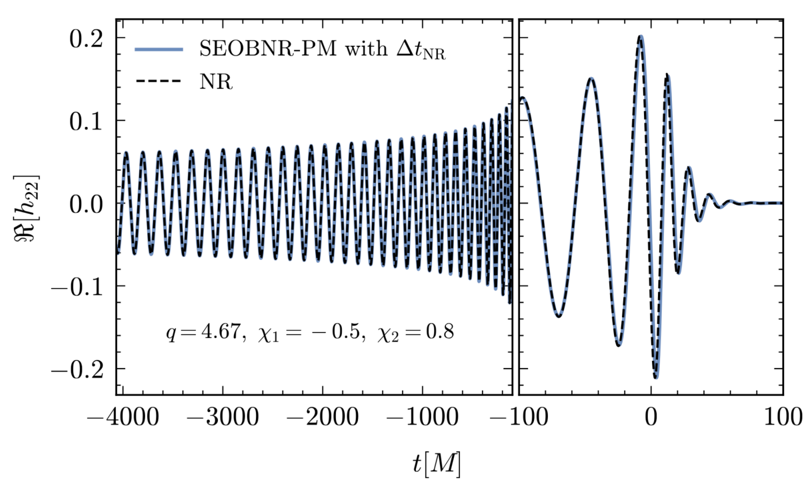
A pical waveform model
nee ls precise Hamiltonians
and Fluxes

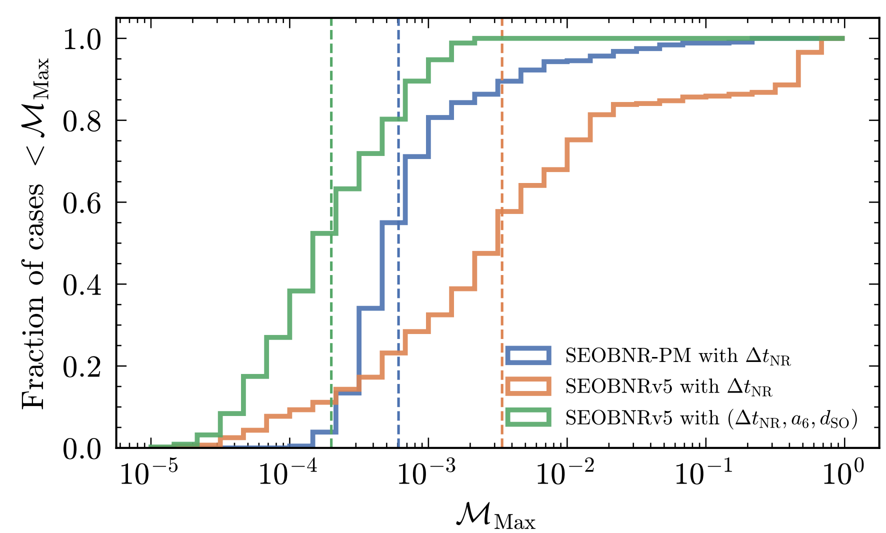
[SEOBNR, TEOBResumS, IMRPhenom, ...]

SEOBNR-PM

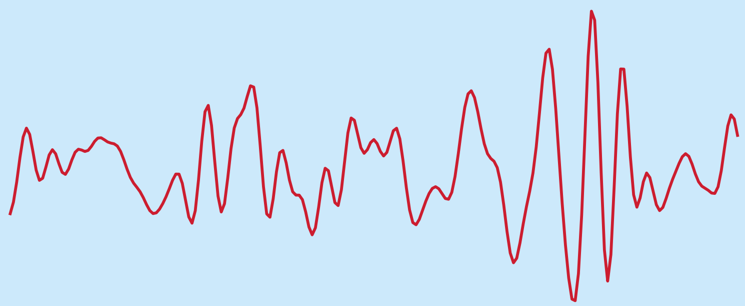
[Buonanno, Mogull, **RP**, Pompili (2024)]

First bound orbit waveform model based on post-Minkowskian data





Why precision gravity?

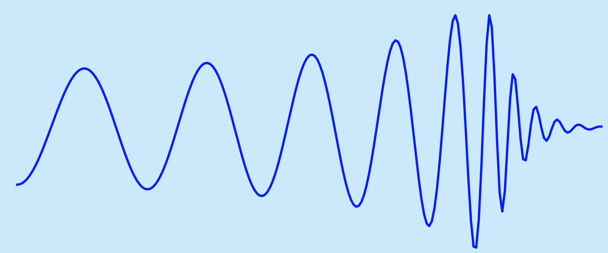


Strain in the detectors

Precise and accurate parameter estimation

Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

GW150914



Waveform templates

Hamiltonians

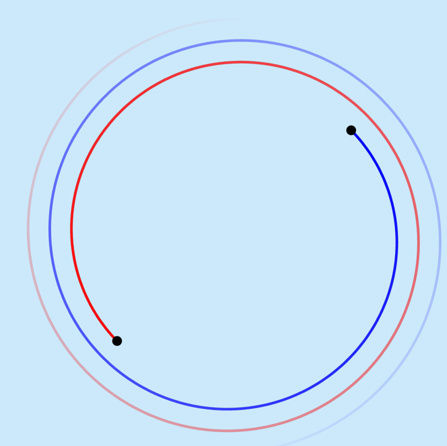
conservative dynamics

Fluxes

dissipative dynamics

$$\dot{r} = \frac{dH}{dp_r} \qquad \dot{p}_r = -\frac{dH}{dr} + F_r$$

$$\dot{\phi} = \frac{dH}{dp_\phi} \qquad \dot{p}_\phi = -\frac{dH}{d\phi} + F_\phi$$



A typical waveform model needs precise Hamiltonians and Fluxes

[SEOBNR, TEOBResumS, IMRPhenom, ...]

Bound state - Inspiral phase

Inspiral :

the components of the binary are moving at **non-relativistic velocities** and their orbital separation is slowly decaying

Merger :

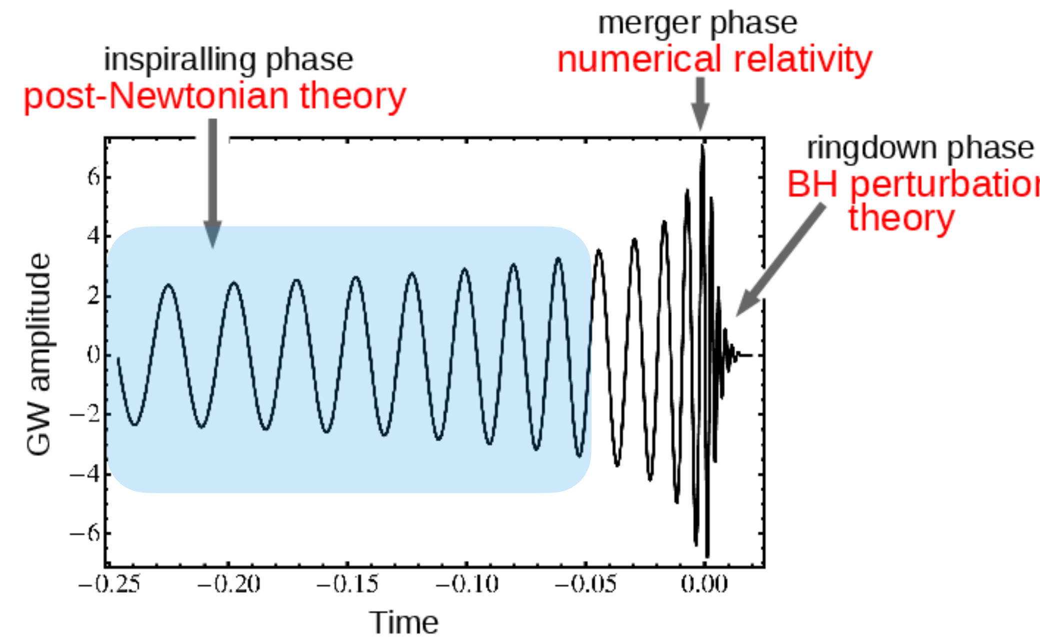
the separation between the components falls roughly below the innermost stable orbit of each other, and the objects reach **relativistic velocities** and **merge into final object**

Ringdown :

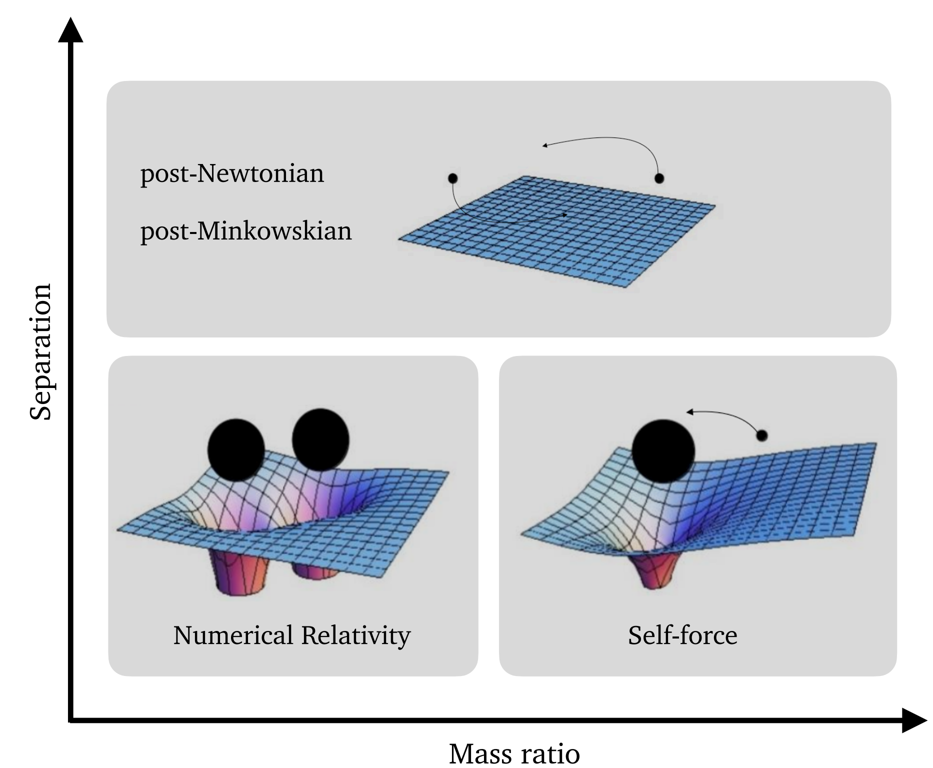
where spacetime **settles down to that of a Kerr** black hole

		1687	1938	1980	2000	2014			
		0PN	1PN	2PN	3PN	4PN	5PN	6PN	
1PM	G_N	1 +	v^2 +	v^4 +	v^6 +	v^8 +	v^{10} +	v^{12} +	⋯
2PM	G_N^2		1 +	v^2 +	v^4 +	v^6 +	v^8 +	v^{10} +	⋯
3PM	G_N^3			1 +	v^2 +	v^4 +	v^6 +	v^8 +	⋯
4PM	G_N^4				1 +	v^2 +	v^4 +	v^6 +	⋯
5PM	G_N^5					1 +	v^2 +	v^4 +	⋯

[Newton, Einstein, Infeld, Hoffman, Ohta, Okamura, Kimura, Hiida, Jaranowski, Schäfer, Damour, Buonanno, Blanchet, Faye, Iyer, Will, Wiseman, Poisson, Flanagan, Deruelle, Thorne, Sathyaprakash, Bini, Geralico, . . . Goldberger, Rothstein, Porto, Ross, Foffa, Sturani, Levi, Steinhoff, Mastrolia, Blumlein, Marquard, . . .]



[Blanchet (2019)]



[adapted from Barack and Pound (2018)]

Bound state - Inspiral phase

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Merger :

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the innerm

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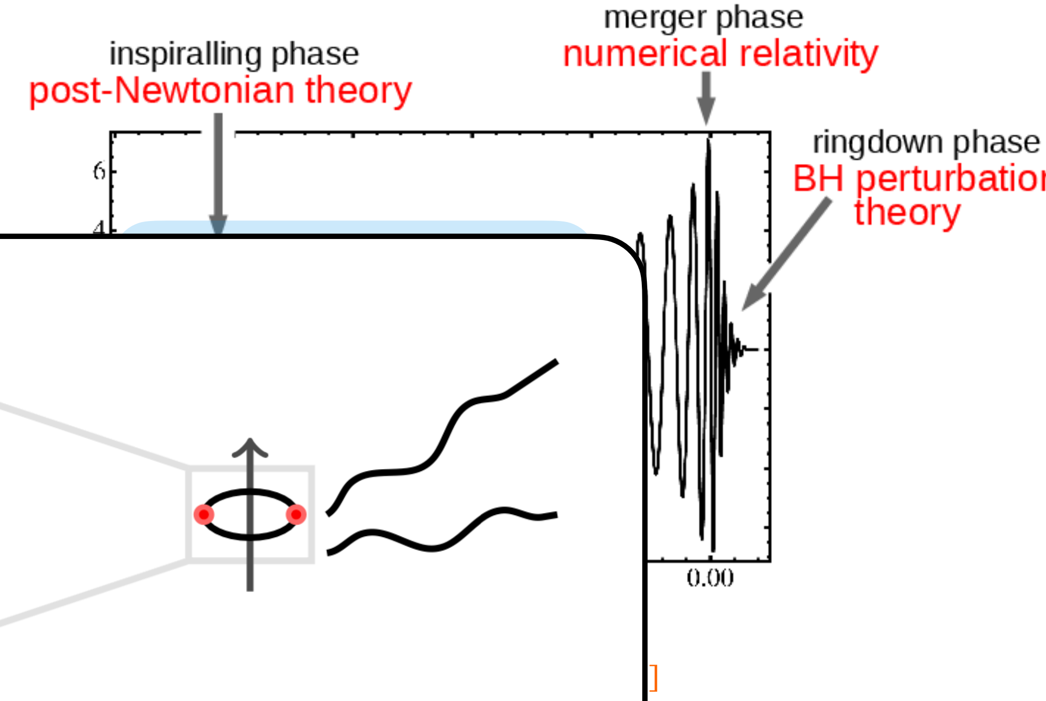
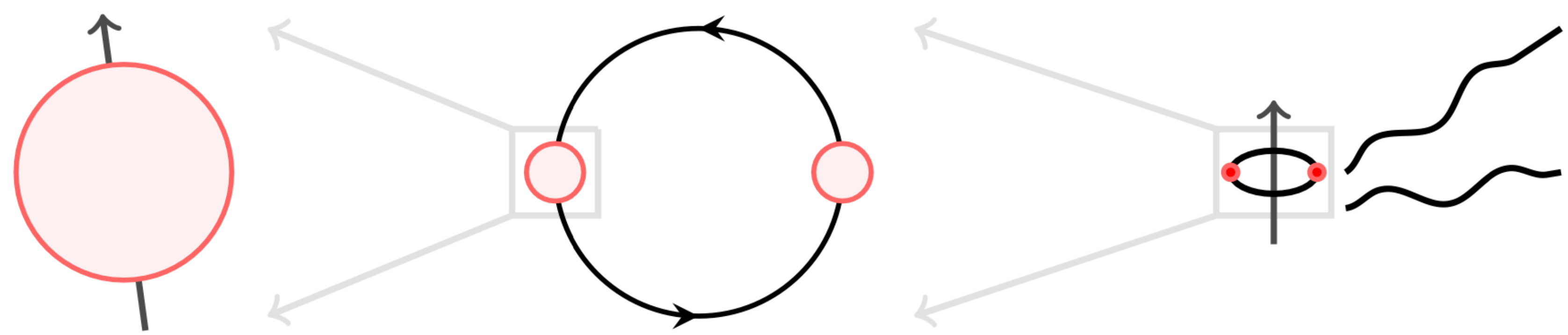
Ringdown :

where spa





EFTs provide a simple framework
to capture the essential physics at different scales

[Goldberger and Rothstein (2006)]

1687

		0PN										
1PM	G_N	1										
2PM	G_N^2											
3PM	G_N^3	1	+	v^2	+	v^4	+	v^6	+	v^8	+	$\cdots$
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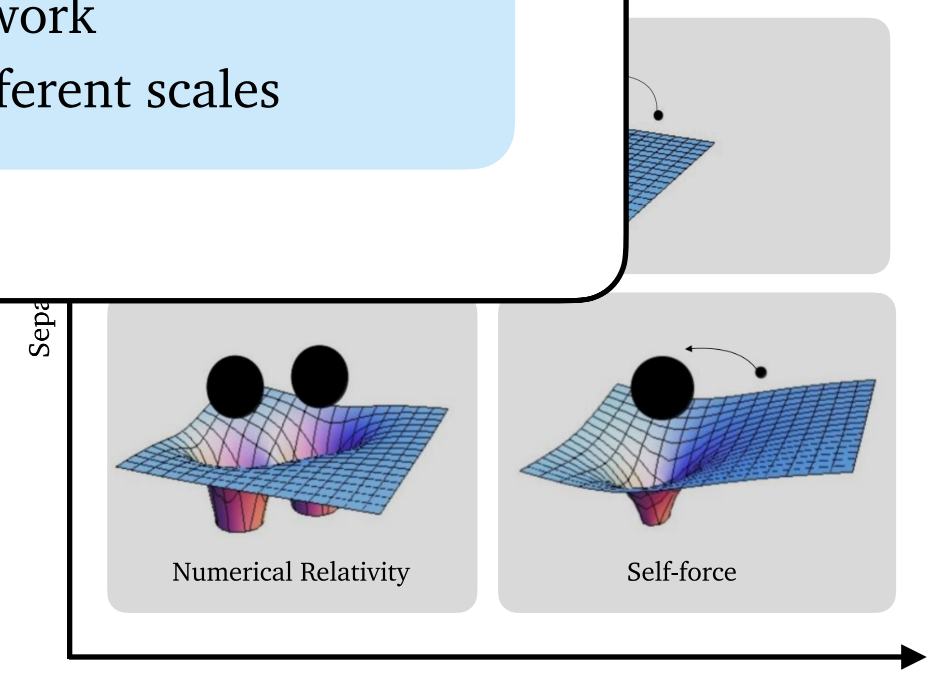


[adapted from Barack and Pound (2018)]

This talk

What are EFTs? Why do we need EFTs?

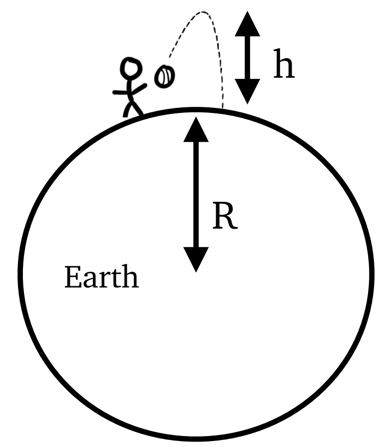
How are EFTs used for two-body problems?

What did we do with it?

Effective Field Theories

"To any given order in perturbation theory, and for a given set of asymptotic states, the ***most general Lagrangian*** *containing all terms allowed by the assumed symmetries* ***will yield the most general S-matrix elements*** *consistent with analyticity, perturbative unitarity, cluster decomposition and assumed symmetries."*

[Weinberg (1979)]



Top down approach

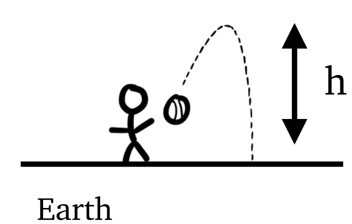
$$L_N = \frac{1}{2}mv^2 + \frac{G_N m M}{r}$$
$$= \frac{1}{2}mv^2 + \frac{G_N m M}{R+h}$$

$$\downarrow$$

$$= \frac{1}{2}mv^2 - m\left(\frac{G_N M}{R^2}\right)h + m\left(\frac{G_N M}{R^3}\right)h^2 + \cdots$$

Flat earth limit

$h \ll R$



Bottom up approach

?

$$\downarrow$$

Wilson coefficients

$$L_{Flat} = \frac{1}{2}mv^2 - mg_1 h - mg_2 h^2 + \cdots$$

RG flow of the coefficients: $g_1 = \left(\frac{G_N M}{R^2}\right) \implies R\frac{dg_1}{dR} = -2g_1$

Effective Field Theories

*"To any given order in perturbation theory, and for a given set of asymptotic states, the **most general Lagrangian** containing all terms allowed by the assumed symmetries **will yield the most general S-matrix elements** consistent with analyticity, perturbative unitarity, cluster decomposition and assumed symmetries."*

[Weinberg (1979)]

To specify a EFT at a certain scale you need:

Degrees of freedom **Symmetries**

Power counting - to evaluate the importance of the operators

Expansion parameter - to set up the perturbation theory

Top down appro

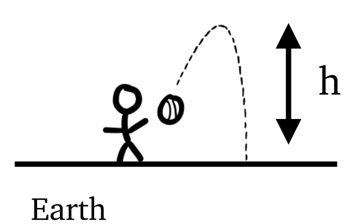
$$L_N = \frac{1}{2}mv^2 + \frac{G_N m}{r}$$

$$= \frac{1}{2}mv^2 + \frac{G_N m}{R +}$$

$$= \frac{1}{2}mv^2 - m\left(\frac{G_N M}{R^2}\right)h + m\left(\frac{G_N M}{R^3}\right)h^2 + \cdots$$

pproach

limit



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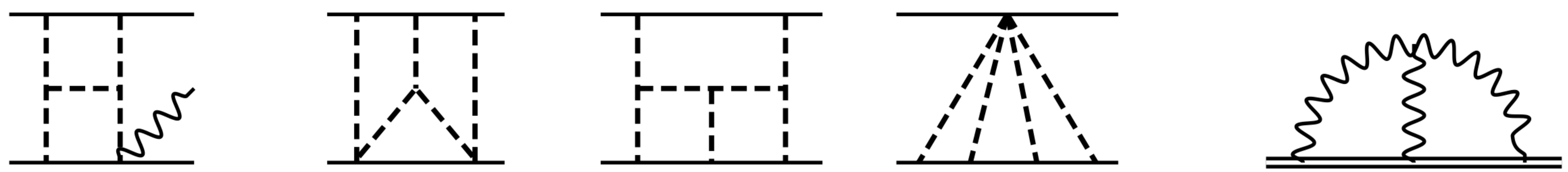
RG flow of the coefficients: $g_1 = \left(\frac{G_N M}{R^2}\right) \implies R\frac{dg_1}{dR} = -2g_1$

Why EFT/QFT for classical problem?

