A KINETIC MODEL OF QUORUM SENSING

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Bacterial communication poses many purely scientific and applied problems. An important component of social interaction of bacteria is quorum sensing; which refers to the regulation of bacterial function in response to fluctuations in cell-population density. Such regulation operates via the production and release of signal micromolecules (autoinducers).

Here, we present a conceptually different model from the literature, based on kinetic theory, and derive a population equation in a measure setting. We study its well-posedness first, then we show the set of steady solutions is a continuum of convex combination of Dirac masses. In the case where only one Dirac masses can exist, we are able to construct a Lyapunov functional, and prove convergence in Wasserstein metric.

We then introduce a variant of this model, where we account for small fluctuations in the membrane transport of autoinducers, which results in a parabolic regularization of the population model. We prove an existence result in $\mathrm{L}^1 \cap \mathrm{L}^\infty$ first, then we provide numerical evidence of bistability and steady solutions consists in at most 3 distributions, and lastly, we prove linear stability in a particular case thanks to a Poincaré-Wirtinger's type inequality. A challenging question still open is the connection between the stationary solutions of the parabolic regularization and the limit model with vanishing fluctuation.