Bulk realisation of anisotropic conformal Carroll symmetries

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In this work, we provide a realisation of infinite-dimensional **anisotropic conformal Carroll algebras** as the **asymptotic symmetries of plane wave spacetimes**.

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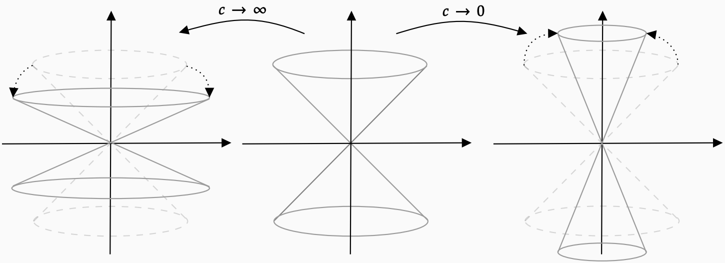
Introduction

A funky limit of the Poincaré algebra

In the $c \to \infty$ limit, the Poincaré group is contracted to the Galilean group. In the $c \to 0$ limit, a group contraction results in the **Carroll algebra** [Lévy-Leblond, 1965; Sen Gupta, 1966; Bacry, Lévy-Leblond, 1968] :

$$[C_a, P_b] = \delta_{ab} H,\ [J_{ab}, P_c] = 2\delta_{c[b} P_{a]},\ [J_{ab}, J_{cd}] = 4\delta_{[a[c} J_{d]b]}, [C_a, J_{bc}] = 2\delta_{a[b} C_{c]}.$$

In this limit, the **light cone closes**:



Geometric realisation in Carroll structures

The Carroll algebra appears geometrically as the isometries of **flat Carroll manifolds**, smooth manifolds endowed with a twice-symmetric and covariant two-tensor h whose kernel is generated by a non-vanishing vector field τ [Henneaux, 1979]:

$$h_{\mu\nu} = (0, \delta_{ij}), \quad \tau^{\mu} = (1, 0, .., 0).$$

This is usually considered as the **weak** definition of a Carroll structure. A **strong** definition requires an affine connection ∇ preserving the weak triplet $(\mathcal{M}, h, \tau)$ [Duval, Gibbons, Horváthy, Zhang, 2014].

In the strong case, the isometries are the $c \to 0$ limit of Poincaré described earlier.

In the weak case, the Carroll algebra is supplemented with an infinite set of **supertranslations**:

$$x_i \to x_i' = x_i, \qquad t \to t' = t + f(x_i).$$

Conformal extensions of the Carroll algebra

Non-relativistic symmetries **decouple space and time**. Conformal extensions of non-relativistic transformations thus allow for **anisotropic scale transformations**:

$$t \to \lambda^z t, \quad x \to \lambda x.$$

At the level of the Carroll structure, this is implemented by

$$\mathcal{L}_\xi h_{\mu\nu} = \lambda(x) h_{\mu\nu}, \qquad \mathcal{L}_\xi \tau^\mu = -\frac{\lambda(x)}{k}\tau^\mu,$$

with k the **level**, and $z = 2/k$ the **scaling exponent** [Duval, Gibbons, Horváthy, 2014].

The diffeomorphisms ξ solving these equations *for a given* k generate the $z = 2/k$ **conformal Carroll algebra,** $\mathfrak{ccarr}_z$.

Conformal extensions of the Carroll algebra

In particular, we will be interested in the $0 \leq |z| < +\infty$ cases in $d = 2, 3$ [Afshar, Bekaert, Najafizadeh, 2024].

- In $d = 2$,

$$L_n = -z(n+1)x^n t\partial_t - x^{n+1}\partial_x, \quad M_r = x^r \partial_t,$$

with

$$[L_n, L_m] = (n-m)L_{m+n}, \quad [L_n, M_r] = ((n+1)z - r)\, M_{r+n,s}.$$

Conformal extensions of the Carroll algebra

In particular, we will be interested in the $0 \leq |z| < +\infty$ cases in $d = 2, 3$ [Afshar, Bekaert, Najafizadeh, 2024].