Advantages of QFT/EFT approach for classical problem:

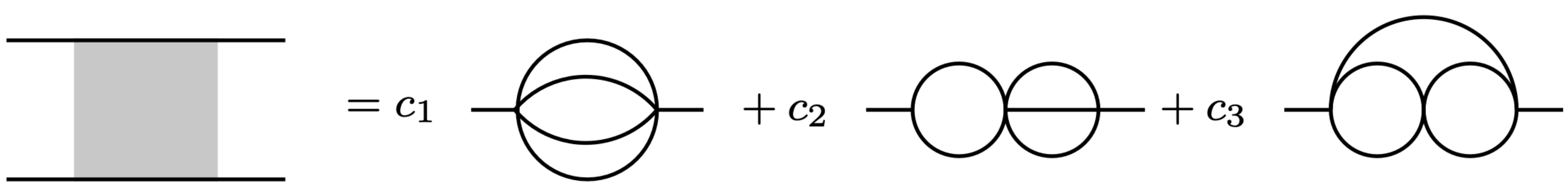
- ➡ **Feynman diagram** help organise the large computations



- ➡ **Dimensional regularisation** to handle the spurious divergences

- ➡ **Multi-loop techniques** for solving integrals (IBPs, differential and difference equations for MIs, etc.)

[Kol, Shir, Foffa, Sturani, Mastrolia, Strum, . . .]



- ➡ Also: double-copy, supersymmetry, massive-higher spins, spinor helicity, etc.

[Bern, Parra-Martinez, Roiban, Ruf, Solon, Shen, Zeng, Dlapa, Kälin, Porto, Jakobsen, Mogull, Plefka, Steinhoff, Damgaard, Bjerrum-Bohr, Aoude, Haddad, Helset, Guevara, Ochirov, Vines, Arkani-Hamed, Huang, Kosower, O'Connel, . . .]

This talk

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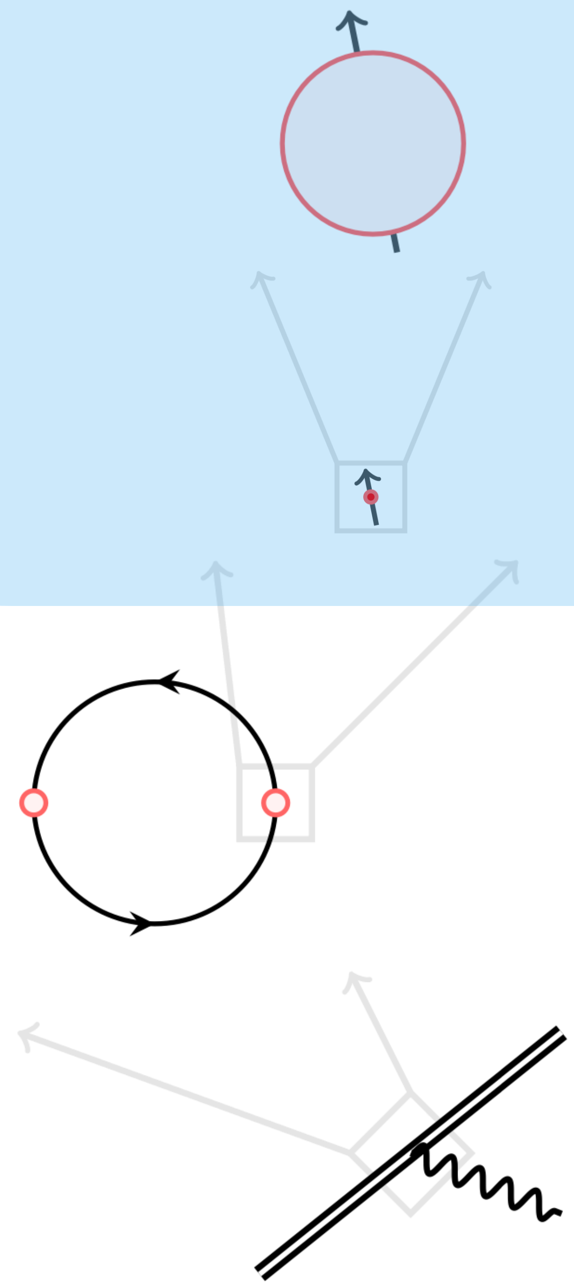
Questions?

Effective Field Theories for PN



	Black holes	Neutron stars
R_S	Black holes	Neutron stars
$\gg R_S$	$\dfrac{\text{Black holes perturbation theory}}{\text{Point particle + internal structure}}$	$\dfrac{\text{Numerical simulations}}{\text{Point particle + internal structure}}$

r — Binary
+ Near zone + Far zone

$\gg r$ — $\dfrac{\text{Binary + Far zone}}{\text{Multipoles + Far zone}}$

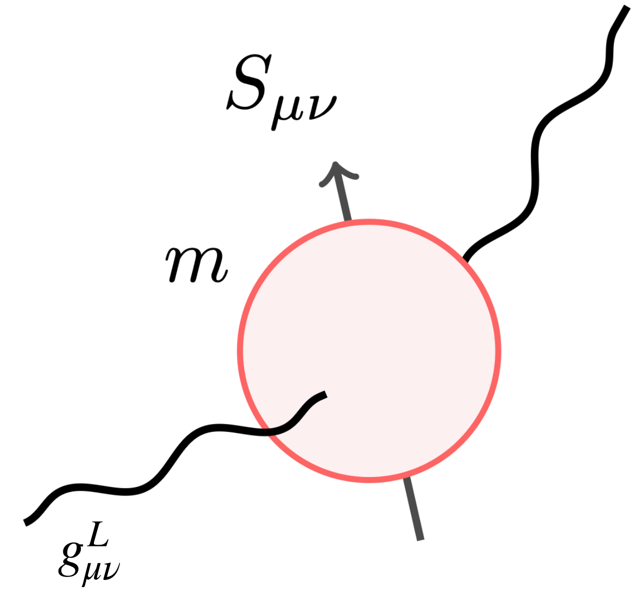
Gravitational waves

[Goldberger and Rothstein (2006)]

Compact object

We decompose the metric as:

- ➡ $g^S_{\mu\nu}$ - Short scale metric (encodes interactions $\ll R_S$)
- ➡ $g^L_{\mu\nu}$ - Long scale metric (encodes interactions $\gg R_S$)



To construct a EFT at length scales $\gg R_S$ (integrate out $g^S_{\mu\nu}$)

$$e^{iS_{EH}[g_L^{\mu\nu}]+iS_{eff}[J^{(a)},g_L^{\mu\nu}]} = \int D[g_S^{\mu\nu}]\ D[\delta x]\ e^{iS_{EH}[g^{\mu\nu}]+iS_{int}[\delta x,g^{\mu\nu}]}$$

Degrees of freedom:

- ➡ A point particle worldline - centre of mass of the compact object
- ➡ Multipole moments $J^{(a)}$ - internal structure of the compact object
- ➡ Long scale metric $g^L_{\mu\nu}$

Inherent multipole moments :

m- mass

$S_{\mu\nu}$ - spin

$Q^I_{\mu\nu}$ - quadrupole

Response multipole moments :

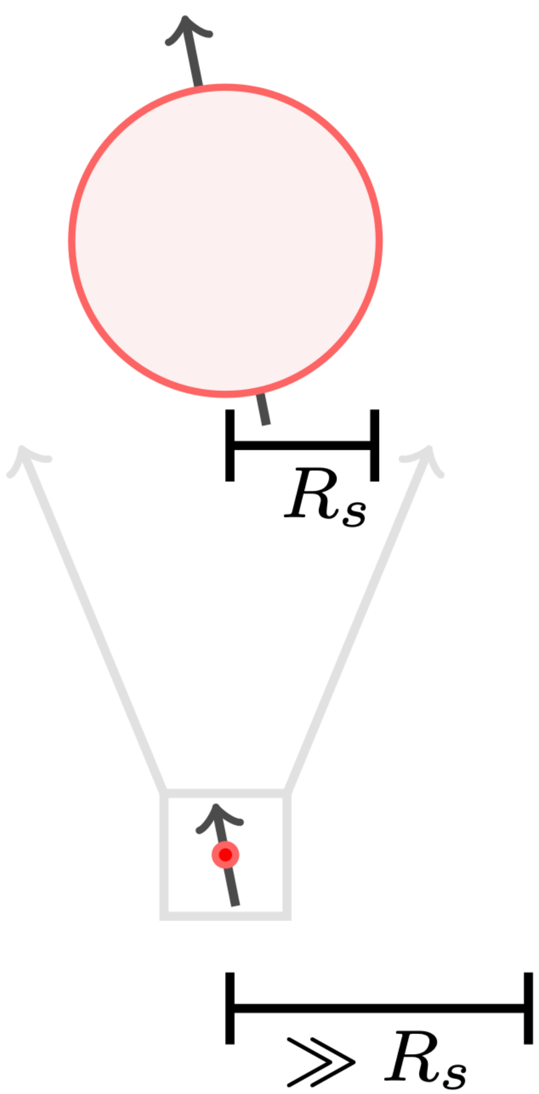
$Q^R_{\mu\nu}$ - quadrupole

Symmetries:

- ➡ Diffeomorphism invariance for the $g^L_{\mu\nu}$
- ➡ Re-parameterization invariance for the worldline

Point particle EFT

A compact object could be approximated by a point particle with internal structure (Wilson coefficients) at large scales as compared to R_s



Mass: $\int dt\; m\sqrt{g^{L}_{\mu\nu}u^{\mu}u^{\nu}}$

Spin induced quadrupole

Spin: $\int dt \left\{ -\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{2mc}\left(C^{(0)}_{\mathrm{ES}^2}\right)\frac{E_{\mu\nu}}{u}\left[S^{\mu}S^{\nu}\right]_{\mathrm{STF}} + \cdots \right\}$

Point particle EFT

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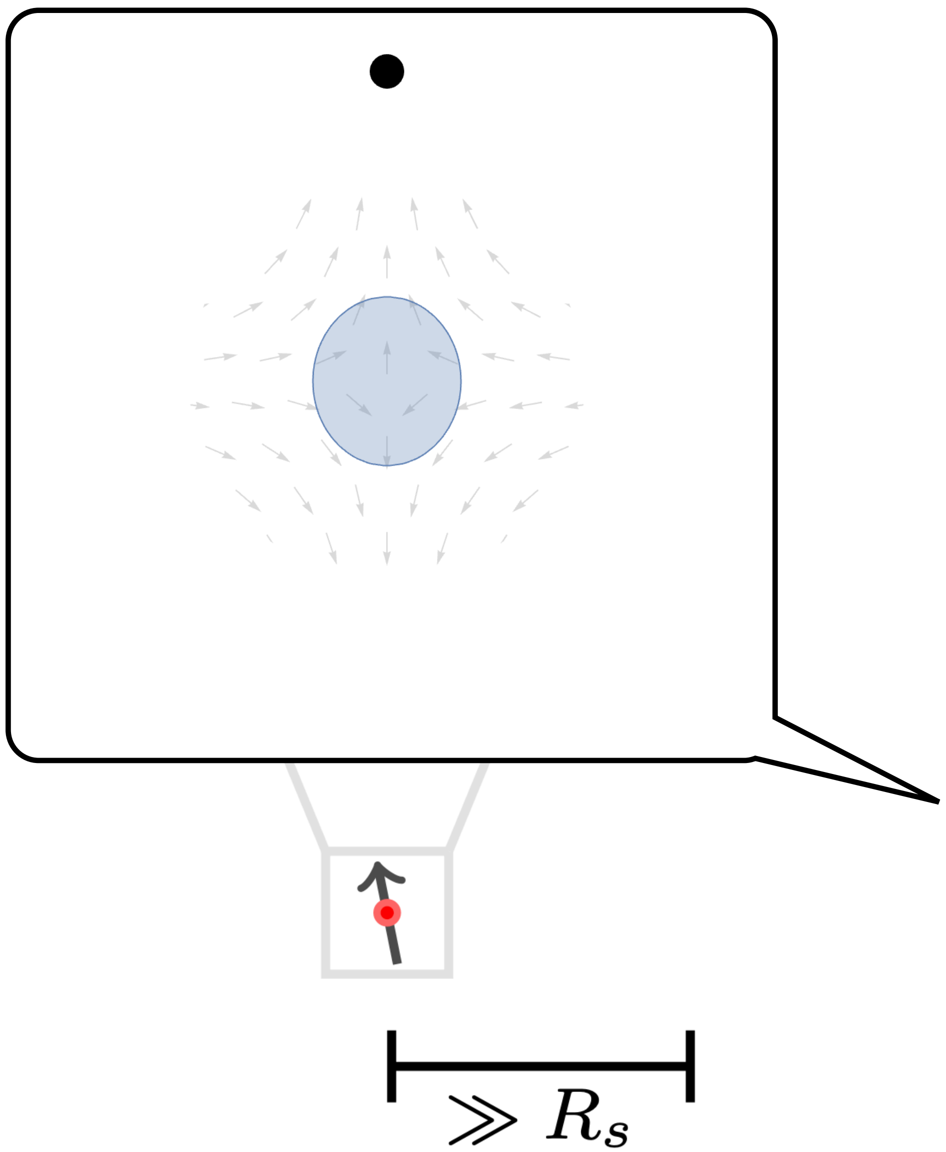

Mass: $\int dt\ m\sqrt{g^L_{\mu\nu}u^\mu u^\nu}$

Spin: $\int dt \left\{ -\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{2mc}\left(C^{(0)}_{\mathrm{ES}^2}\right)\frac{E_{\mu\nu}}{u}\left[S^\mu S^\nu\right]_{\mathrm{STF}} + \cdots \right\}$

Adiabatic Tides: $\int dt \left\{ \lambda\ \frac{z}{4}E_{\mu\nu}E^{\mu\nu} + \lambda\kappa\frac{G_N^2 m^2}{c^6}\frac{z}{2}\frac{dE_{\mu\nu}}{dt}\frac{dE^{\mu\nu}}{dt} + \cdots \right\}$

Post-adiabatic Love number

Static Love number

Point particle EFT

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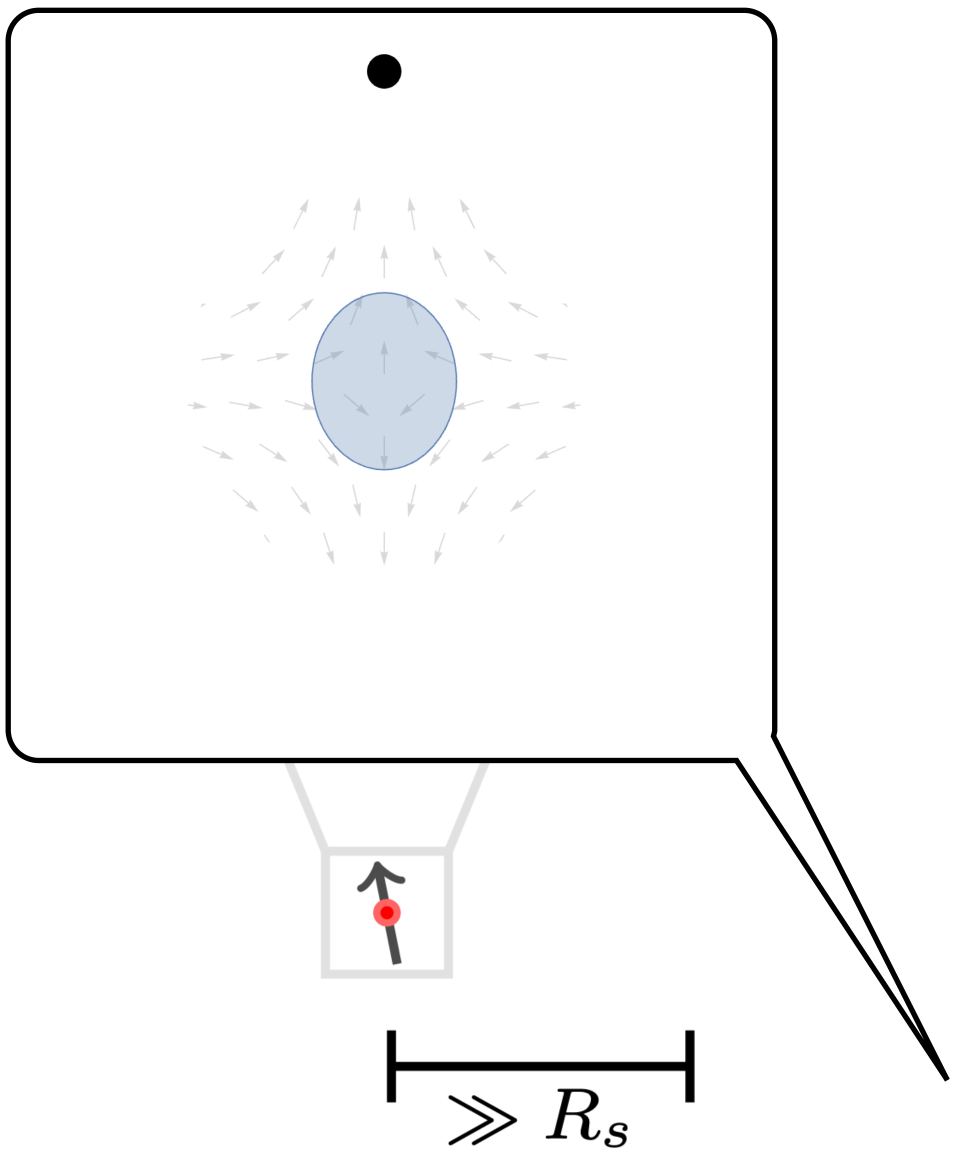

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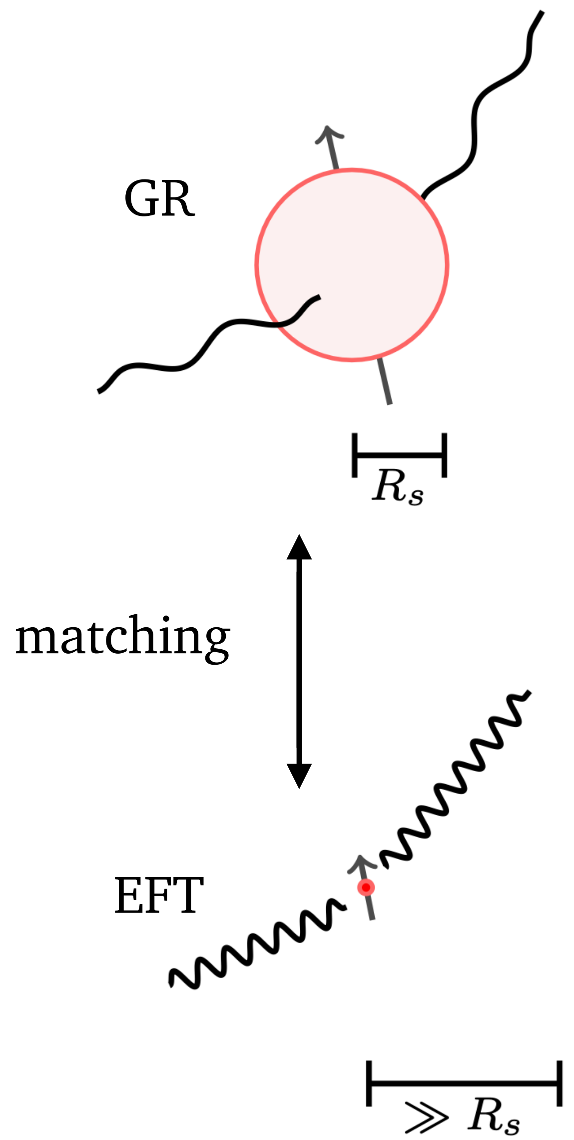
Dynamic Tides: $$\int dt \left\{ \frac{z}{4\lambda\omega_f^2}\left[\frac{c^2}{z^2}\dot{Q}_{\mu\nu}\dot{Q}^{\mu\nu} - \omega_f^2 Q_{\mu\nu}Q^{\mu\nu}\right] - \frac{z}{2}Q^{\mu\nu}E_{\mu\nu} + \cdots \right\}$$

