ID de Contribution: **5** Type: **Non spécifié**

The Wronskians over Multidimension and Homotopy Lie Algebras

mercredi 19 novembre 2025 14:50 (50 minutes)

The Wronskian determinant of N functions in one variable x helps us verify that they are linearly independent on an interval (a, b) in $\mathbb{R}$. Can we have an equally convenient procedure over multidimensional spaces $\mathbb{R}^d$ with Cartesian coordinates $x, y, z, \ldots$? Yes we can; let us study the definition of Wronskians for functions in many variables, and let us explore which differential-algebraic identities these structures satisfy.
To see why the Wronskian determinants actually do satisfy a set of quadratic, Jacobi-type identities, we observe first that the Wronskian of size 2×2 results from commutation of vector fields on the real line $\mathbb{R}$. From differential geometry we know that vector fields are differential operators of strict order $p = 1$. By taking the alternated composition of $N = 2p$ differential operators of strict order $p > 0$ on the affine line, we obtain the operator of same order p with the Wronskian determinant for coefficient. As we had the Jacobi identity for the Lie algebra of vector fields, so we establish the (table of) quadratic, Jacobi-type identities for higher-order Wronskians of $N > 1$ arguments. (In string theory, these identities govern homotopy deformations of Lie algebras, here of vector fields.) We prove that the new Wronskians over multidimensional base with d coordinates $x, y, z, \ldots$ do satisfy the (table of) identities for strongly homotopy Lie algebras.
The problem is to understand how fast the dimension grows under iterated N-ary brackets. We spot a countable chain of finite-dimensional homotopy Lie algebras that generalize the vector field realization of $sl(2)$ on $\mathbb{R}$; we explicitly calculate all the structure constants. Yet the four-dimensional analogue of $sl(2)$ over the plane $\mathbb{R}^2$ with Cartesian coordinates (x, y), now with ternary bracket from the Wronskian, is so far the only known finite-dimensional homotopy Lie algebra of this type over base dimension > 1. The hunt is on; we conclude that Lie algebra $sl(2)$ is the prototype not only for semisimple complex Lie algebras encoded by root systems but also for countably many N-ary homotopy Lie algebras.

https://arxiv.org/abs/2511.03848

https://arxiv.org/abs/2510.02145

https://arxiv.org/abs/math/0410185

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