- In $d = 3$, with complex coordinates $w = x + iy$,

$$L_n = -\frac{z}{2}(n+1)w^n t\partial_t - w^{n+1}\partial_w, \qquad \bar{L}_n = -\frac{z}{2}(n+1)\bar{w}^n t\partial_t - \bar{w}^{n+1}\partial_{\bar{w}},$$
$$M_{rs} = w^r \bar{w}^s \partial_t,$$

with

$$[L_n, L_m] = (n-m)L_{m+n}, \quad [\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{m+n}$$
$$[L_n, M_{rs}] = \left(\frac{z}{2}(n+1) - r\right) M_{r+n,s}, \quad [\bar{L}_n, M_{rs}] = \left(\frac{z}{2}(n+1) - s\right) M_{r,s+n}.$$

A few special cases

For $z = 1$, in $d = 2$ and 3, we find the $\mathfrak{bms}_3$ **and** $\mathfrak{bms}_4$ **algebras**, respectively.

- The $d = 2$, $z = 0$ case corresponds to the symmetries of **warped CFTs** [Detournay, Hartman, Hofman, 2012]
- The $d = 2, 3$, $z = 0$ symmetries appeared in **near-horizon region of black holes** in [Donnay, Giribet, Gonzalez, Pino; Afshar, 2016] and subsequently for any z in $d = 3$ in [Grumiller, 2019].

Remarkable conformal Carroll subalgebras

For future reference, we mention **minimal conformal extensions** of the Carroll algebra [Afshar, Bekaert, Najafizadeh, 2024].

In $d = 2$, the Carroll algebra is generated by $\{L_{-1}, M_0, M_1\}$:

$$L_{-1} = -\partial_x, \quad M_0 = \partial_t, \quad M_1 = x\partial_t.$$

- **Type D conformal Carroll** (*scalar* Carroll): $L_0 = -zt\partial_t - x\partial_x$ (generates a z−dilation)
- **Type K conformal Carroll**: $M_2 = x^2\partial_t$ (temporal SCT).
- **Type $D-K$ conformal Carroll** (*conformal* Carroll): L_0, M_2.

Remarkable conformal Carroll subalgebras

For future reference, we mention **minimal conformal extensions** of the Carroll algebra [Afshar, Bekaert, Najafizadeh, 2024].

In $d = 3$, the Carroll algebra is generated by $\{L_{-1}, L_{-1}, L_0 - \bar{L}_0, M_{00}, M_{01}, M_{10}\}$:

$$\begin{gathered} L_{-1} = -\partial_w,\ \bar{L}_{-1} = -\partial_{\bar{w}},\ M_{00} = \partial_t, \\ M_{10} = w\partial_t,\ M_{01} = \bar{w}\partial_t, L_0 - \bar{L}_0 = -(w\partial_w - \bar{w}\partial_{\bar{w}}). \end{gathered}.$$

- **Type** D **conformal Carroll** (*scalar* Carroll): $L_0 + L_0 = -zt\partial_t - (w\partial_w + \bar{w}\partial_{\bar{w}})$ (generates a $z-$dilation).
- **Type** K **conformal Carroll**: $M_{11} = w\bar{w}\partial_t$ (temporal SCT).
- **Type** $D - K$**-type conformal Carroll** (*conformal* Carroll): $L_0 + \bar{L}_0, M_{11}$.

Plane wave spacetimes

Plane waves are a subclass of **pp-wave spacetimes**, geometries that allow for a null, nowhere-vanishing and covariantly constant vector field [Blau, 2011].
In d dimensions, two usual coordinate systems:

- **Rosen coordinates**:

$$ds^2 = 2dudv + C_{ij}(u)dy^i dy^j,$$

  $i, j = 1, .., d-2$.

- **Brinkmann coordinates**:

$$ds^2 = 2dx^+ dx^- + A_{ab}(x^+)x^a x^b (dx^+)^2 + d\vec{x}^2,$$

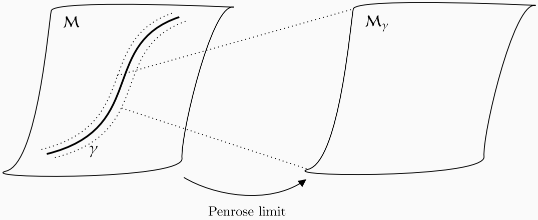
  $a, b = 1, .., d-2$.