Fundamental frequency

Point particle EFT - matching

A compact object could be approximated by a point particle with internal structure (Wilson coefficients) at large scales as compared to R_s



$$C^{(0)}_{\mathrm{ES}^2} = 1$$
$$\lambda = 0$$

BH

$$C^{(0)}_{\mathrm{ES}^2} \approx 2-8$$
$$\lambda \approx 500$$

NS

Mass: $$\int dt\ m\sqrt{g^{L}_{\mu\nu}u^{\mu}u^{\nu}}$$

Spin induced quadrupole

Spin: $$\int dt\ \left\{ -\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{2mc}\left(C^{(0)}_{\mathrm{ES}^2}\right)\frac{E_{\mu\nu}}{u}\left[S^{\mu}S^{\nu}\right]_{\mathrm{STF}} + \cdots \right\}$$

Post-adiabatic Love number

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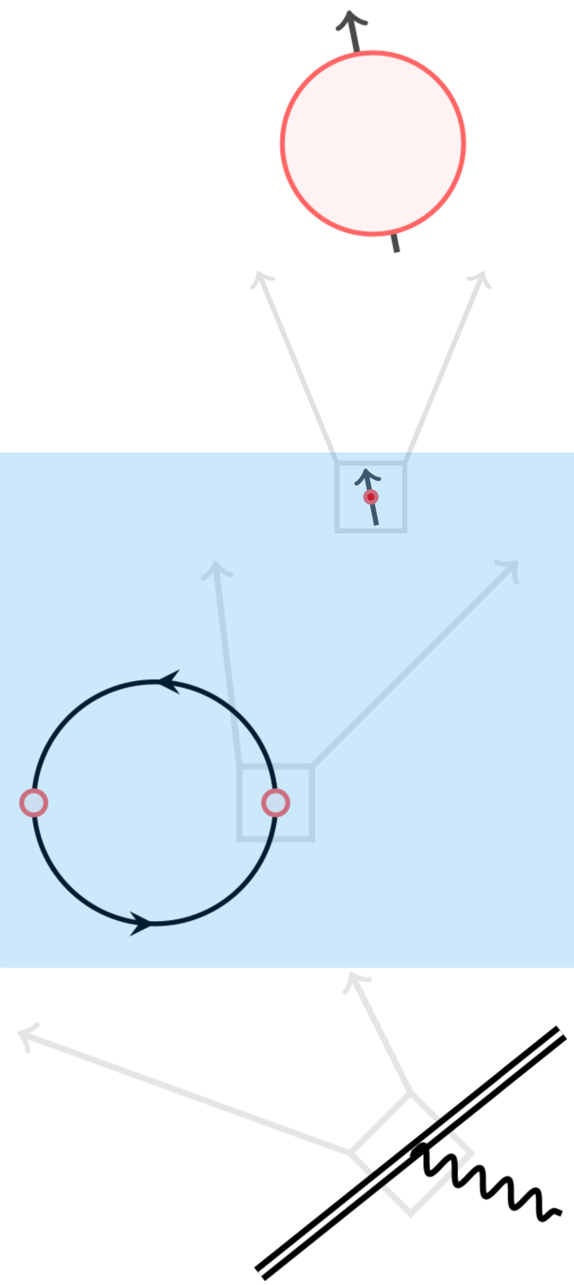
Static Love number

Dynamic Tides: $$\int dt\ \left\{ \frac{z}{4\lambda\omega_f^2}\left[\frac{c^2}{z^2}\dot{Q}_{\mu\nu}\dot{Q}^{\mu\nu} - \omega_f^2 Q_{\mu\nu}Q^{\mu\nu}\right] - \frac{z}{2}Q^{\mu\nu}E_{\mu\nu} + \cdots \right\}$$

Fundamental mode frequency

Effective Field Theories for PN



R_S — Black holes → Black holes perturbation theory; Neutron stars → Numerical simulations

$\gg R_S$ — Point particle + internal structure | Point particle + internal structure

r — Binary + Near zone + Far zone

$\gg r$ — Binary + Far zone → Multipoles + Far zone

Gravitational waves

Post-Newtonian setup

Binary is approximated by two point particles (weak field) $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Particle move at non-relativistic velocities (slow velocity) $\mathrm{v} \ll c$

$$\mathrm{nPN} \rightarrow \left(\frac{\mathrm{v}^2}{c^2}\right)^n$$

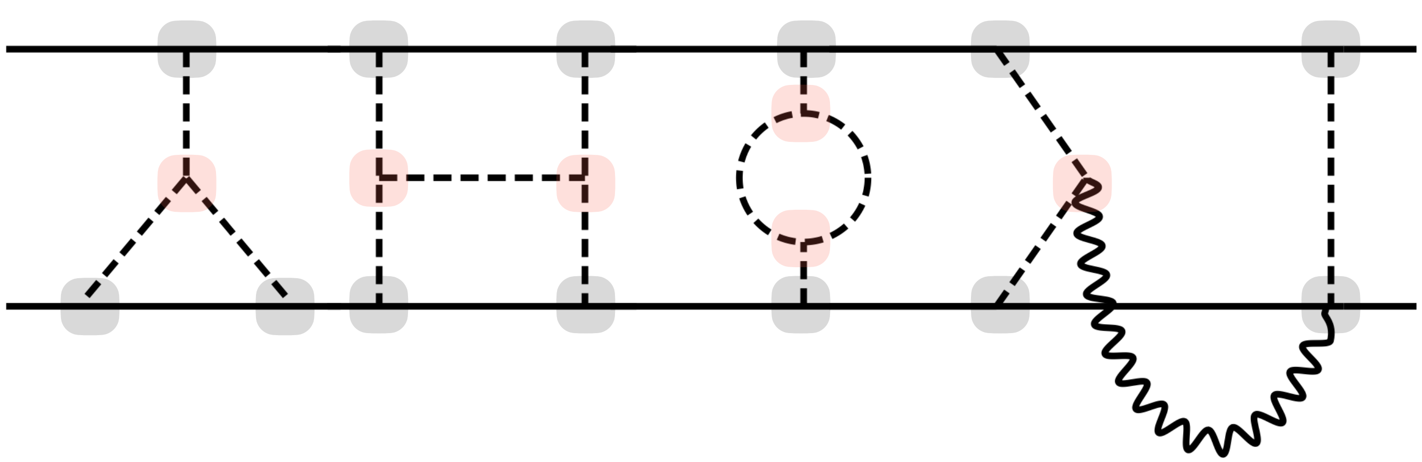
To separate the physics at scale r from the physics at scales ≫ r, we decompose the gravitons as -

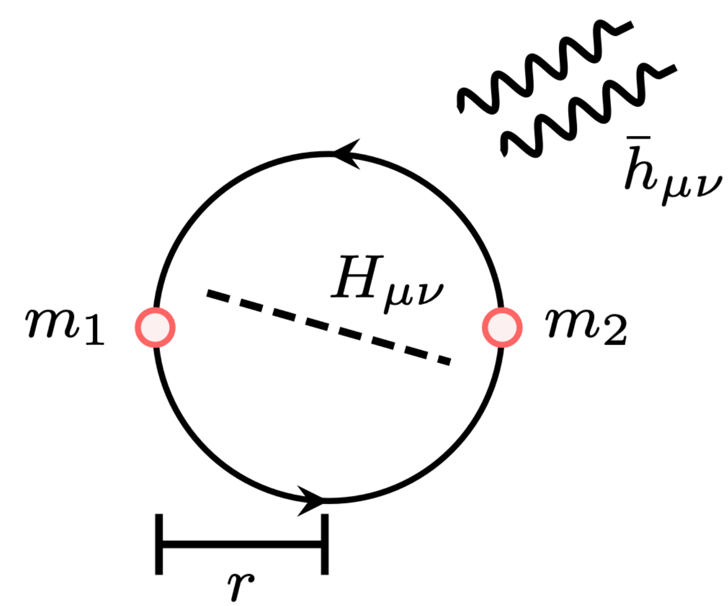
[Goldberger and Rothstein (2006)]

Potential gravitons ($H_{\mu\nu}$): $\left(\omega, \frac{1}{\lambda}\right) \approx \left(\frac{\mathrm{v}}{r}, \frac{1}{r}\right)$

Radiation gravitons ($\bar{h}_{\mu\nu}$): $\left(\omega, \frac{1}{\lambda}\right) \approx \left(\frac{\mathrm{v}}{r}, \frac{\mathrm{v}}{r}\right)$

The action for the binary is: $$e^{iS_{eff}[x_{pp}^{(a)}]} = \int D[H_{\mu\nu}]\, D[\bar{h}_{\mu\nu}]\, e^{iS_{EH}[h_{\mu\nu}] + iS_{pp}[x_{pp}^{(a)}, h_{\mu\nu}]}$$





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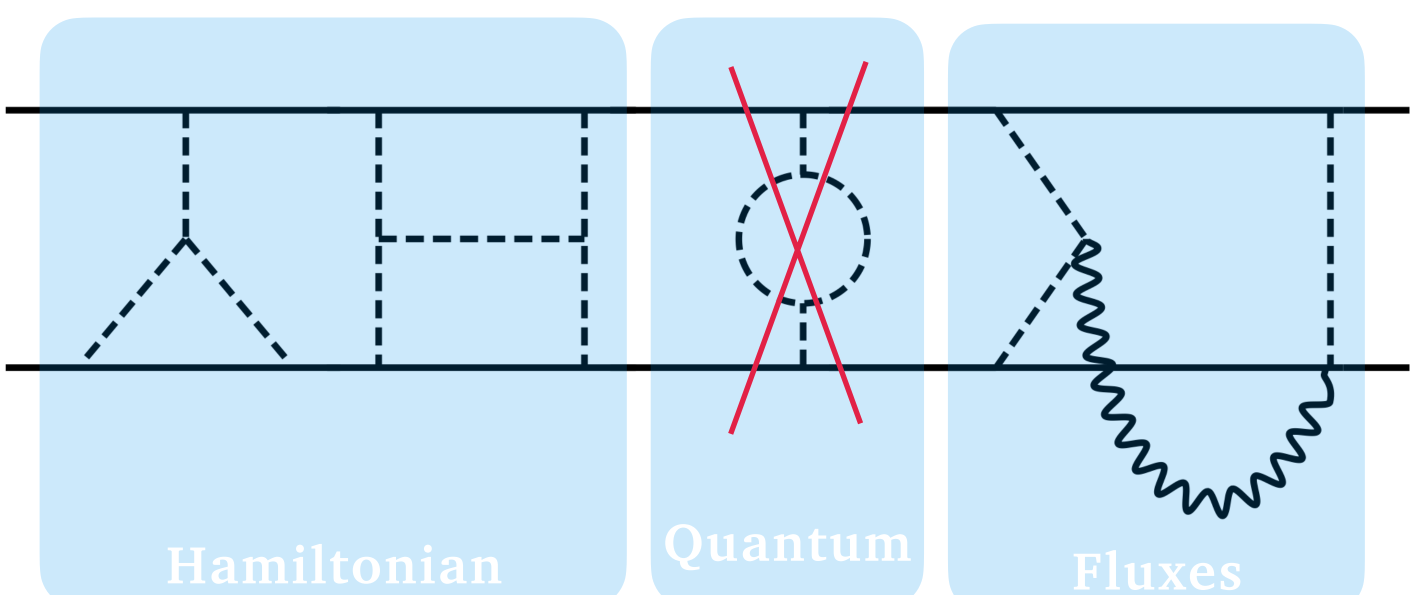
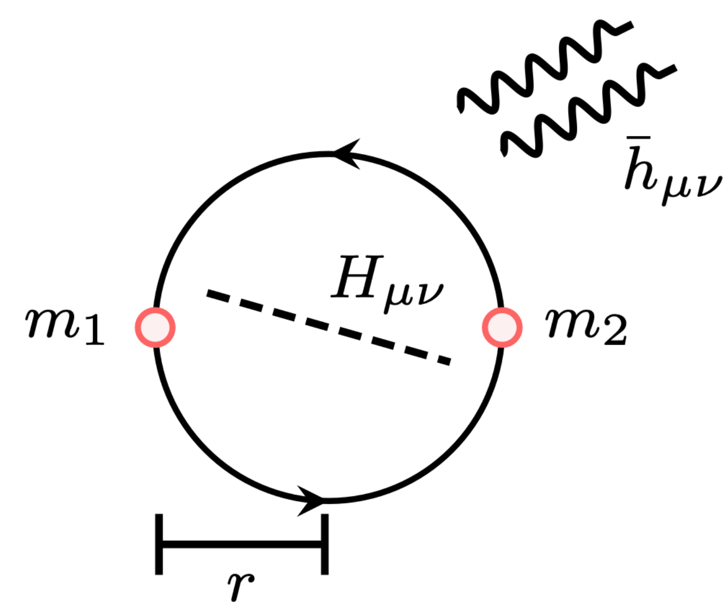
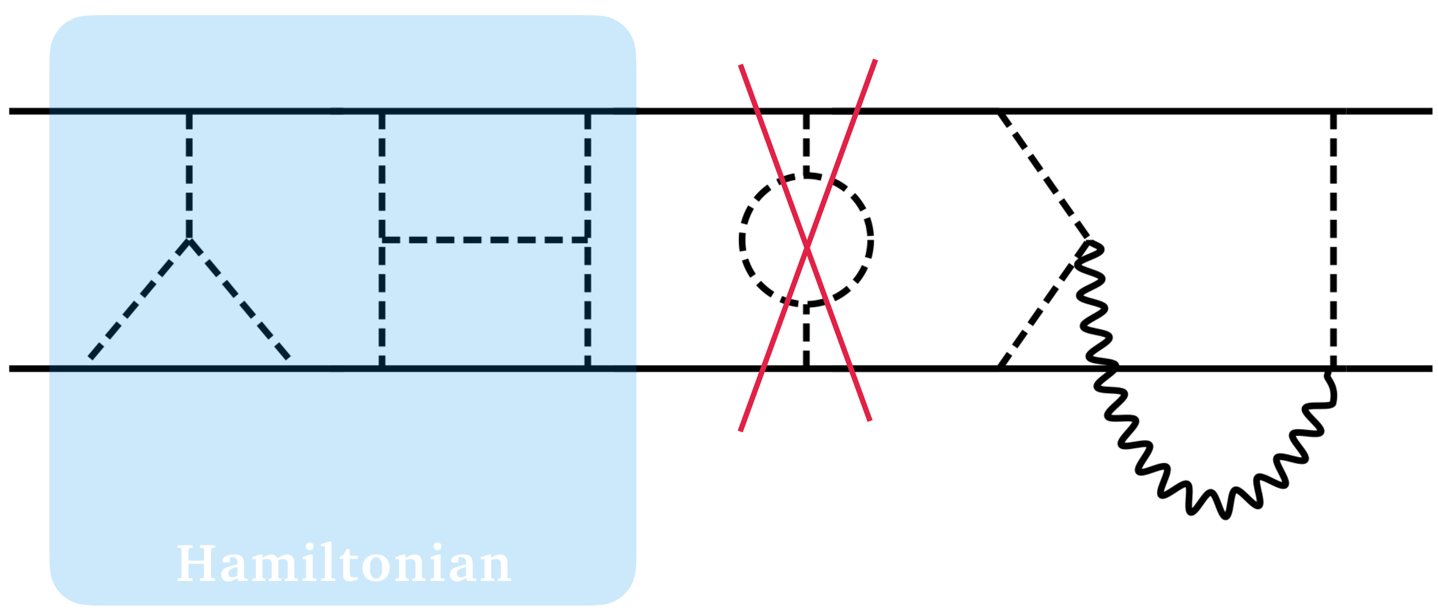
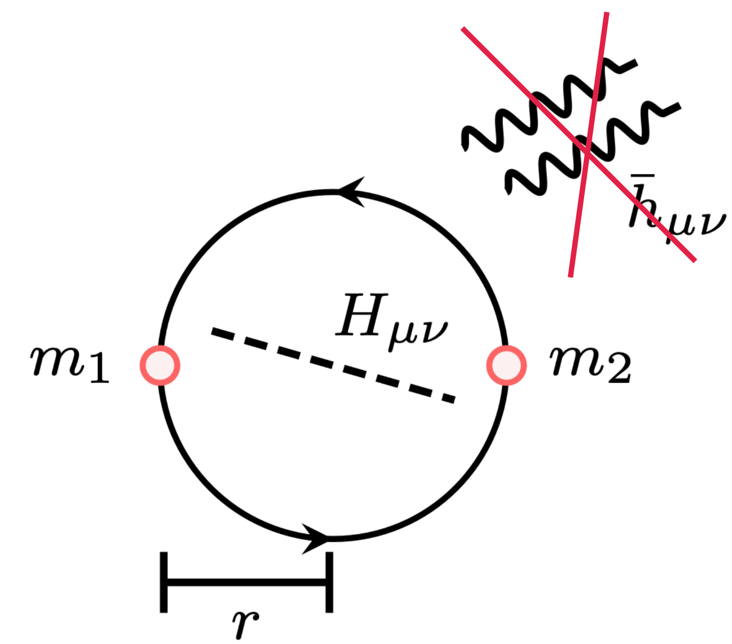




Conservative effects





Instantaneous conservative effects given by (integrating out potential gravitons with $\bar{h}_{\mu\nu} = 0$)

$$e^{i\int dt\, L_{eff}[x_{pp}^{(a)}]} = \int D[H_{\mu\nu}]\; e^{iS_{EH}[H_{\mu\nu}]+iS_{pp}[x_{pp}^{(a)},H_{\mu\nu}]}$$

$$V_{\text{eff}} = -\,i \lim_{d\to 3} \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)} \times$$

Effective potential

Removing higher order time derivatives

Removing spurious poles

Effective conservative Hamiltonian

Effective two body Hamiltonian

$$V_{\text{eff}} = -\,i \lim_{d\to 3} \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}$$

Effective potential

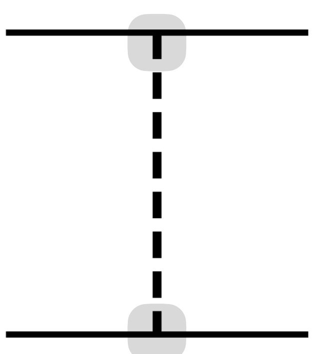
Removing higher order time derivatives

Removing spurious poles

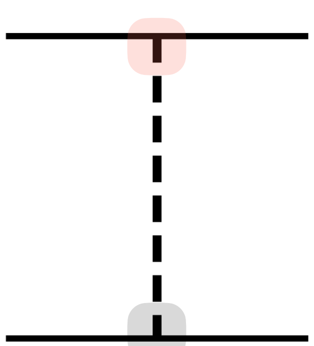
Effective conservative Hamiltonian

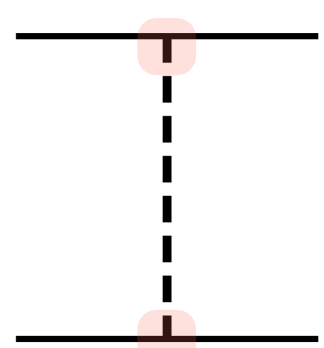
At leading order,

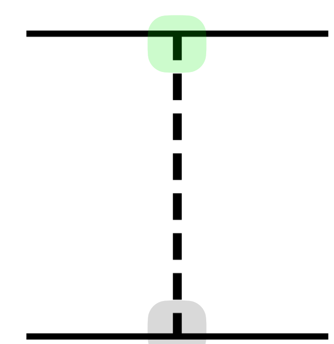
$$S_{\text{pp}} = \int dt\, \left\{ m\sqrt{g^{L}_{\mu\nu}u^{\mu}u^{\nu}} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + \lambda\, Q^{\mu\nu}E^{\mu\nu} + \cdots \right\}$$



$$\equiv\ G_N m_1 m_2 \frac{4\pi i}{\mathbf{p}^2} \longrightarrow V^{\text{LO}}_{\text{pp}} = -\frac{G_N m_1 m_2}{r} \longrightarrow \hat{H}^{\text{LO}}_{\text{pp}} = -\frac{1}{r} + \frac{1}{2}p_r^2 + L^2\left(\frac{1}{2r^2}\right)$$







$$\hat{H}^{\text{LO}}_{\text{SO}} = (\mathbf{S}_1\cdot\mathbf{L})\left\{\frac{2\nu}{r^3} + \frac{1}{q}\frac{3\nu}{2r^3}\right\}$$

$$\hat{H}^{\text{LO}}_{\text{S}_1\text{S}_2} = (\mathbf{S}_1\cdot\mathbf{S}_2)\left\{-\frac{\nu}{r^3}\right\} + \left((\mathbf{S}_1\cdot r)(\mathbf{S}_2\cdot r)\right)\left\{\frac{3\nu}{r^5}\right\}$$

$$\hat{H}^{\text{LO}}_{\text{T}} = \lambda_1 \frac{1}{q}\left(-\frac{3}{2r^6}\right)$$

