

# Phase Transitions and Mittag-Leffler Functions for Critical Schemes Under the Gibbs Model

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Composition schemes are ubiquitous in combinatorics, number theory, statistical mechanics and probability theory.

We give a unifying explanation to various phenomena observed in the combinatorial and statistical physics literature in the context of  $q$ -enumeration (models where objects with a parameter of value  $k$  have a Gibbs measure/Boltzmann weight  $q^k$ ).

For structures enumerated by a composition scheme, we prove a phase transition for any parameter having such a Gibbs measure: for a critical value  $q = q_c$ , the limit law of the parameter is a two-parameter Mittag-Leffler distribution, while it is Gaussian in the supercritical regime ( $q > q_c$ ), and it is a Boltzmann distribution in the subcritical regime ( $0 < q < q_c$ ). We apply our results to fundamental statistics of lattice paths and quarter-plane walks.

We also explain previously observed limit laws for pattern-restricted permutations, and a phenomenon uncovered by Krattenthaler for the wall contacts in watermelons.

(Based on the article <https://arxiv.org/abs/2311.17226> by Cyril Banderier, Markus Kuba, Stephan Wagner, Michael Wallner).

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