Plane waves as the geometry of null geodesics

Plane waves are the generic result of the **Penrose limit** of any spacetime [Penrose, 1976]. Roughly:

- Take any null geodesic γ on a smooth Lorentzian manifold $\mathcal{M}$.
- Perform a succession of **rescalings on the coordinates and the metric** and take a limit for the **scale parameter**.
- Identically, can be understood covariantly as a null Fermi coordinate expansion on the metric [Blau, Frank, Weiss, 2006]

The result is a **blown-up infinitesimal neighbourhood of the geodesic.**



Isometries

We focus on $d = 4$. Plane waves always have **at least five isometries**.
In Rosen coordinates, they act as

$$u \to u, \quad y^i \to y^i + H_{ij}(u)b^j + c^i, \quad v \to v - b^j y_j - \frac{1}{2} b^j H_{ij}(u) b_j + f.$$

It was shown that these five isometries form the **Carroll group in three dimensions without rotations** [Duval, Gibbons, Horváthy, Zhang, 2017].

This number can be enhanced:

- to **six** if the plane wave is conformally flat: $A_{ab}(x^+) = \delta_{ab} A(x^+)$, which **restores the Carroll rotation**,
- to **seven** if. . . ?

What plane waves are preserved by conformal extensions of the Carroll algebra?

Plane waves as backgrounds for physical theories

Plane waves are solution of the vacuum Einstein equations **iff the Brinkmann wave profile is traceless**: the isometry algebra of Einstein plane waves **cannot contain the full Carroll algebra**.

However, plane waves are **string theory backgrounds** [Amati, Climcik, 1989; Horowitz, Steif; 1990]: for $C_{ij}(u) = \delta_{ij} C(u)$, the equations of motion of the low-energy effective action

$$\mathcal{S} = \int d^4x \sqrt{-g}\, e^{-2\phi} \left(R - \frac{1}{12} H^2 + 4 \nabla_\mu \phi \nabla^\mu \phi \right)$$

reduce to

$$R_{uu}(u) - \frac{1}{2} (C(u))^{-2} H^2_{uxy}(u) + 2\phi''(u) = 0.$$

Plane waves are **exact string theory backgrounds**: α' corrections vanish for the same reason as plane wave curvature invariants.

Plane waves in the literature

Plane waves sit at the intersection of many areas of high-energy physics:

- Provided the first test of the AdS/CFT correspondence beyond the supergravity approximation (BMN limit) [Berenstein, Maldacena, Nastase, 2002; Sadri, Sheikh-Jabbari, 2003] .
- Relation to Carroll symmetry [Despontin, Detournay, **DF**, 2025]
- Extensions of **infrared triangle physics** to non-trivial curved spacetimes [Ben Achour, Uzan, 2024; Klisch, 2025].
- **Quasinormal mode** spectrum of black holes and **photon ring holography** through the Penrose limit [Hadar, Kapec, Lupsasca, Strominger, 2022; Fransen, 2023; Kapec, Sheta, 2024].
- Plane wave **holography**?

Bulk realisation of conformal Carroll symmetries

The $z = 0$ conformal Carroll algebra

We begin with the $z = 0$ **conformal Carroll algebra**. In this $(k \to \infty)$ limit, scale transformations yield

$$t \to t, \quad x^i \to \lambda x^i.$$

The $z = 0$ conformal Carroll algebra is given by

$$L_n = -x^{n+1}\partial_x,\ M_r = x^r\partial_t, \quad [L_n, L_m] = (n-m)L_{m+n},\ \ [L_n, M_r] = -rM_{n+r}$$

in $d = 2$, and

$$\begin{aligned} &L_n = -w^{n+1}\partial_w,\ \bar{L}_n = -\bar{w}^{n+1}\partial_{\bar{w}},\ M_{rs} = w^r\bar{w}^s\partial_t, \\ &[L_n, L_m] = (n-m)L_{m+n},\ [\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{m+n}, \\ &[L_n, M_{rs}] = -rM_{n+r,s},\ [\bar{L}_n, M_{rs}] = -sM_{r,n+s} \end{aligned}$$

in $d = 3$.

A $z = 0$ plane wave

We now consider the plane wave

$$
\begin{aligned}
ds^2 &= 2dudv + e^u(dx^2 + dy^2) && \text{(Rosen)} \\
&= 2dx^+dx^- + \frac{1}{4}(\vec{x})^2(dx^+)^2 + (d\vec{x})^2 && \text{(Brinkmann).}
\end{aligned}
$$

In $d = 3$, this plane wave is the Penrose limit of self-dual warped AdS_3. In $d = 4$, this is the Penrose limit of $\mathrm{dS}^2 \times \mathbb{H}^2$. It is also a double analytic continuation of the Nappi-Witten spacetime.

This constitutes a string theory background with a **non-trivial dilaton**.