Effective two body Hamiltonian

$$V_{\text{eff}} = -i \lim_{d\to 3} \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}$$

Effective potential

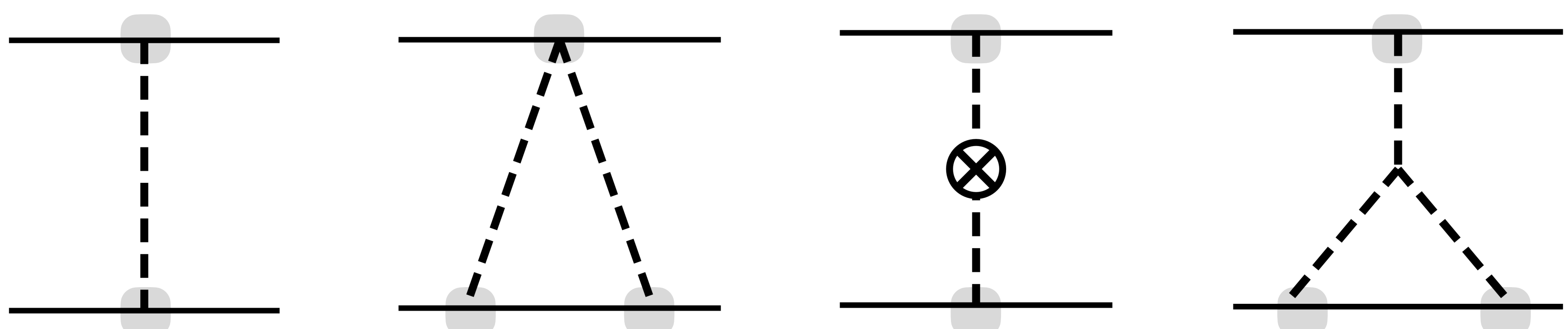
Removing higher order time derivatives

Removing spurious poles

Effective conservative Hamiltonian

At next-to leading order,

$$S_{\text{pp}} = \int dt \left\{ m\sqrt{g^{L}_{\mu\nu}u^{\mu}u^{\nu}} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \lambda\, Q^{\mu\nu}E^{\mu\nu} + \cdots \right\}$$



$$V^{\text{NLO}}_{\text{pp}} = \frac{G_N m_1 m_2}{2|\mathbf{r}|}\left[3\left(\mathbf{v}_1^2+\mathbf{v}_2^2\right) - 7\left(\mathbf{v}_1\cdot\mathbf{v}_2\right) - \frac{\left(\mathbf{v}_1\cdot\mathbf{r}\right)\left(\mathbf{v}_2\cdot\mathbf{r}\right)}{\mathbf{r}^2}\right] - \frac{G_N^2 m_1 m_2\left(m_1+m_2\right)}{2\mathbf{r}^2}$$

$$\hat{H}^{\text{NLO}}_{\text{pp}} = \frac{1}{2r^2} + p_r^2\left(-\frac{\nu}{r} - \frac{3}{2r}\right) + p_r^4\left(\frac{3\nu}{8} - \frac{1}{8}\right) + L^2\left(-\frac{\nu}{2r^3} - \frac{3}{2r^3} + p_r^2\left(\frac{3\nu}{4r^2} - \frac{1}{4r^2}\right)\right) + L^4\left(\frac{3\nu}{8r^4} - \frac{1}{8r^4}\right)$$

Effective two body Hamiltonian

$$V_{\text{eff}} = -i \lim_{d \to 3} \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1 - \mathbf{x}_2)}$$

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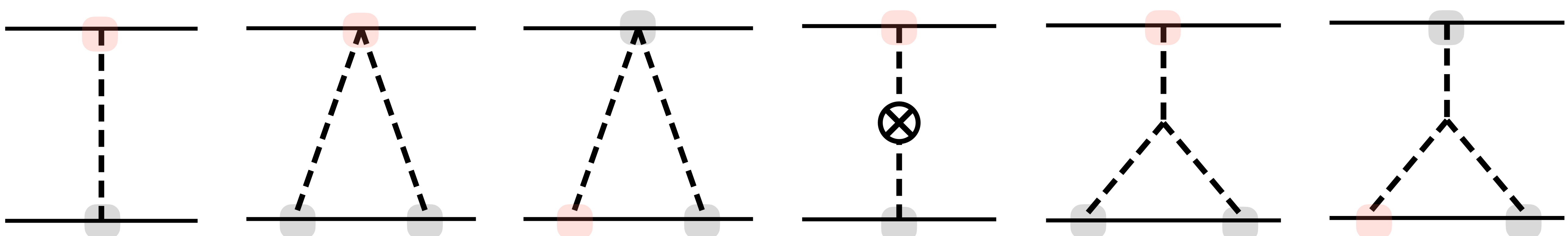
At next-to leading order,

$$S_{\text{pp}} = \int dt \left\{ m\sqrt{g^L_{\mu\nu} u^\mu u^\nu} - \frac{1}{2} S_{\mu\nu} \Omega^{\mu\nu} + \lambda\, Q^{\mu\nu} E^{\mu\nu} + \cdots \right\}$$



$$\hat{H}^{\text{NLO}}_{\text{SO}} = (\mathbf{S}_1 \cdot \mathbf{L}) \left\{ -\frac{5\nu^2}{4r^4} - \frac{6\nu}{r^4} + p_r^2 \left(\frac{43\nu^2}{8r^3} \right) + L^2 \left(\frac{13\nu^2}{8r^5} \right) + \frac{1}{q} \left(-\frac{5\nu^2}{4r^4} - \frac{5\nu}{r^4} + p_r^2 \left(\frac{17\nu^2}{4r^3} - \frac{5\nu}{8r^3} \right) + L^2 \left(\frac{5\nu^2}{4r^5} - \frac{5\nu}{8r^5} \right) \right) \right\}$$

Effective two body Hamiltonian

$$V_{\mathrm{eff}} = -\,i \lim_{d\to 3} \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}$$

Effective potential

Removing higher order time derivatives

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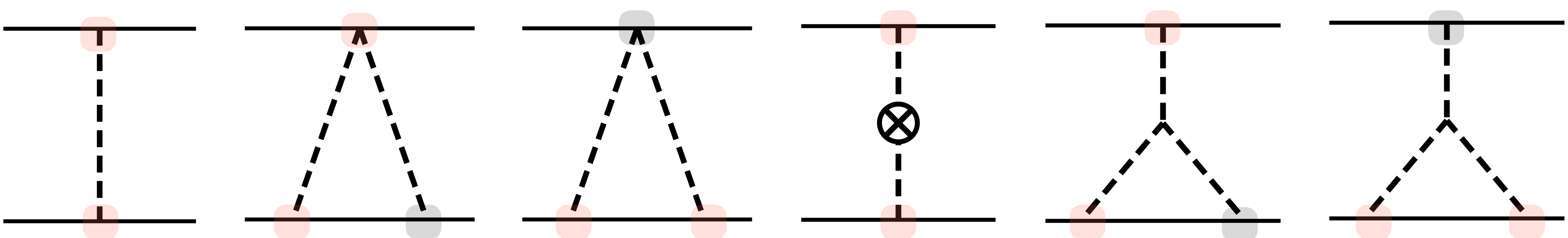
At next-to leading order,

$$S_{\mathrm{pp}} = \int dt \left\{ m\sqrt{g^{L}_{\mu\nu}u^{\mu}u^{\nu}} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \lambda\, Q^{\mu\nu}E^{\mu\nu} + \cdots \right\}$$



$$\hat{H}^{\mathrm{NLO}}_{\mathrm{S_1S_2}} = (\mathbf{S}_1\cdot\mathbf{S}_2)\left\{L^2\left(\frac{\nu}{r^5}-\frac{\nu^2}{4r^5}\right)+p_r^2\left(-\frac{7\nu^2}{4r^3}-\frac{\nu}{2r^3}\right)+\frac{5\nu}{r^4}\right\}+\left((\mathbf{S}_1\cdot\mathbf{r})(\mathbf{S}_2\cdot\mathbf{r})\right)\left\{\frac{3L^2\nu^2}{2r^7}+p_r^2\left(\frac{23\nu^2}{4r^5}+\frac{\nu}{2r^5}\right)-\frac{11\nu}{r^6}\right\}+\cdots$$

$$\hat{H}^{\mathrm{NLO}}_{\mathrm{S_1^2}} = (\mathbf{S}_1\cdot\mathbf{S}_1)\left\{\frac{3L^2\nu^2}{2r^5}-\frac{3\nu^2p_r^2}{8r^3}+\frac{7\nu^2}{8r^4}+\frac{1}{q}\left(L^2\left(\frac{11\nu^2}{8r^5}-\frac{3\nu}{2r^5}\right)+p_r^2\left(\frac{3\nu}{8r^3}-\frac{\nu^2}{2r^3}\right)+\frac{7\nu^2}{8r^4}+\frac{9\nu}{8r^4}\right)\right\}+\cdots$$

Effective two body Hamiltonian

$$V_{\text{eff}} = -\,i \lim_{d\to 3} \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}$$

Effective potential

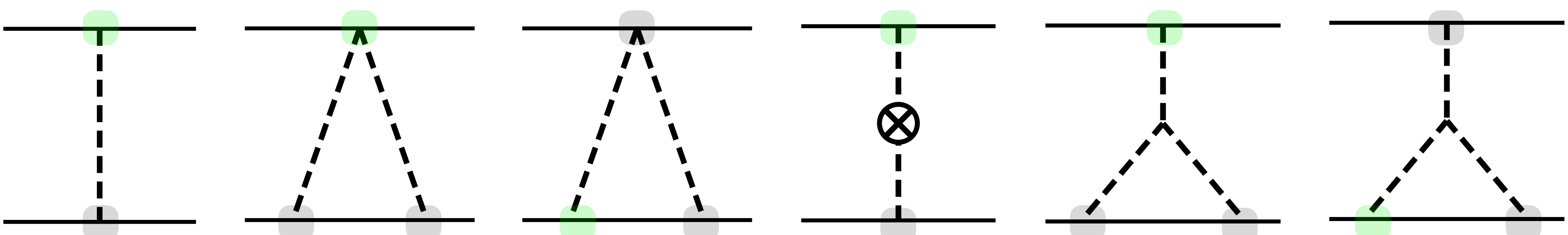
Removing higher order time derivatives

Removing spurious poles

Effective conservative Hamiltonian

At next-to leading order,

$$S_{\text{pp}} = \int dt\, \left\{ m\sqrt{g^{L}_{\mu\nu}u^{\mu}u^{\nu}} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \lambda\; Q^{\mu\nu}E^{\mu\nu} + \cdots \right\}$$



$$\hat{H}^{\text{NLO}}_{\text{T}} = \lambda_1 \left\{ -\frac{3\nu}{r^7} + p_r^2\left(\frac{15\nu}{4r^6}\right) + L^2\left(-\frac{3\nu}{4r^8}\right) + \frac{1}{q}\left[\frac{21}{2r^7} - \frac{3\nu}{r^7} + p_r^2\left(\frac{3}{4r^6} - \frac{3\nu}{2r^6}\right) + L^2\left(-\frac{3\nu}{2r^8} - \frac{15}{4r^8}\right)\right]\right\}$$

State-of-the-art - Hamiltonian

	0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	...
Point particle	**Newtonian**	**1PN**		**2PN**		**3PN**		**4PN + LO (Her)**		**5PN** + NLO (Her)		6PN	
Spin-orbit			**LO-SO**		**NLO-SO**		**NNLO-SO**		**N^3LO-SO**		N^4LO-SO + LO (Her)		
Spin^2				**LO-S^2**		**NLO-S^2**		**NNLO-S^2**		**N^3LO-S^2**		N^4LO-S^2 + LO (Her)	
Spin^3							**LO-S^3**		**NLO-S^3**		NNLO-S^3		
Spin^4								**LO-S^4**		**NLO-S^4**		NNLO-S^4	
Spin^5											**LO-S^5**		

Tree
1 Loop
2 Loop
3 Loop
4 Loop
5 Loop

Her=Hereditary effects

$$
\begin{aligned}
H^{\rm pp}_{\rm cir} = -\frac{m\nu c^2 x}{2}\Bigg\{ & 1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2 \\
& + \left[-\frac{675}{64} + \left(\frac{34445}{576} - \frac{205}{96}\pi^2\right)\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right]x^3 \\
& + \Bigg[-\frac{3969}{128} + \left(-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_{\rm E} + \frac{448}{15}\ln(16x)\right)\nu \\
& + \left(-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right)\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\Bigg]x^4 + \mathcal{O}(x^5)\Bigg\}
\end{aligned}
$$

[Damour, Jaranowski, Schafer (2014);
Jaranowski, Schafer (2015);
Bernard, Blanchet, Bohe, Faye, Marsat (2016, 2017);
Damour, Jaranowski, Schafer (2016);
Foffa, Mastrolia, Sturani, Strum, Bobadilla (2019);
Foffa, Porto, Rothstein, Sturani (2019);
Blumlein, Maier, Marquard, Schafer (2020)]

$$x = \left(GM\,\omega_{\rm orb}\right)^{2/3}$$

State-of-the-art - Hamiltonian

	0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	...
Point particle	**Newtonian**	**1PN**		**2PN**		**3PN**		**4PN** + LO (Her)		**5PN** + NLO (Her)		6PN	
Spin-orbit			**LO-SO**		**NLO-SO**		**NNLO-SO**		**N^3LO-SO**		N^4LO-SO + LO (Her)		
Spin^2				**LO-S^2**		**NLO-S^2**		**NNLO-S^2**		**N^3LO-S^2**		N^4LO-S^2 + LO (Her)	
Spin^3							**LO-S^3**		**NLO-S^3**		NNLO-S^3		
Spin^4								**LO-S^4**		**NLO-S^4**		NNLO-S^4	
Spin^5											**LO-S^5**		

Tree
1 Loop
2 Loop
3 Loop
4 Loop
5 Loop

Her=Hereditary effects

$$
\begin{aligned}
H^{\rm SO}_{\rm cir} &= x^{5/2}\left\{S^*\left(-\nu\right)+S\left(-\frac{4}{3}\nu\right)\right\}\\
&+x^{7/2}\left\{S^*\left(-\frac{3}{2}\nu+\frac{5}{3}\nu^2\right)+S\left(-4\nu+\frac{31}{18}\nu^2\right)\right\}\\
&+x^{9/2}\left\{S^*\left(-\frac{27}{8}\nu+\frac{39}{2}\nu^2-\frac{5}{8}\nu^3\right)+S\left(-\frac{27}{2}\nu+\frac{211}{8}\nu^2-\frac{7}{12}\nu^3\right)\right\}\\
&+x^{11/2}\left\{S^*\left(-\frac{135}{16}\nu+\frac{565}{8}\nu^2-\frac{1109}{24}\nu^3-\frac{25}{324}\nu^4\right)\right.\\
&\qquad\left.+S\left(-45\nu+\left(\frac{19679}{144}+\frac{29\pi^2}{24}\right)\nu^2-\frac{1979}{36}\nu^3-\frac{265}{3888}\nu^4\right)\right\}
\end{aligned}
$$

[Antonelli, Kavanagh, Khalil, Steinhoff, Vines (2020);
Kim, Levi, Yin (2022);
Mandal, Mastrolia, **RP**, Steinhoff (2022)]

$$x=\left(GM\,\omega_{\rm orb}\right)^{2/3}$$

$$S=S_1+S_2$$

$$S^*=\frac{m_2}{m_1}S_1+\frac{m_1}{m_2}S_2$$

State-of-the-art - Hamiltonian

PN orders	0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	...
Point particle	**Newtonian**	**1PN**		**2PN**		**3PN**		**4PN + LO (Her)**		**5PN** + NLO (Her)		6PN	
Spin-orbit			**LO-SO**		**NLO-SO**		**NNLO-SO**		**N^3LO-SO**		N^4LO-SO + LO (Her)		
Spin^2				**LO-S^2**		**NLO-S^2**		**NNLO-S^2**		**N^3LO-S^2**		N^4LO-S^2 + LO (Her)	
Spin^3							**LO-S^3**		**NLO-S^3**		NNLO-S^3		
Spin^4								**LO-S^4**		**NLO-S^4**		NNLO-S^4	
Spin^5											**LO-S^5**		

- Tree
- 1 Loop
- 2 Loop
- 3 Loop
- 4 Loop
- 5 Loop

Her=Hereditary effects

$$
\begin{aligned}
H^{\rm SS}_{\rm cir} \;&= \widetilde{S}_{(1)}\widetilde{S}_{(2)}\left\{x^3\left\{\nu\right\} + x^4\left\{\frac{5}{6}\nu + \frac{5}{18}\nu^2\right\} + x^5\left\{\frac{35}{8}\nu - \frac{1001}{72}\nu^2 - \frac{371}{216}\nu^3\right\}\right.\\
&\qquad\left. + x^6\left\{\frac{243}{16}\nu - \left(\frac{2107}{16} - \frac{123}{32}\pi^2\right)\nu^2 + \frac{147}{8}\nu^3 + \frac{13}{16}\nu^4\right\}\right\}\\
&+ \widetilde{S}^2_{(1)}\left\{x^4\left\{\frac{25}{18}\nu^2 + \frac{1}{q}\left(-\frac{5}{2}\nu + \frac{5}{6}\nu^2\right)\right\}\right.\\
&\quad + x^5\left\{\frac{10}{3}\nu^2 - \frac{749}{108}\nu^3 + \frac{1}{q}\left(-\frac{21}{4}\nu - \frac{7}{6}\nu^2 - \frac{217}{36}\nu^3\right)\right\}\\
&\quad + x^6\left\{\frac{1947}{112}\nu^2 - \frac{48357}{560}\nu^3 + \frac{159}{16}\nu^4\right.\\
&\qquad\left.\left. + \frac{1}{q}\left(-\frac{243}{16}\nu + \left(\frac{747}{16} - \frac{189\pi^2}{2048}\right)\nu^2 - \frac{13731}{280}\nu^3 + \frac{153}{16}\nu^4\right)\right\}\right\}
\end{aligned}
$$

[Antonelli, Kavanagh, Khalil, Steinhoff, Vines (2020);
Kim, Levi, Yin (2022);
Mandal, Mastrolia, **RP**, Steinhoff (2022)]

$$x = \left(GM\,\omega_{\rm orb}\right)^{2/3}$$

State-of-the-art - Hamiltonian

PN orders	1.5	2.5	3.5	4.5	5.5	...

	0	1	2	3	4	5	6
Point particle	**Newtonian**	**1PN**	**2PN**	**3PN**	**4PN + LO (Her)**	**5PN** + NLO (Her)	6PN

PN orders	5	6	7	8	9
$E_{\mu\nu}E^{\mu\nu}$	**LO**	**NLO**	**NNLO**	**N^3LO**	N^4LO + LO (Her)
$B_{\mu\nu}B^{\mu\nu}$		**LO**	**NLO**	NNLO	N^3LO
$\nabla_\alpha E_{\mu\nu}\nabla^\alpha E^{\mu\nu}$			**LO**	NLO	NNLO
$\dot{E}_{\mu\nu}\dot{E}^{\mu\nu}$				**LO**	NLO

Her=Hereditary effects

$$
\begin{aligned}
H^{\mathrm{T}}_{\mathrm{cir}} = \; & x^6\left(\frac{9}{2}\widetilde{\lambda}_{(+)}\right) + x^7\left[\left(-\frac{33}{4}\nu + \frac{121}{8}\right)\widetilde{\lambda}_{(+)} + \left(\frac{55}{8}\right)\delta\widetilde{\lambda}_{(-)}\right] \\
& + x^8\left[\left(\frac{91}{16}\nu^2 - \frac{2717}{42}\nu + \frac{20865}{224}\right)\widetilde{\lambda}_{(+)} + \left(-\frac{715}{48}\nu + \frac{11583}{224}\right)\delta\widetilde{\lambda}_{(-)}\right] \\
& + x^9\left\{\left[-\frac{45}{32}\nu^3 + \frac{74495}{448}\nu^2 + \left(-\frac{823243}{784} + \frac{8895\pi^2}{512}\right)\nu + \frac{894721}{3136} + \frac{321}{7}(2\nu - 1)\log\left(x\widetilde{R}_{\mathrm{orb}}\right)\right]\widetilde{\lambda}_{(+)}\right. \\
& \left. + \left[\frac{825}{64}\nu^2 - \frac{42225}{224}\nu + \frac{378751}{3136} - \frac{321}{7}\log\left(x\widetilde{R}_{\mathrm{orb}}\right)\right]\delta\widetilde{\lambda}_{(-)} - 45(\widetilde{\lambda}\kappa)_{(+)}\right\}
\end{aligned}
$$

[Mandal, Mastrolia, Silva, **RP**, Steinhoff (2024)]

$$x = \left(GM\,\omega_{\mathrm{orb}}\right)^{2/3} \qquad \lambda_\pm = \frac{m_2}{m_1}\lambda_1 \pm \frac{m_1}{m_2}\lambda_2$$

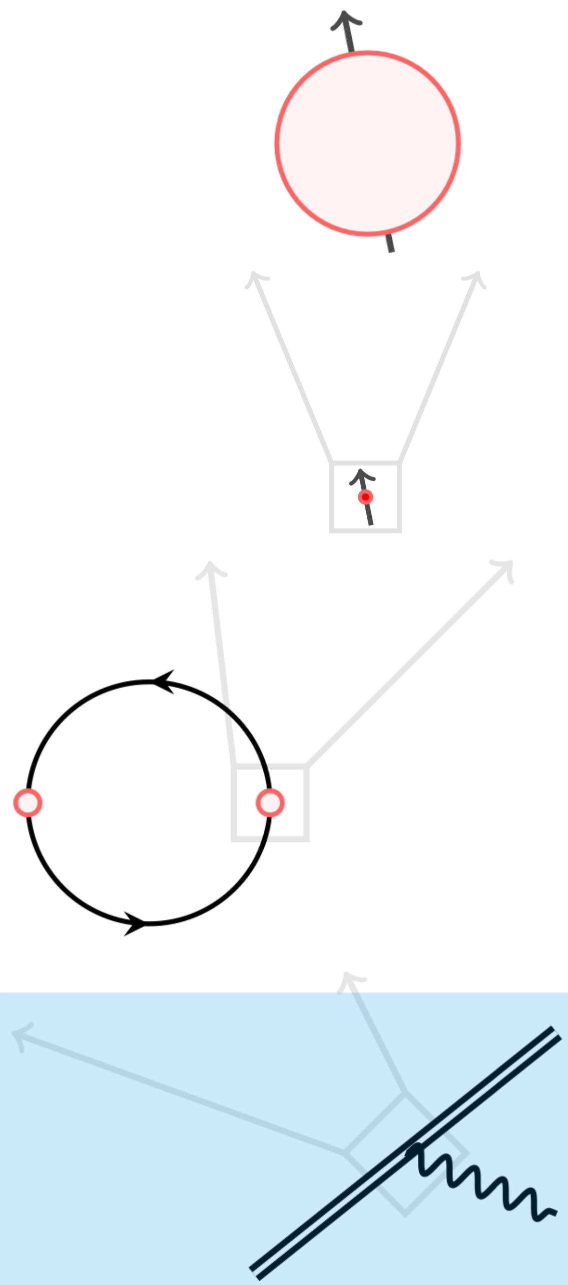
$H^{\mathrm{S}}_{\mathrm{ci}}$

Vines (2020); (2022)]

2/3

$$+\frac{1}{q}\left(-\frac{243}{16}\nu + \left(\frac{747}{16} - \frac{189\pi^2}{2048}\right)\nu^2 - \frac{13731}{280}\nu^3 + \frac{153}{16}\nu^4\right)\Big\}\Big\}$$

Effective Field Theories for PN



R_S

Black holes | Neutron stars

$\gg R_S$

$\dfrac{\text{Black holes perturbation theory}}{\text{Point particle + internal structure}}$ | $\dfrac{\text{Numerical simulations}}{\text{Point particle + internal structure}}$

r

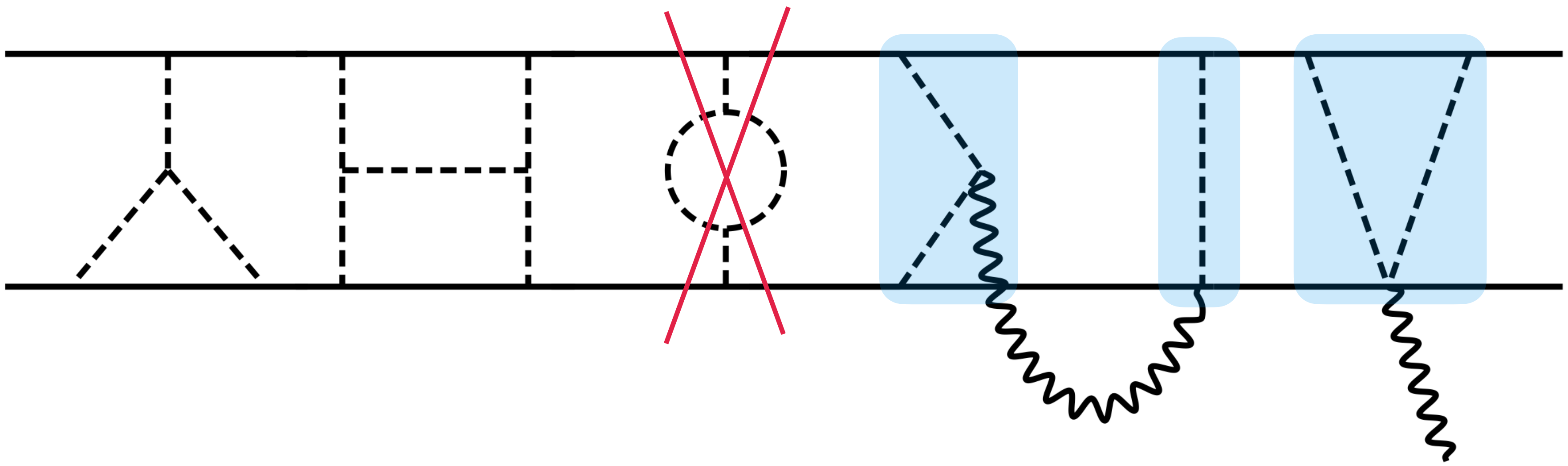
Binary
+ Near zone + Far zone

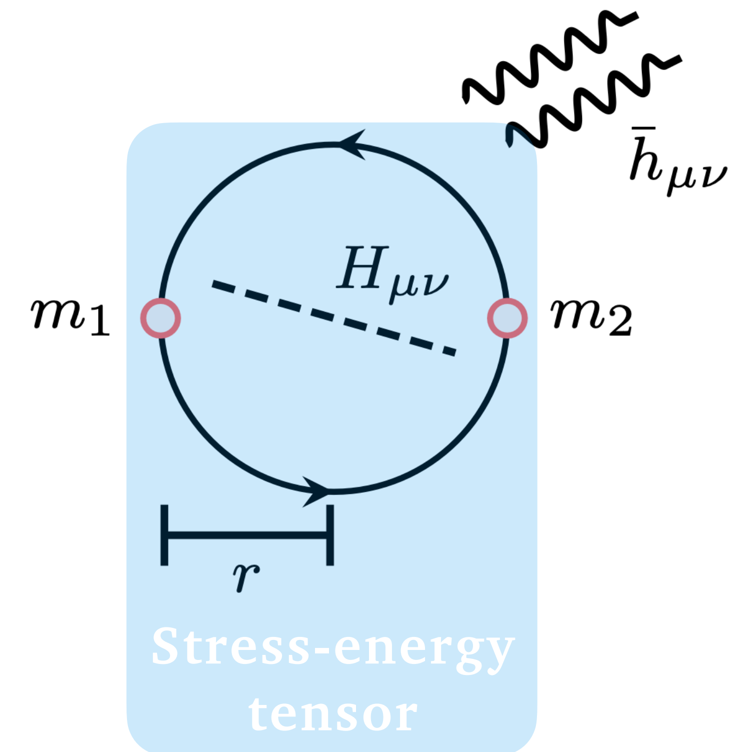
$\gg r$

$\dfrac{\text{Binary + Far zone}}{\text{Multipoles + Far zone}}$

Gravitational waves

Dissipative effects of the binary can be completely described by the Stress-energy tensor of the binary emitting radiation gravitons





Stress-energy tensor is obtained by (integrating out potential gravitons with $\bar{h}_{\mu\nu} \neq 0$)

$$e^{iS_{EH}[\bar{h}^{\mu\nu}]+i\int d^4x\; \boxed{T^{eff}_{\mu\nu}[x^{(a)}_{pp}]}\; \bar{h}^{\mu\nu}} = \int D[H_{\mu\nu}]\; e^{iS_{EH}[H_{\mu\nu},\bar{h}_{\mu\nu}]+iS_{pp}[x^{(a)}_{pp},H_{\mu\nu},\bar{h}_{\mu\nu}]}$$

$$\Gamma_{\text{eff}}[\bar{h}] = -\frac{i}{2}\lim_{d\to 3}\int d^3x \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}$$

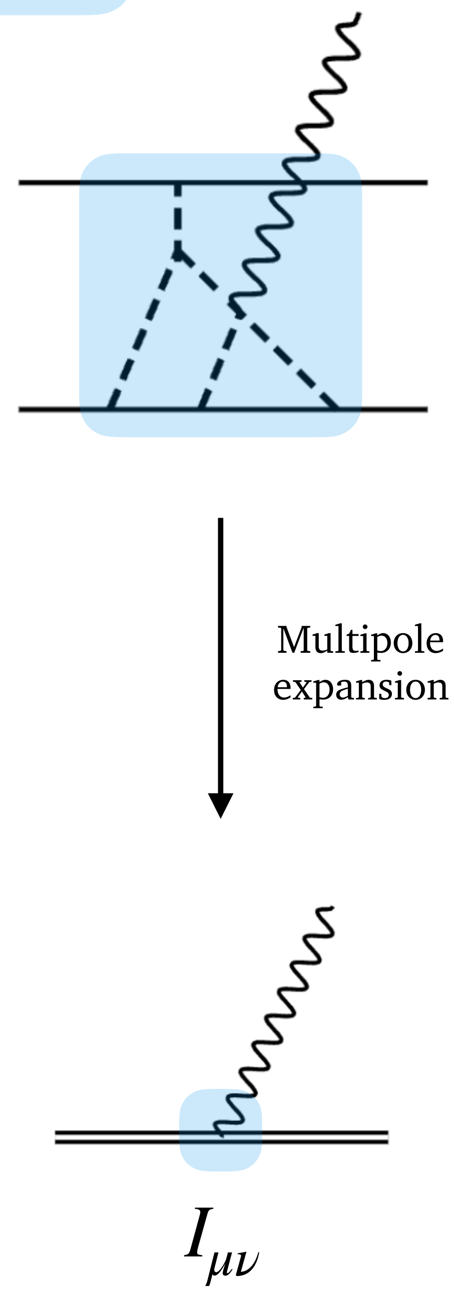
Effective stress-energy tensor

Stress-energy Tensor

Multipole EFT and Matching

A binary is approximated by its multipole moments, when considering its effect very far away from it.

$$S_{\text{binary}} = \int d^4x \, \boxed{T^{eff}_{\mu\nu}[x^{(a)}_{pp}]} \, \bar{h}^{\mu\nu}$$



Matching: $M = \int d\vec{x} T_{00}$ $\quad P_i = \int d\vec{x} T_{0i}$ $\quad L^i = \int d\vec{x} \epsilon^{ijk} T_{0j} x^k$

$$I^{ij} = \int d\vec{x} [T_{00} x^i x^j]_{\text{STF}} + \int d\vec{x} [T_{aa} x^i x^j]_{\text{STF}} + \frac{4}{3} \int d\vec{x} \partial_t [T_{0a} x^a x^i x^j]_{\text{STF}} + \cdots$$

Multipole EFT:

$$S_{\text{Multipoles}} = \int dt \left(\boxed{M} \bar{h}_{00} + \boxed{\mathbf{L}^i} \epsilon_{ijk} \partial_j \bar{h}_{0k} + \boxed{I^{ij}} E_{ij} + \boxed{J^{ij}} B_{ij} + \cdots \right)$$

Multipole moments of binary

$$\Gamma_{\text{eff}}[\bar{h}] = -\frac{i}{2}\lim_{d\to 3}\int d^3x \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}$$

Effective stress-energy tensor

Removing higher order time derivatives

Removing spurious poles

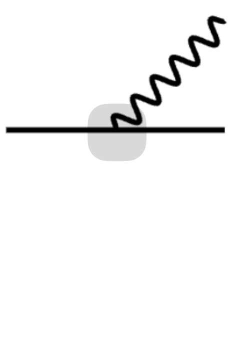
$\longrightarrow$

SO(3) decomposition

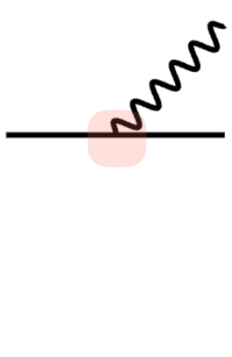
Multipole moments of the binary

At leading order,

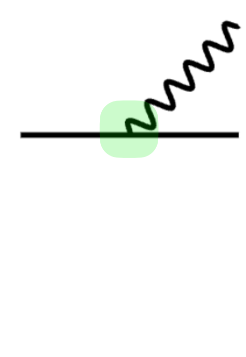
$$S_{\text{pp}} = \int dt \left\{ m\sqrt{g^L_{\mu\nu}u^\mu u^\nu} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \lambda\, Q^{\mu\nu}E^{\mu\nu} + \cdots \right\}$$



$$\equiv\; T_{00} = -\frac{1}{2}\left(m_1 e^{i\vec{k}.\vec{x}_1} + m_2 e^{i\vec{k}.\vec{x}_2}\right) \longrightarrow \left(\hat{I}^{\text{pp}}_{\text{0PN}}\right)^{ij} = \left[\mathbf{n}^i\mathbf{n}^j\right] r^2$$



$$\left(I^{\text{AT}}_{\text{0PN}}\right)^{ij} = \left[2(S_1\times\mathbf{n})^i\mathbf{v}^j - (S_1\times\mathbf{v})^i\mathbf{n}^j\right]\left\{-\frac{4}{3}q\nu r\right\}$$



$$\left(I^{\text{AT}}_{\text{0PN}}\right)^{ij} = \left[\mathbf{n}^i\mathbf{n}^j\right]\frac{\lambda_1}{r^5}\left\{3\left(1+\frac{1}{q}\right)r^2\right\}$$

Multipole moments of binary

$$\Gamma_{\text{eff}}[\bar{h}] = -\frac{i}{2}\lim_{d\to 3}\int d^3x \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}$$

Effective stress-energy tensor

Removing higher order time derivatives

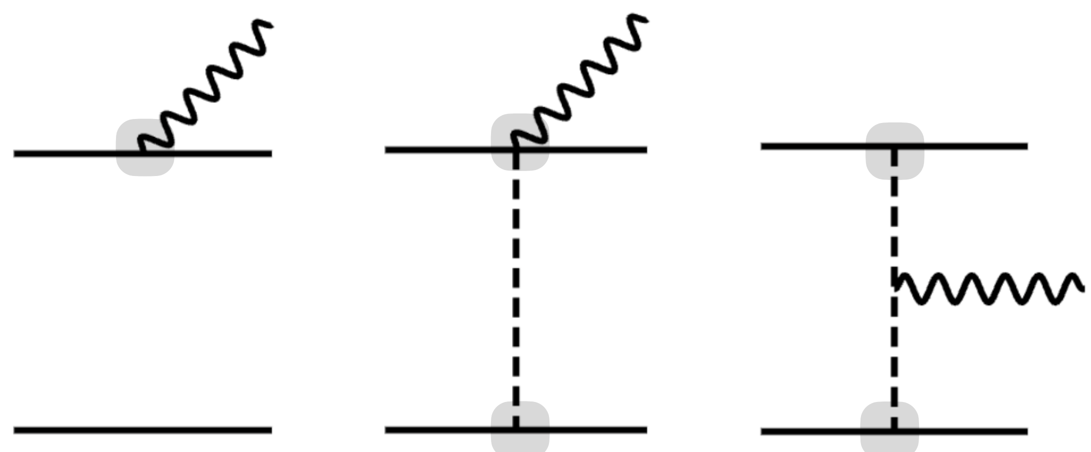
Removing spurious poles

SO(3) decomposition

Multipole moments of the binary

At next-to leading order,

$$S_{\text{pp}} = \int dt \left\{ m\sqrt{g^L_{\mu\nu}u^\mu u^\nu} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \lambda\, Q^{\mu\nu}E^{\mu\nu} + \cdots \right\}$$



$$\left(\hat{I}^{\text{pp}}_{\text{1PN}}\right)^{ij} = \left[\mathbf{n}^i\mathbf{n}^j\right]\left\{\left(\frac{29}{42}-\frac{29\nu}{14}\right)r^2v^2 + \left(\frac{8\nu}{7}-\frac{5}{7}\right)r\right\} + \left[\mathbf{n}^i\mathbf{v}^j\right]\left\{(\mathbf{n}\cdot\mathbf{v})r^2\left(\frac{12\nu}{7}-\frac{4}{7}\right)\right\} + \left[\mathbf{v}^i\mathbf{v}^j\right]\left\{\left(\frac{11}{21}-\frac{11\nu}{7}\right)r^2\right\}$$

Multipole moments of binary

$$\Gamma_{\text{eff}}[\bar{h}] = -\frac{i}{2}\lim_{d\to 3}\int d^3x \int_{\mathbf{p}} e^{i\,\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}$$

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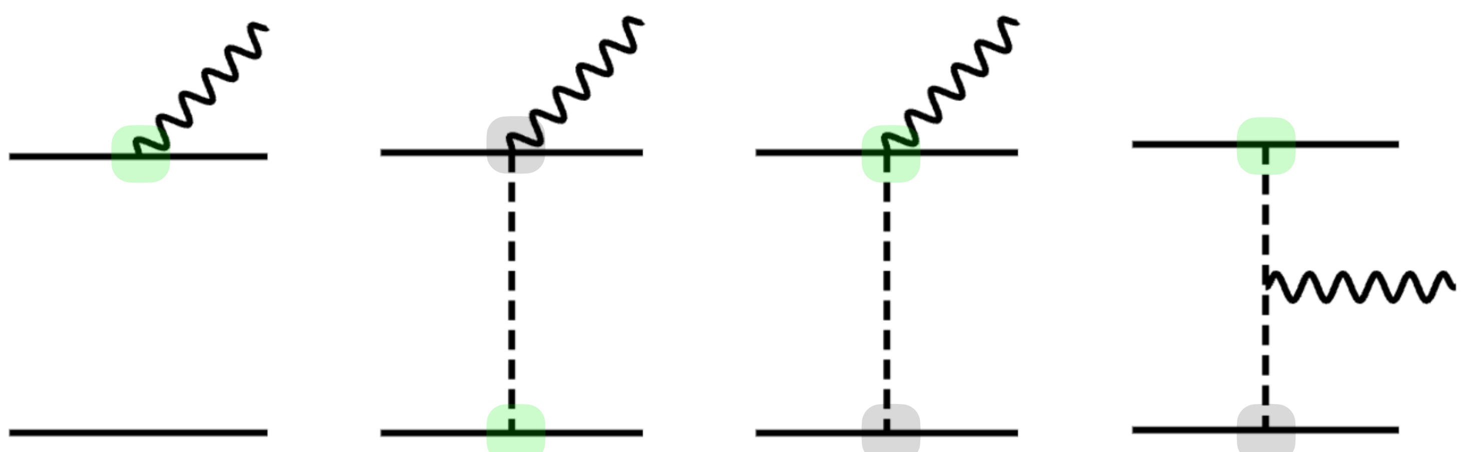


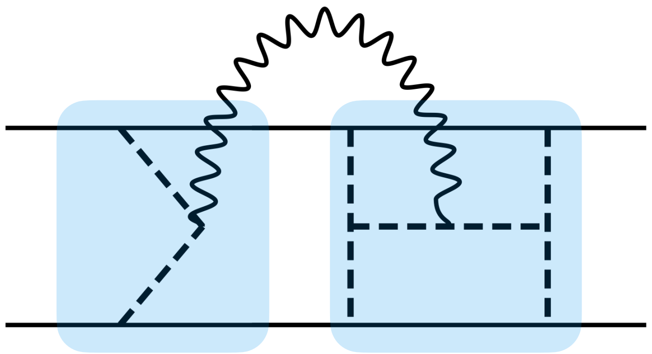
$$\left(I^{\text{AT}}_{\text{1PN}}\right)^{ij} = \left[\mathbf{n}^i\mathbf{n}^j\right]\frac{\lambda_1}{r^5}\left\{ r^2\left(\left(-\frac{40\nu}{7}-\frac{15}{2}\right)(\mathbf{n}\cdot\mathbf{v})^2 + \left(6-\frac{13\nu}{7}\right)v^2\right) + \left(\frac{2\nu}{7}-\frac{15}{2}\right)r \quad + \frac{1}{q}\left[r^2\left(\left(\frac{185}{14}-\frac{40\nu}{7}\right)(\mathbf{n}\cdot\mathbf{v})^2 + \left(\frac{55}{7}-\frac{13\nu}{7}\right)v^2\right) + \left(\frac{43\nu}{14}-\frac{75}{7}\right)r\right]\right\}$$

$$+\left[\mathbf{n}^i\mathbf{v}^j\right]\frac{\lambda_1}{r^5}\left\{\left(\frac{130\nu}{7}-\frac{214}{7}\right)(\mathbf{n}\cdot\mathbf{v})qr^2 + \left(\frac{130\nu}{7}-6\right)(\mathbf{n}\cdot\mathbf{v})r^2\right\} \quad + \left[\mathbf{v}^i\mathbf{v}^j\right]\frac{\lambda_{(1)}}{r^5}\left\{\left(\frac{59}{7}-\frac{38\nu}{7}\right)qr^2 + \left(3-\frac{38\nu}{7}\right)r^2\right\}$$