Isometries

We define complex coordinates

$$w = x + iy, \qquad \bar{w} = x - iy.$$

The isometries of this plane wave are generated by

$$\begin{aligned} L_0 &= \partial_u - w\partial_w, & \bar{L}_0 &= \partial_u - \bar{w}\partial_{\bar{w}}, \\ L_{-1} &= -\partial_w, & \bar{L}_{-1} &= -\partial_{\bar{w}}, \\ M_{10} &= w\partial_v + 2e^{-u}\partial_{\bar{w}}, & M_{01} &= \bar{w}\partial_v + 2e^{-u}\partial_w, \\ M_{00} &= \partial_v. \end{aligned}$$

The isometry algebra is **isomorphic to the type-D, $z = 0$ conformal Carrol extension in $d = 3$.**

Isometries

$$
\begin{aligned}
L_0 &= \partial_u - w\partial_w\,, & \bar{L}_0 &= \partial_u - \bar{w}\partial_{\bar{w}}\,,\\
L_{-1} &= -\partial_w\,, & \bar{L}_{-1} &= -\partial_{\bar{w}}\,,\\
M_{10} &= w\partial_v + 2e^{-u}\partial_{\bar{w}}\,, & M_{01} &= \bar{w}\partial_v + 2e^{-u}\partial_w\,,\\
M_{00} &= \partial_v\,.
\end{aligned}
$$

Fixing $u \to +\infty$, the algebra generators are projected on the exact corresponding type-D vector fields, with v **identified to the Carrollian time** :

$$
\begin{aligned}
&L_0 \overset{u\to+\infty}{\longrightarrow} -w\partial_w\,, && \bar{L}_0 \overset{u\to+\infty}{\longrightarrow} -\bar{w}\partial_{\bar{w}}\,,\\
&L_{-1} \overset{u\to+\infty}{\longrightarrow} -\partial_w\,, && \bar{L}_{-1} \overset{u\to+\infty}{\longrightarrow} -\partial_{\bar{w}}\,,\\
&M_{10} \overset{u\to+\infty}{\longrightarrow} w\partial_v\,, \quad M_{01} \overset{u\to+\infty}{\longrightarrow} \bar{w}\partial_v\,, && M_{00} \overset{u\to+\infty}{\longrightarrow} \partial_v\,.
\end{aligned}
$$

Asymptotic symmetries: $z = 0$ conf. Carroll algebra

Consider the space of geometries

$$ds^2 = 2dudv + A_a(v,x^b)dudx^a + B_a(v,x^b)dvdx^a + \left(H(v,x^a)dw^2 + \bar{H}(v,x^a)d\bar{w}^2\right) + \left(e^u + G(v,x^a)\right)dwd\bar{w}.$$

In the $u \to \infty$ region, this space of geometries is preserved by

$$\xi = -\left(\partial_w f(w) + \partial_{\bar{w}}\bar{f}(\bar{w})\right)\partial_u + \alpha(w,\bar{w})\partial_v + f(w)\partial_w + \bar{f}(w)\partial_{\bar{w}}.$$

Asymptotic symmetries: $z = 0$ conf. Carroll algebra

In the $u \to \infty$ region, the $z = 0$ space of geometries is preserved by

$$\xi = -\left(\partial_w f(w) + \partial_{\bar{w}} \bar{f}(\bar{w})\right) \partial_u + \alpha(w, \bar{w}) \partial_v + f(w) \partial_w + \bar{f}(w) \partial_{\bar{w}}.$$

Upon mode expansion,

$$L_n = (n+1) w^n \partial_u - w^{n+1} \partial_w, \quad L_n = (n+1) \bar{w}^n \partial_u - \bar{w}^{n+1} \partial_{\bar{w}}, \quad M_{rs} = w^r \bar{w}^s \partial_v.$$

These reproduce the $z = 0$ conf. Carroll algebra: bulk uplift of $\mathfrak{ccarr}_0(3)$!

The plane wave conformal boundary conundrum

The Penrose limit of $\mathrm{AdS}_p \times S^q$ is given by

$$ds^2 = 2dx^+dx^- - \mu^2(\vec{x})^2(dx^+)^2 + (d\vec{x})^2.$$

- Not an example of AdS holography: the **conformal boundary of AdS** is **not part of the plane wave**.
- Not an example of plane wave holography: the conformal boundary of the plane wave is a **one-dimensional null line** [Berenstein, Maldecena, Nastase, 2002; Marolf, 2002; Hubeny, Rangamani, 2003;...].