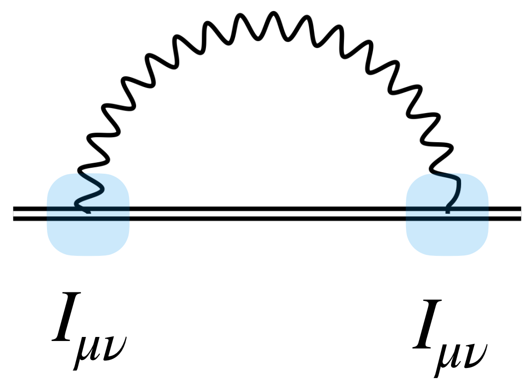
Radiation effects and waveform

Complete dynamics of the binary:

$$e^{iS_{eff}[x_{pp}^{(a)}]} = \int D[H_{\mu\nu}]\; D[\bar{h}_{\mu\nu}]\; e^{iS_{EH}[h_{\mu\nu}]+iS_{pp}[x_{pp}^{(a)},h_{\mu\nu}]}$$



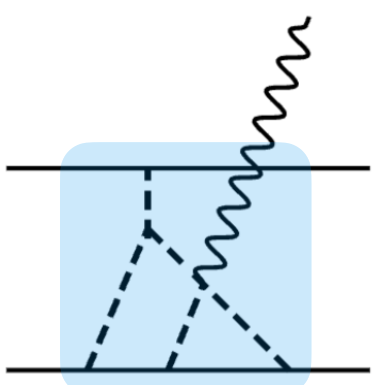
Multipole expansion



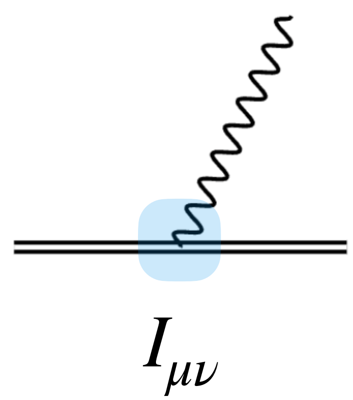
$$F^i_{rr} = -\frac{2m_a G_N}{5}\; x^j_a(t)\; \frac{d^5 I^{ij}(t)}{dt^5}$$

$$P = \frac{G_N}{5}\left\langle \left(\frac{d^3}{dt^3} I^{ij}(t)\right)^2 \right\rangle$$

Gravitational waves from a given dynamics of the binary:



Multipole expansion



$$h^{ij} = -\frac{4G_N}{R}\Lambda^{ijkl}\frac{d^2 I^{kl}(t)}{dt^2}$$

State-of-the-art - Flux

PN orders	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	...
Point Particl	LO		NLO	LO (Her)	NNLO	NLO (Her)	N^3LO +NNLO (Her)	N^3LO (Her)	N^4LO +N^4LO (Her)	N^5LO (Her)	
Spin-orbiit				LO		NLO	LO (Her)	NNLO	NLO (Her)	N^3LO +NNLO (Her)	
Spin^2					LO		NLO	LO (Her)	NNLO	NLO (Her)	
Spin^3								LO		NLO	
Spin^4									LO		

- Disconnected
- Tree
- 1 Loop
- 2 Loop
- 3 Loop

Her=Hereditary effects

$$\mathcal{F}^{\rm pp}_{cir} = \frac{32c^5}{5G}\nu^2x^5\Bigg\{1+\left(-\frac{1247}{336}-\frac{35}{12}\nu\right)x+4\pi x^{3/2}+\left(-\frac{44711}{9072}+\frac{9271}{504}\nu+\frac{65}{18}\nu^2\right)x^2+\left(-\frac{8191}{672}-\frac{583}{24}\nu\right)\pi x^{5/2}$$
$$+\left[\frac{6643739519}{69854400}+\frac{16}{3}\pi^2-\frac{1712}{105}\gamma_{\rm E}-\frac{856}{105}\ln(16\,x)+\left(-\frac{134543}{7776}+\frac{41}{48}\pi^2\right)\nu-\frac{94403}{3024}\nu^2-\frac{775}{324}\nu^3\right]x^3$$
$$+\left(-\frac{16285}{504}+\frac{214745}{1728}\nu+\frac{193385}{3024}\nu^2\right)\pi x^{7/2}$$
$$+\Bigg[-\frac{323105549467}{3178375200}+\frac{232597}{4410}\gamma_{\rm E}-\frac{1369}{126}\pi^2+\frac{39931}{294}\ln 2-\frac{47385}{1568}\ln 3+\frac{232597}{8820}\ln x$$
$$+\left(-\frac{1452202403629}{1466942400}+\frac{41478}{245}\gamma_{\rm E}-\frac{267127}{4608}\pi^2+\frac{479062}{2205}\ln 2+\frac{47385}{392}\ln 3+\frac{20739}{245}\ln x\right)\nu$$
$$+\left(\frac{1607125}{6804}-\frac{3157}{384}\pi^2\right)\nu^2+\frac{6875}{504}\nu^3+\frac{5}{6}\nu^4\Bigg]x^4$$
$$+\left[\frac{265978667519}{745113600}-\frac{6848}{105}\gamma_{\rm E}-\frac{3424}{105}\ln(16\,x)+\left(\frac{2062241}{22176}+\frac{41}{12}\pi^2\right)\nu-\frac{133112905}{290304}\nu^2-\frac{3719141}{38016}\nu^3\right]\pi x^{9/2}+\mathcal{O}(x^5)\Bigg\}$$

[Blanchet, Faye, Henry, Larrouturou, Trestini (2023)]

$$x=\left(GM\,\omega_{\rm orb}\right)^{2/3}$$

State-of-the-art - Flux

PN orders	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	...
Point Particle	**LO**		**NLO**	**LO (Her)**	**NNLO**	**NLO (Her)**	**N^3LO +NNLO (Her)**	**N^3LO (Her)**	**N^4LO +N^4LO (Her)**	**N^5LO (Her)**	
Spin-orbiit				**LO**		**NLO**	**LO (Her)**	**NNLO**	**NLO (Her)**	N^3LO +NNLO (Her)	
Spin^2											
Spin^3											
Spin^4											

Her=Hereditary effects

$$\mathcal{F}^{\rm pp}_{cir} = \frac{32c^5}{5G}\nu^2 x$$

PN orders	0	0.5	1	1.5	2	2.5	3
$E_{\mu\nu}E^{\mu\nu}$	**LO**		**NLO**	**LO (Her)**	**NNLO**	**NLO (Her)**	N^3LO +NNLO (Her)
$B_{\mu\nu}B^{\mu\nu}$			**LO**		**NLO**	**LO (Her)**	NNLO
$\nabla_\alpha E_{\mu\nu}\nabla^\alpha E^{\mu\nu}$					**LO**		NLO
$\dot{E}_{\mu\nu}\dot{E}^{\mu\nu}$							LO

$$\begin{aligned}\mathcal{F}_E^{\rm AT} =&\left\{\left(\frac{768\nu^2}{5}+\frac{192\nu}{5}\right)\widetilde{\lambda}_{(+)}+\frac{192}{5}\delta\widetilde{\lambda}_{(-)}\right\}x^{10}\\&+\left\{\left(-992\nu^3-\frac{9736\nu^2}{35}-\frac{1408\nu}{35}\right)\widetilde{\lambda}_{(+)}+\left(-\frac{184\nu^2}{5}-\frac{1408\nu}{35}\right)\delta\widetilde{\lambda}_{(-)}\right\}x^{11}\\&+4\pi\left\{\left(\frac{768\nu^2}{5}+\frac{192\nu}{5}\right)\widetilde{\lambda}_{(+)}+\frac{192}{5}\delta\widetilde{\lambda}_{(-)}\right\}x^{23/2}\\&+\left\{\left(3088\nu^4+\frac{63692\nu^3}{35}-\frac{1299706\nu^2}{945}+\frac{5344\nu}{45}\right)\widetilde{\lambda}_{(+)}\right.\\&\quad\left.+\left(-\frac{11116\nu^3}{15}+\frac{149566\nu^2}{105}+\frac{5344\nu}{45}\right)\delta\widetilde{\lambda}_{(-)}\right\}x^{12}+\mathcal{O}\left(x^{25/2}\right).\end{aligned}$$

[Henry, Faye, Blanchet(2020),
Mandal, Mastrolia, **RP**, Steinhoff (2024)]

$$x = (GM\,\omega_{\rm orb})^{2/3}$$

$$\lambda_\pm = \frac{m_2}{m_1}\lambda_1 \pm \frac{m_1}{m_2}\lambda_2$$

urou, Trestini (2023)]

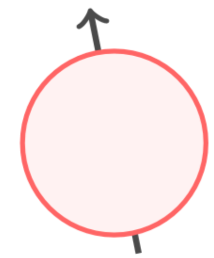
$$\left.{}_{\rm orb}\right)^{2/3}$$

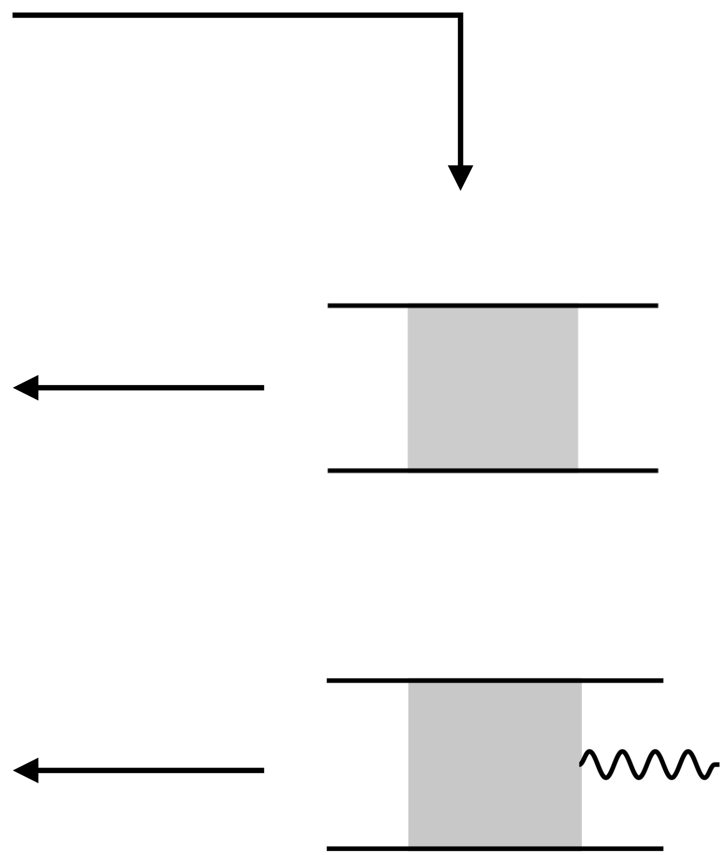
$$+\left[\frac{265978667519}{745113600}-\frac{6848}{105}\gamma_{\rm E}-\frac{3424}{105}\ln(16\,x)+\left(\frac{2062241}{22176}+\frac{41}{12}\pi^2\right)\nu-\frac{133112905}{290304}\nu^2-\frac{3719141}{38016}\nu^3\right]\pi x^{9/2}+\mathcal{O}(x^5)\right\}$$

Summary - PN pipeline

EFT around a point particle for compact objects with internal structure

$$\underbrace{m\sqrt{u^2}}_{\text{pp}} + \underbrace{\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu}}_{\text{spin}} + \underbrace{c_{E^2}\, E_{\mu\nu}E^{\mu\nu}}_{\text{tides}} + \cdots$$





Two point particle EFT to describe a bound state like binary at scales r

$$\sum_{a=1,2}\frac{1}{2}m_a\mathbf{v}_a^2 + \frac{G_N m_1 m_2}{|\mathbf{r}|} + 1\text{PN} + 2\text{PN} + \cdots$$

Computing Hamiltonian

EFT of **multipole moments** to describe the binary at scale λ

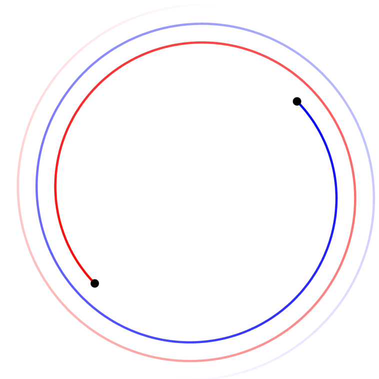
$$\underbrace{M}_{\substack{\text{total}\\\text{mass}}}\bar{h}_{00} + \underbrace{\mathbf{L}^i}_{\substack{\text{angular}\\\text{mom.}}}\epsilon_{ijk}\partial_j h_{0k} + \underbrace{I^{ij}}_{\substack{\text{mass}\\\text{quad.}}}E_{ij} + \underbrace{J^{ij}}_{\substack{\text{current}\\\text{quad.}}}B_{ij} + \cdots$$

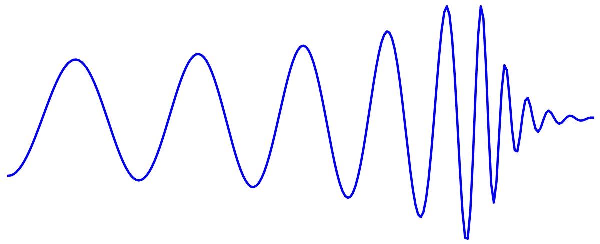
Radiation reaction force / Flux

Equation of motion

$$\dot{r} = \frac{dH}{dp_r} \qquad \dot{p}_r = -\frac{dH}{dr} + F_r$$

$$\dot{\phi} = \frac{dH}{dp_\phi} \qquad \dot{p}_\phi = -\frac{dH}{d\phi} + F_\phi$$





This talk

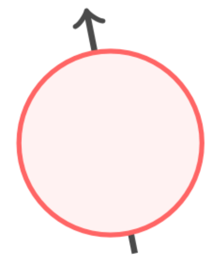
What are EFTs? Why do we need EFTs?

How are EFTs used for
two-body problems?

What did we do with it?

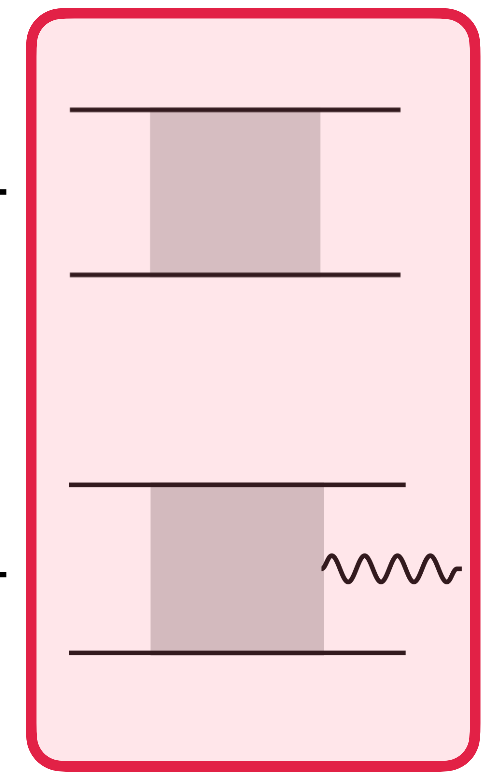
Questions?

Summary - PN pipeline



EFT around a point particle for compact objects with internal structure

$$\underbrace{m\sqrt{u^2}}_{\text{pp}} + \underbrace{\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu}}_{\text{spin}} + \underbrace{c_{E^2}\, E_{\mu\nu}E^{\mu\nu}}_{\text{tides}} + \cdots$$



Two point particle EFT to describe a bound state like binary at scales r

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$$\sum_{a=1,2}\frac{1}{2}m_a\mathbf{v}_a^2 + \frac{G_N m_1 m_2}{|\mathbf{r}|} + 1\text{PN} + 2\text{PN} + \cdots$$

EFT of **multipole moments** to describe the binary at scale λ

Radiation reaction force / Flux

$$\underbrace{M}_{\substack{\text{total}\\\text{mass}}}\bar{h}_{00} + \underbrace{L^i}_{\substack{\text{angular}\\\text{mom.}}}\epsilon_{ijk}\partial_j h_{0k} + \underbrace{I^{ij}}_{\substack{\text{mass}\\\text{quad.}}}E_{ij} + \underbrace{J^{ij}}_{\substack{\text{current}\\\text{quad.}}}B_{ij} + \cdots$$

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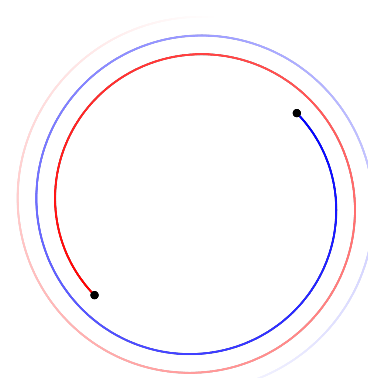
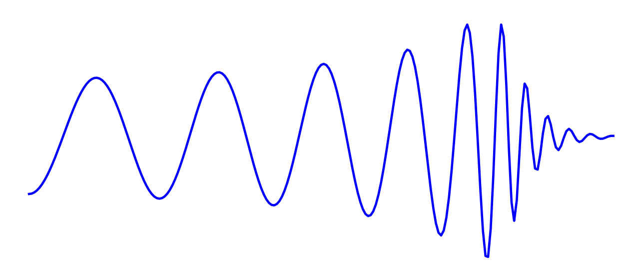
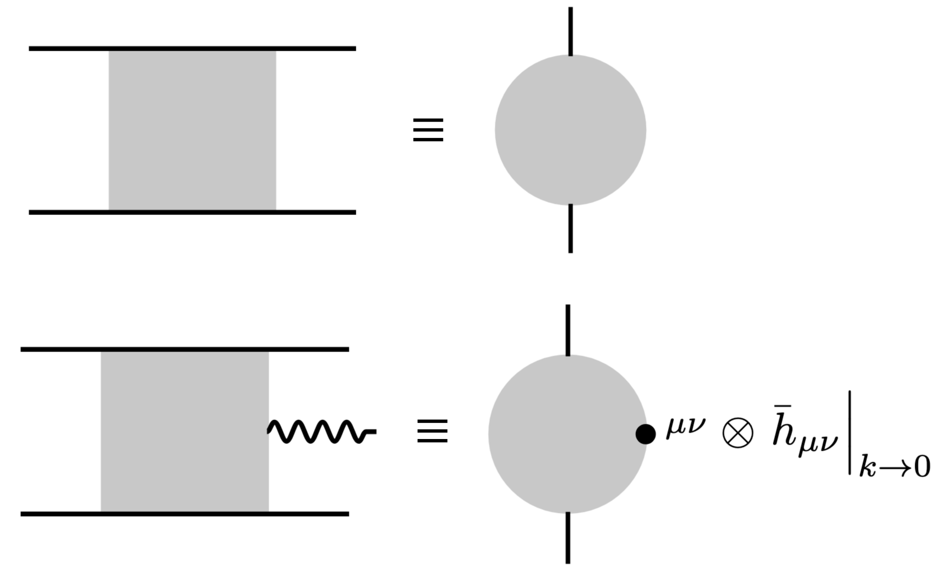