However, **not all plane waves** have such a conformal boundary.

The plane wave conformal boundary conundrum

The asymptotic region we have identified is $u \to +\infty$. On a **fixed u surface**, the geometry is that of a **three-dimensional Carroll structure**:

$$ds^2 = e^u dw\, d\bar{w}, \quad \tau = \partial_v .$$

With $\tilde{u} = -\tanh(x^+) = -\tanh(u)$, our plane wave becomes

$$ds^2 = \frac{1}{1-\tilde{u}^2}\left(2d\tilde{u}d\tilde{v} + d\tilde{x}^2 + d\tilde{y}^2\right).$$

This is **conformally equivalent to a slice of Minkowski space**. This indicates a **null, codimension-one conformal boundary at $u \to \infty$**.

Summary

So far:

- We have identified a plane wave with seven isometries.
- The isometry algebra is isomorphic to the $z = 0$, type-D Carroll algebra.
- In the $u \to +\infty$ limit, the isometries are projected on the exact type-D generators.
- The plane wave is the vacuum metric of a space of geometries that is preserved by the full $z = 0$ conformal Carroll algebra.
- The asymptotic region $u \to +\infty$ is the conformal boundary of the plane wave.

Holographic relationship between plane waves and $z = 0$ $\mathfrak{ccarr}$ field theories?

Can we extend this to other values of z?

A $z \neq 0$ family of plane waves

We consider the one-parameter family of plane waves

$$
\begin{aligned}
ds^2_{(k)} &= 2du dv + u^k dw\, d\bar{w} && \text{(Rosen)}, \\
&= 2dx^+ dx^- + \frac{k(k-2)}{4(x^+)^2}(\vec{x})^2 (dx^+)^2 + (\vec{x})^2 && \text{(Brinkmann)}.
\end{aligned}
$$

- In Brinkmann coordinates, obvious that $k = 0, 2$ **correspond to flat space**.
- The geometries are **reflective around the** $k = 1$ **axis**.

A $z \neq 0$ family of plane waves

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\begin{aligned}
ds^2_{(k)} &= 2dudv + u^k dw\, d\bar{w} && \text{(Rosen)}, \\
&= 2dx^+dx^- + \frac{k(k-2)}{4(x^+)^2}(\vec{x})^2(dx^+)^2 + (\vec{x})^2 && \text{(Brinkmann)}.
\end{aligned}
$$

These are **scale-invariant plane waves**:

$$(x^+, x^-, x^i) \to (\lambda x^+, \lambda^{-1}x^-, x^i)$$

is a symmetry. They are typical of **Penrose limits around singularities** [Blau, Borunda, O'Loughlin, Papadopoulos, 2003,2004], but also

- of FRW spacetimes ,
- near-horizon region of Dp-branes,
- near-horizon region of fundamental strings,
- ...

Isometries for $z \neq 0$

Consider $k \neq 0, 1, 2$. These plane waves have seven isometries:

$$
\begin{aligned}
L_0 &= \frac{u}{k}\partial_u - \frac{v}{k}\partial_v - w\partial_w, & \bar{L}_0 &= \frac{u}{k}\partial_u - \frac{v}{k}\partial_v - \bar{w}\partial_{\bar{w}}, \\
L_{-1} &= -\partial_w, & \bar{L}_{-1} &= -\partial_{\bar{w}}, \\
M_{10} &= w\,\partial_v + \frac{2}{k-1}u^{1-k}\partial_{\bar{w}}, & M_{01} &= \bar{w}\partial_v + \frac{2}{k-1}u^{1-k}\partial_w, \\
M_{00} &= \partial_v.
\end{aligned}
$$

For each value of k, the isometry algebra is **isomorphic to the type-D, $z = 2/k$ Carroll algebra**. The asymptotic region we are looking at now depends on k:

- For $k > 1$, we take $u \to +\infty$.
- For $k < 1$, we take $u \to 0$.

In both cases, the isometries are **projected on the corresponding exact type-D symmetry generators**.

Reflectivity around $k = 1$

As mentioned, the plane waves are reflective around the $k = 1$ axis: **the type-D $z = \frac{2}{1+\alpha}$ and $z = \frac{2}{1-\alpha}$ algebras are isomorphic**.