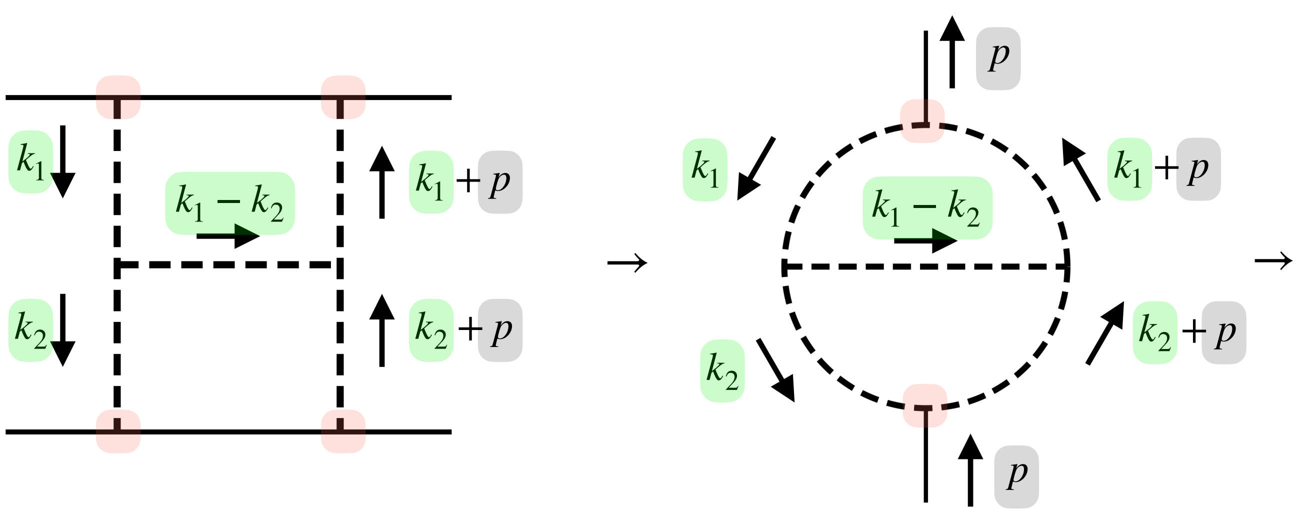


Multi-loop Feynman diagrams

the momentum space Feynman diagrams are mapped onto two-point massless integrals



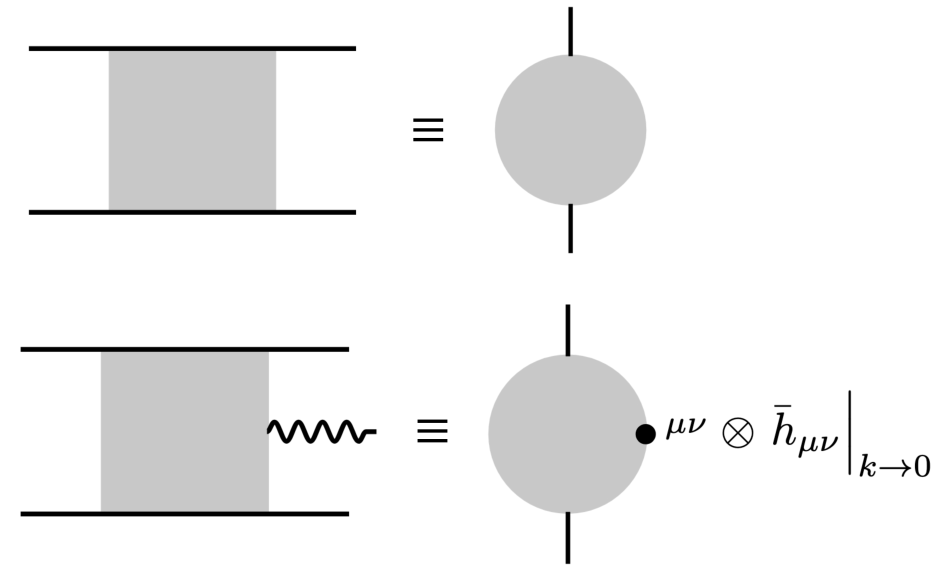
$$\mathcal{FD}^{(l)}_{\{g\}} = \underbrace{N_{\mathrm{C}}^{\mu_1,\mu_2,\cdots}\left(x_{(a)}, S_{(a)}\right)}_{\text{Coefficient that depends on orbital variables}} \underbrace{\int_p e^{\mathbf{i}p_\mu x^\mu_{(12)}} N_{\mathrm{F}}^{\alpha_1,\alpha_2,\cdots}(p)}_{\text{Fourier integral}} \underbrace{\prod_{i=1}^{l} \int_{k_i} \frac{N_{\mathrm{M}}^{\nu_1,\nu_2,\cdots}(k_i)}{\prod_{\sigma\in g} D_\sigma(p,k_i)}}_{\text{Multi-loop integral}}$$



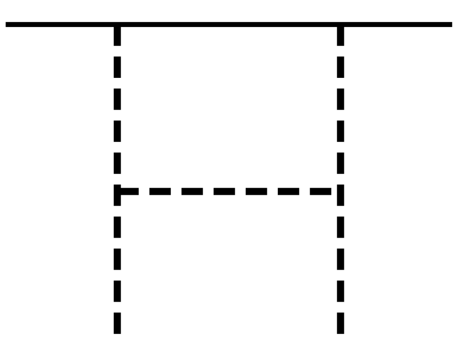
$$\to \; N_C \int d^Dp\; e^{ip\cdot(x_1-x_2)} \int d^Dk_1 d^Dk_2\, \frac{1}{k_1^2}\,\frac{1}{k_2^2}\,\frac{1}{(k_1+p)^2}\,\frac{1}{(k_2+p)^2}\,\frac{1}{(k_1-k_2)^2}$$

Multi-loop Feynman diagrams

the momentum space Feynman diagrams are mapped onto two-point massless integrals



$$\mathcal{FD}^{(l)}_{\{g\}} = \underbrace{N_{\mathrm{C}}^{\mu_1,\mu_2,\cdots}\left(x_{(a)}, S_{(a)}\right)}_{\text{Coefficient that depends on orbital variables}} \underbrace{\int_p e^{\mathrm{i}p_\mu x^\mu_{(12)}} N_{\mathrm{F}}^{\alpha_1,\alpha_2,\cdots}(p)}_{\text{Fourier integral}} \underbrace{\prod_{i=1}^{l}\int_{k_i} \frac{N_{\mathrm{M}}^{\nu_1,\nu_2,\cdots}(k_i)}{\prod_{\sigma\in g} D_\sigma(p,k_i)}}_{\text{Multi-loop integral}}$$



Point particle 3PN	$\longrightarrow$	8 diagrams	$\longrightarrow$	106 different integrals
Spin-orbit 4.5PN	$\longrightarrow$	288 diagrams	$\longrightarrow$	8270 different integrals
Quadratic in spin 5 PN	$\longrightarrow$	142 diagrams	$\longrightarrow$	19512 different integrals

No all integrals are linearly independent $\Longrightarrow$ Integration-by-parts identities

Integration By Parts - Examples

Integral family $I_n(a) = \int_0^\infty dx\ x^n\ e^{-ax}$

$$\longrightarrow \quad = \int_0^\infty d\left(\frac{x^{n+1}}{n+1}\right) e^{-ax}$$

$$= \frac{1}{n+1} x^{n+1} e^{-ax} \Big|_0^\infty - \int_0^\infty x^{n+1} d\left(e^{-ax}\right)$$

$$= \frac{a}{n+1} I_{n+1}(a)$$

Integration-by-parts identity $I_n(a) = \frac{n}{a} I_{n-1}(a)$

Master Integral $I_0(a) = \int_0^\infty dx\ e^{-ax}$

Differential equation $$\frac{\partial}{\partial a} I_0(a) = \int_0^\infty dx\ (-x)\ e^{-ax} = -I_1 = -\frac{1}{a} I_0$$

Solution $I_0(a) = -\frac{c}{a}$

Boundary condition $I_0(a=1) = \boxed{\int_0^\infty e^{-x} = 1} \implies \boxed{c = 1}$

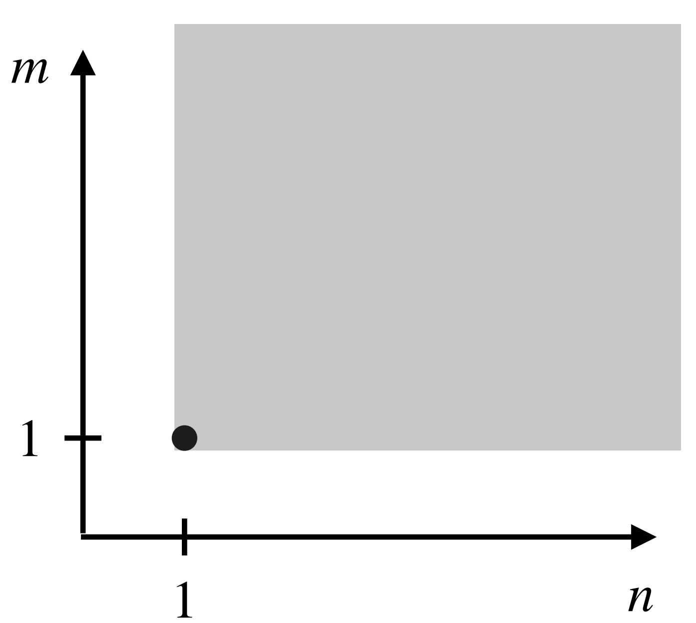
Integration By Parts - Examples

Integral family $$I_{m,n} = \int_0^\infty dx \frac{1}{(x+a)^m}\frac{1}{(x+b)^n}$$

Integration-by-parts identity $$I_{m,n} = \begin{cases} -\dfrac{1}{(n-1)(a-b)}\left[-\dfrac{1}{a^{m-1}b^{n-1}} + (m+n-2)\ I_{m,n-1}\right] \\ -\dfrac{1}{(m-1)(a-b)}\left[-\dfrac{1}{a^{m-1}b^{n-1}} + (m+n-2)\ I_{m-1,n}\right] \end{cases}$$

Master Integral $$I_{1,1} = \int_0^\infty dx \frac{1}{(x+a)}\frac{1}{(x+b)}$$

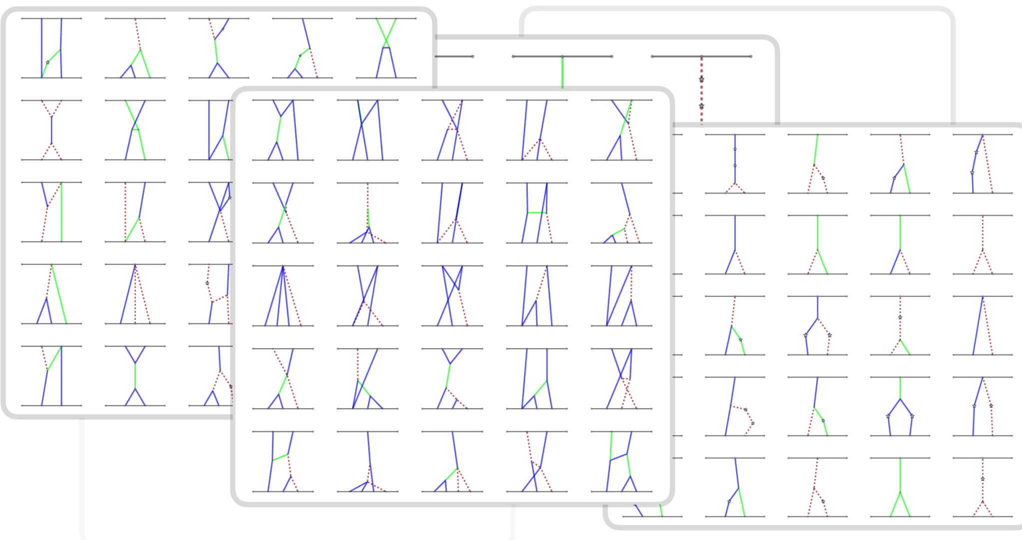
Differential equation $$\frac{\partial}{\partial a} I_{1,1} = \int_0^\infty dx \frac{-1}{(x+a)^2}\frac{1}{(x+b)} = -\,I_{2,1}$$
$$= \frac{1}{a-b}\left[-\frac{1}{a} + I_{1,1}\right]$$



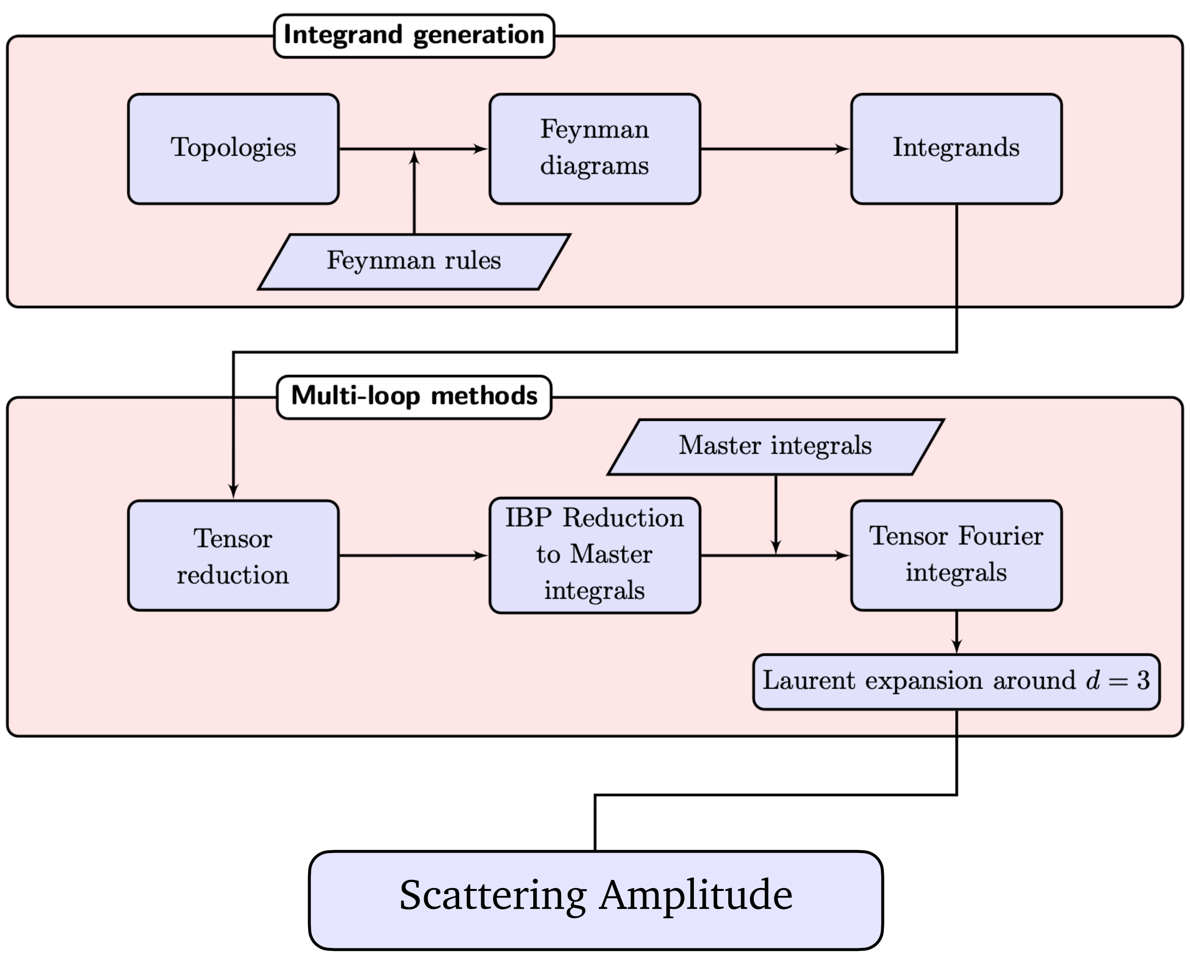
Solution $$I_{1,1} = \frac{-c + \log(a)}{a-b}$$

Boundary condition $$I_{1,1}^{a=b} = \int_0^\infty dx \frac{1}{(x+b)^2} = \boxed{\int_b^\infty dy \frac{1}{y^2} = \frac{1}{b}} \implies c = \log(b)$$

In-house Mathematica code



[Mandal, Mastrolia, **RP**, Steinhoff]



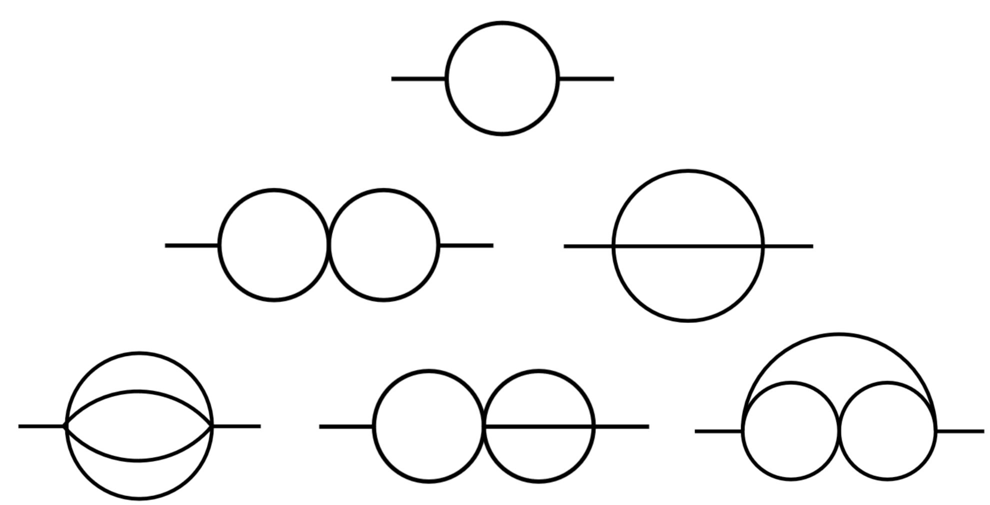
$$\mathcal{FD}^{(l)}_{\{g\}} = \underbrace{N_{\mathrm{C}}^{\mu_1,\mu_2,\cdots}(x_{(a)},S_{(a)})}_{\text{Coefficient that depends on orbital variables}} \underbrace{\int_p e^{i p_\mu x^\mu_{(12)}} N_{\mathrm{F}}^{\alpha_1,\alpha_2,\cdots}(p)}_{\text{Fourier integral}} \underbrace{\prod_{i=1}^{l}\int_{k_i} \frac{N_{\mathrm{M}}^{\nu_1,\nu_2,\cdots}(k_i)}{\prod_{\sigma\in g} D_\sigma(p,k_i)}}_{\text{Multi-loop integral}}$$

Computing Feynman diagrams

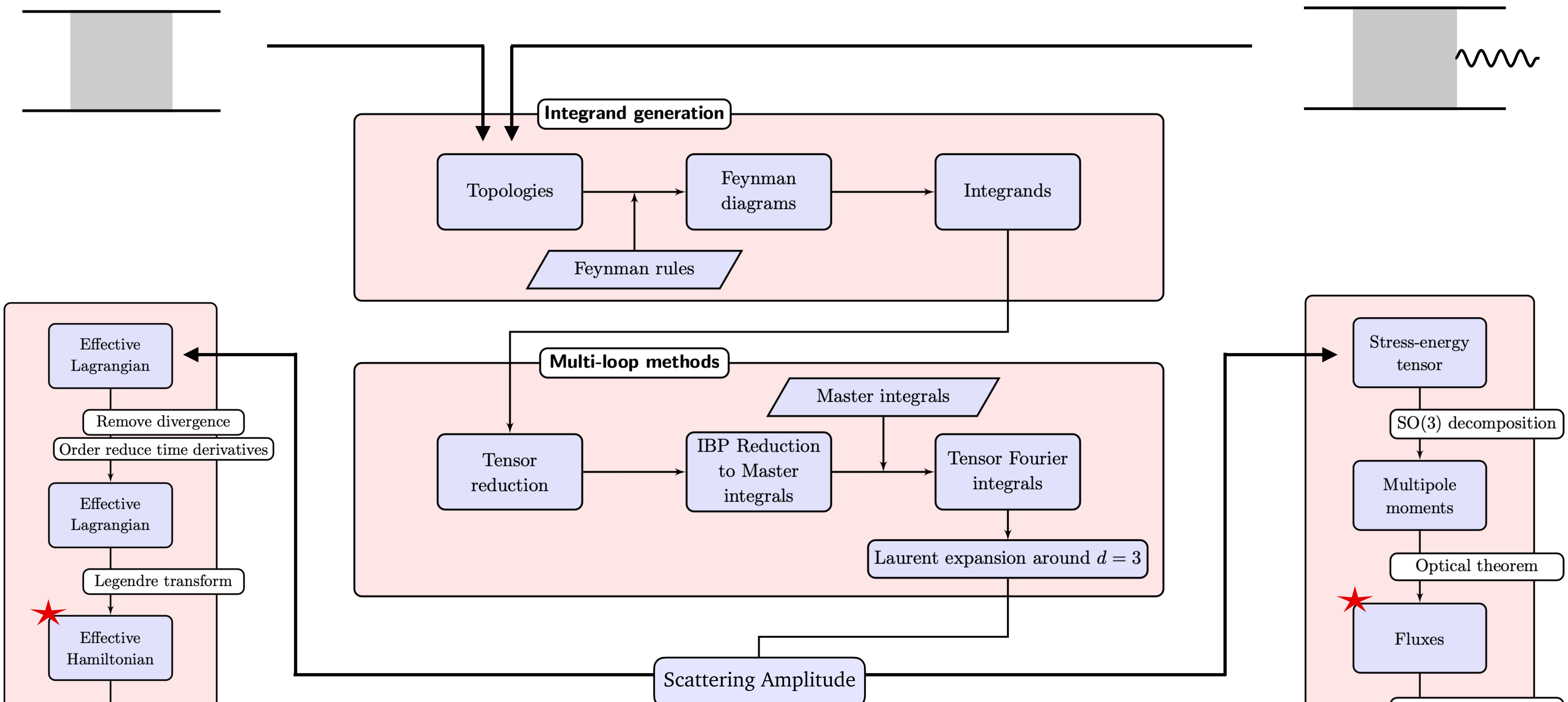
- Diagrams with QGRAF,
- Tensor Algebra with xTensor,
- IBPs with LiteRed/KIRA/FIRE,
- Can compute up to 3 loops or up to G_N^4

  (Prelimenary results at 4 loops or upto G_N^5)

Master Integrals



In-house Mathematica code



Very flexible to add more degrees of freedom (spin, tides, beyond GR, enviornmental effects, . . .)