Explicitly, starting from $k = 1 + \alpha$, one goes to $k = 1 - \alpha$ by defining

$$u' = u, \quad v' = v - \frac{\alpha}{2} u^\alpha \vec{x}^2, \quad \vec{x}' = u^\alpha \vec{x}.$$

Then, the new generators

$$L_0' = \frac{1}{1-\alpha}(\alpha L_0 + \bar{L}_0), \qquad \bar{L}_0' = \frac{1}{1-\alpha}(\alpha \bar{L}_0 + L_0),$$
$$L_{-1}' = M_{10}, \quad \bar{L}_{-1}' = M_{01}, \quad M_{10}' = L_{-1}, \quad M_{01}' = \bar{L}_{-1}, \quad M_{00}' = M_{00}$$

satisfy the $z = \dfrac{2}{1-\alpha}$ type-D Carroll algebra.

Asymptotic symmetries: $z \neq 0$ conf. Carroll algebra

We consider, for each value of k, the following space of geometries:

$$ds^2_{(k)} = 2dudv + A_a(v,x^b)dudx^a + uB_a(v,x^b)dvdx^a + \left(uH(v,x^a)dw^2 + u\bar{H}(v,x^a)d\bar{w}^2\right) + \left(u^k + uG(v,x^a)\right) dwd\bar{w}.$$

This geometry is preserved, in the corresponding asymptotic region, by

$$L_n = \frac{n+1}{k} w^n \left(u\partial_u - v\partial_v\right) - w^{n+1}\partial_w, \quad \bar{L}_n = \frac{n+1}{k} \bar{w}^n \left(u\partial_u - v\partial_v\right) - \bar{w}^{n+1}\partial_{\bar{w}},$$
$$M_{rs} = w^r \bar{w}^s \partial_v.$$

These reproduce the $z = \frac{2}{k}$ conf. Carroll algebra: bulk uplift of $\mathfrak{ccarr}_{\frac{2}{k}}(3)$!

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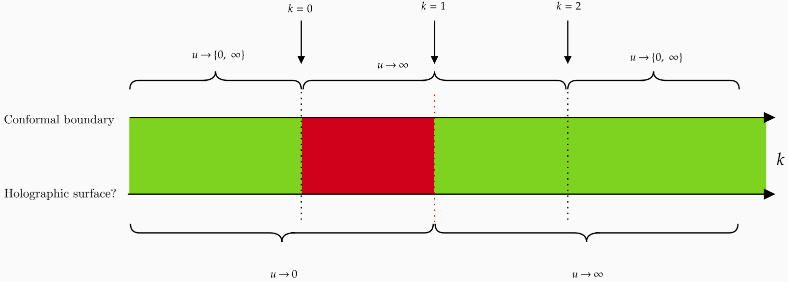
Conformal boundary of $z \neq 0$ plane waves

The one-parameter family of plane waves being **conformally flat**, we studied the conformal boundary by conformally relating to the ESU. Again, the conformal boundary is given by a **null, co-dimension one hypersurface**.



The case of flat space

For $k = 2$, we have **ten isometries**, the Poincaré algebra:

$$L_1 = uw\partial_u - vw\partial_v - w^2\partial_w - 2\frac{v}{u}\partial_{\bar{w}}, \qquad \bar{L}_1 = u\bar{w}\partial_u - v\bar{w}\partial_v - \bar{w}^2\partial_{\bar{w}} - 2\frac{v}{u}\partial_w,$$

$$M_{11} = -2\partial_u + w\bar{w}\partial_v + \frac{2w}{u}\partial_w + \frac{2\bar{w}}{u}\partial_{\bar{w}}.$$

These coordinates are known as **planar Bondi coordinates**. One reaches past and future null infinities by sending $u \to \mp\infty$.

Takeaway message

In our paper, we:

- Identified $D = 3, 4$ plane wave spacetimes whose isometries are conformal extensions of the Carroll algebra in $d = 2, 3$.
- Identified families of geometries that realise the full, infinite-dimensional conformal Carroll algebras in selected asymptotic regions.
- Showed that these asymptotic regions mostly coincide with conformal boundaries of the plane waves.
- Commented on the correlation functions of $z = 0$ conformal Carroll field theories.

This work hints towards a holographic description of gravity in plane wave spacetimes.

Outlook

This is step zero in holography for plane waves. Now, we aim to:

- Identify solutions of physical interest in the various spaces of geometries.
- Compute surface charges.
- Match correlators in the bulk with conformal Carroll correlation functions.

Other questions:

- Do other plane waves realise other conformal extensions of the Carroll algebra?
- Relation to photon ring physics?
- A conformal Carrollian infrared triangle?

Thank you!