★ = generic orbits

$$E_{\rm pp}(x) = -x\frac{1}{2} + x^2\left\{\frac{3}{8} + \frac{\nu}{24}\right\} + x^3\left\{\frac{27}{16} - \frac{19}{16}\nu + \frac{1}{48}\nu^2\right\} + x^4\left\{\frac{675}{128} + \left(-\frac{34445}{1152} + \frac{205\pi^2}{192}\right)\nu + \frac{155}{192}\nu^2 + \frac{35}{10368}\nu^3\right\} + \mathcal{O}(x^5)$$

$$\mathcal{F}_E^{\rm pp} = \left\{\frac{32\nu^2}{5}\right\}x^5 + \left\{-\frac{56\nu^3}{3} - \frac{2494\nu^2}{105}\right\}x^6 + 4\pi\left\{\frac{32\nu^2}{5}\right\}x^{3/2} + \left\{\frac{208\nu^4}{9} + \frac{37084\nu^3}{315} - \frac{89422\nu^2}{2835}\right\}x^7 + \mathcal{O}\left(x^{15/2}\right),$$

Hamiltonians - NNNLO spin

$$\int dt \left\{ m\sqrt{g^L_{\mu\nu}u^\mu u^\nu} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \cdots \right\}$$

PN orders	0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	...
Point particle	Newtonian	1PN		2PN		3PN		4PN + LO (Her)		5PN + NLO (Her)		6PN	
Spin-orbit			LO-SO		NLO-SO		NNLO-SO		N^3LO-SO		N^4LO-SO + LO (Her)		
Spin^2				LO-S^2		NLO-S^2		NNLO-S^2		N^3LO-S^2		N^4LO-S^2 + LO (Her)	
Spin^3							LO-S^3		NLO-S^3		NNLO-S^3		
Spin^4								LO-S^4		NLO-S^4		NNLO-S^4	
Spin^5											LO-S^5		

Tree
1 Loop
2 Loop
3 Loop
4 Loop
5 Loop

➡ $C^{(0)}_{\mathrm{ES}^2} = 1$ for Kerr BHs.

$C^{(2)}_{\mathrm{E}^2}$ and $C^{(0)}_{\mathrm{E}^2\mathrm{S}^2}$ are yet unknown for Kerr BHs

Spin-orbit coupling at 4.5PN: $\left(S_{(a)} \cdot L\right)$

arXiv:2209.00611 [Mandal, Mastrolia, **RP**, Steinhoff (2022)]

- ➡ Analogous to fine structure correction to Hydrogen atom
- ➡ 894 Feynman diagrams up to 3 loops

Quadratic-in-spin coupling at 5PN: $\left(S^2_{(a)}\right), \left(S_{(1)} \cdot S_{(2)}\right)$

arXiv:2210.09176 [Mandal, Mastrolia, **RP**, Steinhoff (2022)]

- ➡ Analogous to hyperfine structure correction to Hydrogen atom
- ➡ 723 Feynman diagrams up to 3 loops

Wilson coefficients

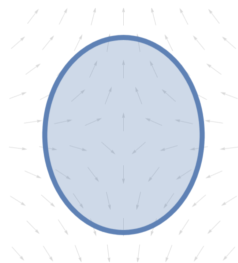
$$L^{(R,S^2)} = -\frac{1}{2mc}\left(C^{(0)}_{\mathrm{ES}^2}\right)\frac{E_{\mu\nu}}{u}\left[S^\mu S^\nu\right]_{\mathrm{STF}} + \cdots$$

$$L^{(R^2,S^0)} = \frac{1}{2}\left(C^{(2)}_{\mathrm{E}^2}\right)\frac{G_N^2 m}{c^5}\frac{E_{\mu\nu}E^{\mu\nu}}{u^3}S^2 + \cdots$$

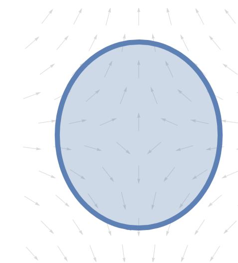
$$L^{(R^2,S^2)} = \frac{1}{2}\left(C^{(0)}_{\mathrm{E}^2\mathrm{S}^2}\right)\frac{G_N^2 m}{c^5}\frac{E_{\mu\alpha}E_\nu^{\ \alpha}}{u^3}\left[S^\mu S^\nu\right]_{\mathrm{STF}} + \cdots$$

Results also computed by [Kim, Levi, Yin (2022)] using EFTs, and [Antonelli, Kavanagh, Khalil, Steinhoff, Vines (2020)] using self-force

Hamiltonians - 3PN tides



Dynamic tides



Adiabatic tides

$$\mathscr{L}_{\mathrm{dy}} = \frac{z}{4\lambda\omega_f^2}\left[\frac{c^2}{z^2}\dot{Q}_{\mu\nu}\dot{Q}^{\mu\nu} - \omega_f^2 Q_{\mu\nu}Q^{\mu\nu}\right] - \frac{z}{2}Q^{\mu\nu}E_{\mu\nu} - \boxed{\kappa\frac{G_N^2 m^2}{c^6}\frac{z}{2}Q_{\mu\nu}\ddot{E}^{\mu\nu}}$$

$$\mathscr{L}_{\mathrm{ad}} = \lambda\frac{z}{4}E_{\mu\nu}E^{\mu\nu} - \boxed{\lambda\kappa\frac{G_N^2 m^2}{c^6}\frac{z}{2}\dot{E}_{\mu\nu}\dot{E}^{\mu\nu}}$$

Adiabatic tides

PN orders	5	6	7	8	9
$E_{\mu\nu}E^{\mu\nu}$	LO	NLO	NNLO	N^3LO	N^4LO + LO (Her)
$B_{\mu\nu}B^{\mu\nu}$		LO	NLO	NNLO	N^3LO
$\nabla_\alpha E_{\mu\nu}\nabla^\alpha E^{\mu\nu}$			LO	NLO	NNLO
$\dot{E}_{\mu\nu}\dot{E}^{\mu\nu}$				LO	NLO

- Tree
- 1 Loop
- 2 Loop
- 3 Loop
- 4 Loop
- 5 Loop

Dynamic tides

PN orders	0	1	2	3	4
$Q_{\mu\nu}E^{\mu\nu}$	LO	NLO	NNLO	N^3LO	N^4LO + LO (Her)
$Q_{\mu\nu}\dot{B}^{\mu\nu}$		LO	NLO	NNLO	N^3LO
$Q_{\mu\nu}\ddot{E}^{\mu\nu}$				LO	

Adiabatic and Dynamic tides up to 3PN

arXiv:2304.02030 [Mandal, Mastrolia, Silva, **RP**, Steinhoff (2023)]
arXiv:2308.01865 [Mandal, Mastrolia, Silva, **RP**, Steinhoff (2023)]

- Modelling of mode oscillation of NS
- 290 Feynman diagrams up to 3 loops

Requires renormalisation!!

$$\beta(\kappa) \equiv R\frac{d\kappa}{dR} = -\frac{214}{105}$$

$$\kappa(R) = \kappa(R_0) - \frac{214}{105}\log\left(\frac{R}{R_0}\right)$$

Fluxes and waveform - 2PN tides

Energy and Angular momentum flux:

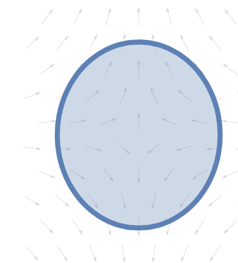
$$
\begin{aligned}
\mathcal{F}_E^{\mathrm{AT}} =& \left\{ \left(\frac{768\nu^2}{5} + \frac{192\nu}{5} \right) \widetilde{\lambda}_{(+)} + \frac{192}{5} \delta\widetilde{\lambda}_{(-)} \right\} x^{10} \\
&+ \left\{ \left(-992\nu^3 - \frac{9736\nu^2}{35} - \frac{1408\nu}{35} \right) \widetilde{\lambda}_{(+)} + \left(-\frac{184\nu^2}{5} - \frac{1408\nu}{35} \right) \delta\widetilde{\lambda}_{(-)} \right\} x^{11} \\
&+ 4\pi \left\{ \left(\frac{768\nu^2}{5} + \frac{192\nu}{5} \right) \widetilde{\lambda}_{(+)} + \frac{192}{5} \delta\widetilde{\lambda}_{(-)} \right\} x^{23/2} \\
&+ \left\{ \left(3088\nu^4 + \frac{63692\nu^3}{35} - \frac{1299706\nu^2}{945} + \frac{5344\nu}{45} \right) \widetilde{\lambda}_{(+)} \right. \\
&\quad \left. + \left(-\frac{11116\nu^3}{15} + \frac{149566\nu^2}{105} + \frac{5344\nu}{45} \right) \delta\widetilde{\lambda}_{(-)} \right\} x^{12} + \mathcal{O}\left(x^{25/2} \right) .
\end{aligned}
$$

Gravitational waveform phase:

$$
\begin{aligned}
\phi^{\mathrm{AT}} = -\frac{3x^{5/2}}{16\nu^2} \Bigg[& \left\{ (1+22\nu)\widetilde{\lambda}_{(+)} + \delta\widetilde{\lambda}_{(-)} \right\} + x \left\{ \left(\frac{195}{56} + \frac{1595\nu}{14} + \frac{325\nu^2}{42} \right) \widetilde{\lambda}_{(+)} + \left(\frac{195}{56} + \frac{4415\nu}{168} \right) \delta\widetilde{\lambda}_{(-)} \right\} \\
&+ x^{3/2} \left\{ -\frac{5\pi}{2}(1+22\nu)\widetilde{\lambda}_{(+)} - \frac{5\pi}{2}\delta\widetilde{\lambda}_{(-)} \right\} \\
&+ x^2 \left\{ \left(\frac{136190135}{9144576} + \frac{978554825\nu}{1524096} - \frac{281935\nu^2}{2016} + 5\nu^3 \right) \widetilde{\lambda}_{(+)} \right. \\
&\quad \left. + \left(\frac{136190135}{9144576} + \frac{213905\nu}{864} + \frac{1585\nu^2}{432} \right) \delta\widetilde{\lambda}_{(-)} \right\} + \mathcal{O}\left(x^{5/2} \right) \Bigg]
\end{aligned}
$$

Gravitational waveform modes:

$$
\begin{aligned}
\mathcal{H}_{22}^{\mathrm{AT}} =& \left\{ (12\nu+3)\,\widetilde{\lambda}_{(+)} + 3\delta\widetilde{\lambda}_{(-)} \right\} + x \left\{ \left(\frac{45\nu^2}{7} - 20\nu + \frac{9}{2} \right) \widetilde{\lambda}_{(+)} + \left(\frac{125\nu}{7} + \frac{9}{2} \right) \delta\widetilde{\lambda}_{(-)} \right\} \\
&+ x^{3/2} \left\{ (24\nu+6)\,\pi\widetilde{\lambda}_{(+)} + 6\pi\delta\widetilde{\lambda}_{(-)} \right\} \\
&+ x^2 \left\{ \left(-\frac{274\nu^3}{21} - \frac{19367\nu^2}{168} - \frac{7211\nu}{168} + \frac{1403}{56} \right) \widetilde{\lambda}_{(+)} + \left(\frac{103\nu^2}{24} + \frac{1559\nu}{56} + \frac{1403}{56} \right) \delta\widetilde{\lambda}_{(-)} \right\} + \mathcal{O}\left(x^{5/2} \right)
\end{aligned}
$$



Adiabatic tides

$$
\mathscr{L}_{\mathrm{ad}} = \lambda \frac{z}{4} E_{\mu\nu} E^{\mu\nu} - \lambda\kappa \frac{G_N^2 m^2}{c^6} \frac{z}{2} \dot{E}_{\mu\nu} \dot{E}^{\mu\nu}
$$

Adiabatic tides up to 2PN

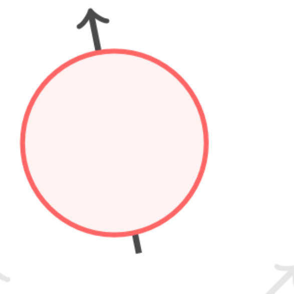
arXiv:2412.01706 [Mandal, Mastrolia, **RP**, Steinhoff (2024)]

- 85 Feynman diagrams up to 2 loops
- Corrected the previous result of [Henry, Faye, and Blanchet (2020)]

PN orders	0	0.5	1	1.5	2	2.5	3
$E_{\mu\nu}E^{\mu\nu}$	LO		NLO	LO (Her)	NNLO	NLO (Her)	N^3LO +NNLO (Her)
$B_{\mu\nu}B^{\mu\nu}$			LO		NLO	LO (Her)	NNLO
$\nabla_\alpha E_{\mu\nu}\nabla^\alpha E^{\mu\nu}$					LO		NLO
$\dot{E}_{\mu\nu}\dot{E}^{\mu\nu}$							LO

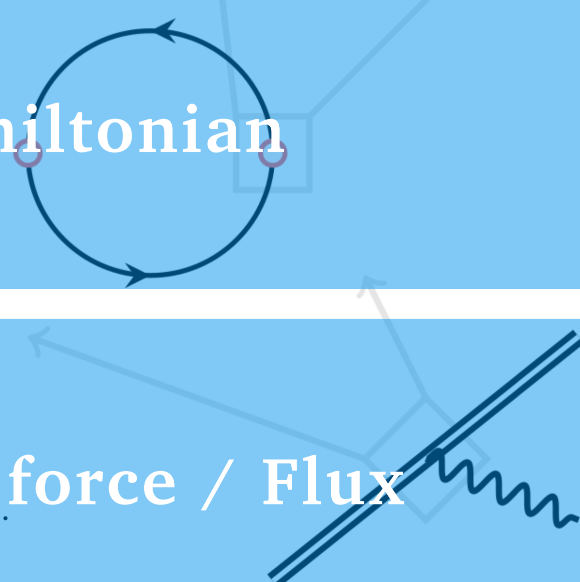
Tree
1 Loop
2 Loop

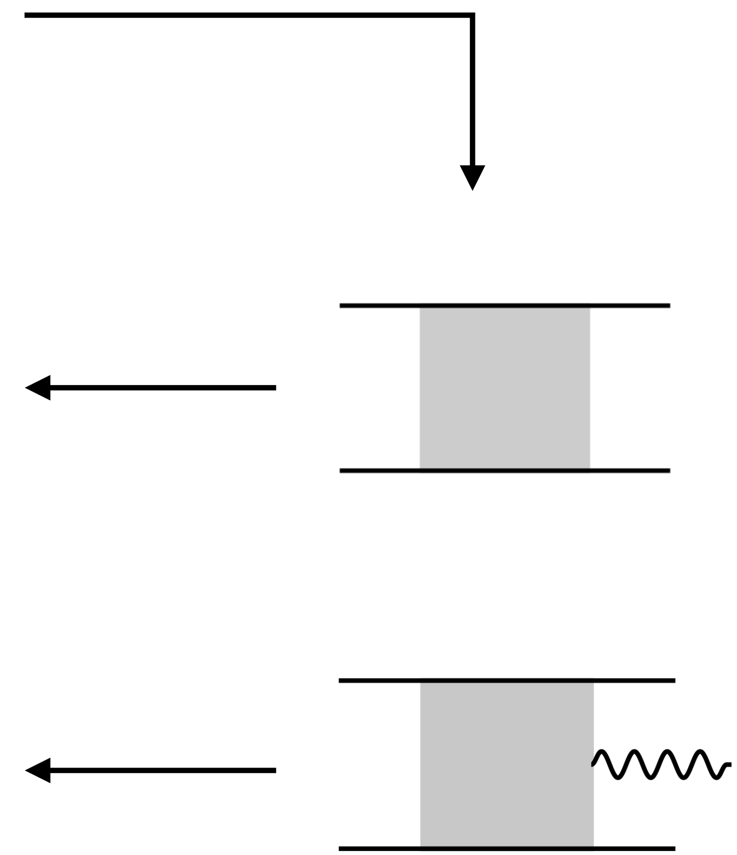
Summary - PN pipeline



EFT around a point particle for compact objects with internal structure

$$\underbrace{m\sqrt{u^2}}_{\text{pp}} + \underbrace{\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu}}_{\text{spin}} + \underbrace{c_{E^2}\,E_{\mu\nu}E^{\mu\nu}}_{\text{tides}} + \cdots$$





Two point particle EFT to describe a bound state like binary at scales r

Computing Hamiltonian

$$\sum_{a=1,2}\frac{1}{2}m_a\mathbf{v}_a^2 + \frac{G_N m_1 m_2}{|\mathbf{r}|} + 1\text{PN} + 2\text{PN} + \cdots$$

EFT of **multipole moments** to describe the binary at scale λ

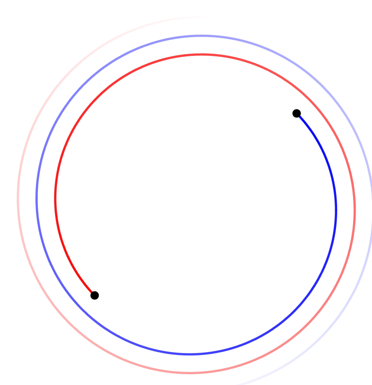
Radiation reaction force / Flux

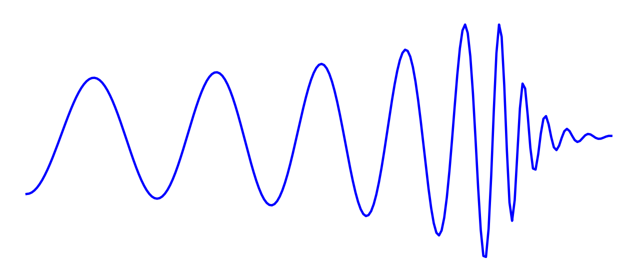
$$\underbrace{M}_{\substack{\text{total}\\\text{mass}}}\,\bar{h}_{00} + \underbrace{\mathbf{L}^i}_{\substack{\text{angular}\\\text{mom.}}}\,\epsilon_{ijk}\partial_j h_{0k} + \underbrace{I^{ij}}_{\substack{\text{mass}\\\text{quad.}}}\,E_{ij} + \underbrace{J^{ij}}_{\substack{\text{current}\\\text{quad.}}}\,B_{ij} + \cdots$$

Equation of motion

$$\dot{r} = \frac{dH}{dp_r} \qquad \dot{p}_r = -\frac{dH}{dr} + F_r$$

$$\dot{\phi} = \frac{dH}{dp_\phi} \qquad \dot{p}_\phi = -\frac{dH}{d\phi} + F_\phi$$





Summary - PN pipeline

EFT around a point particle for compact objects

State-of-the-art results!

Spin-orbit Hamiltonian at 4.5PN (NNNLO): $\left(S_{(a)} \cdot L\right)$

arXiv:2209.00611 [Mandal, Mastrolia, **RP**, Steinhoff (2022)]

Quadratic-in-spin Hamiltonian at 5PN (NNNLO): $\left(S^2_{(a)}\right), \left(S_{(1)} \cdot S_{(2)}\right)$

arXiv:2210.09176 [Mandal, Mastrolia, **RP**, Steinhoff (2022)]

Adiabatic and Dynamic tides Hamiltonian up to 3PN:

arXiv:2304.02030 [Mandal, Mastrolia, Silva, **RP**, Steinhoff (2023)]
arXiv:2308.01865 [Mandal, Mastrolia, Silva, **RP**, Steinhoff (2023)]

Adiabatic tides Fluxes and modes up to 2PN: arXiv:2412.01706 [Mandal, Mastrolia, **RP**, Steinhoff (2024)]

In house Mathematica code

Can compute 4 point and 5 point diagrams

Can compute up to 3 loops

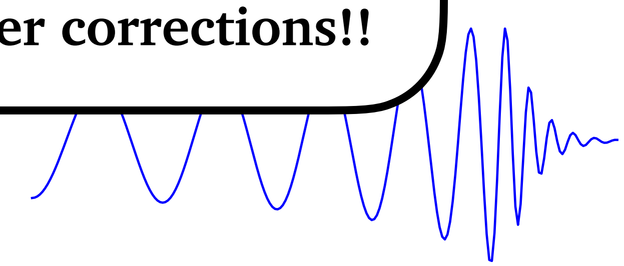
Very flexible to add more degrees of freedom

Renormalisation of post-adiabatic Love number!!

$$\beta(\kappa) \equiv R\,\frac{d\kappa}{dR} = -\frac{214}{105}$$

Automatic computational framework of QFT/EFT prove very effective in going to higher order corrections!!

Equa... of motion $\quad \dot{\phi} = \frac{dH}{dp_\phi} \qquad \dot{p}_\phi = -\frac{dH}{d\phi} + F_\phi$



Thank you