
A strong approximation in L^2 for the solutions of the Maxwell system with highly oscillating periodic coefficients.

NOURELHOUDA KHEDHIRI [†], JUAN CASADO DIAZ[‡], M.-L. TAYEB[§]

Abstract

We consider a Maxwell system on \mathbf{R}^3 with highly oscillating periodic coefficients. It is known that the solutions converge in the weak-* topology of $L^\infty(0, T; L^2(\mathbf{R}^3))$ to the solution of a similar problem with constant coefficients given as the H -limits of the electric permittivity and the magnetic permeability respectively, i.e. the limit in the sense of the homogenization of linear elliptic equations with varying coefficients. However, it is not true that the elliptic corrector also provides a corrector for the solution of the Maxwell system, i.e. an approximation of the solutions in the strong topology of L^2 . We shall prove that the oscillations in the space variable also produce oscillations in the time variable. We get a corrector consisting of adding to the elliptic corrector the sum of infinitely plane waves in the fast variable. Note that related results have been previously proved for the wave equation. Our proof is based on the two-scale convergence theory for almost periodic functions. One of the novelties is to show how this contains the classical Bloch decomposition.

References

- [1] G. ALLAIRE. *Homogenization and two-scale convergence*. SIAM J. Math. Analysis 23 (1992), 1482-1518.
- [2] G. ALLAIRE, L. FRIZ. *Localization of high-frequency waves propagating in a locally periodic medium*. Proc. Roy. Soc. Edinburgh 140A (2010), 897-926.
- [3] G. ALLAIRE, M. PALOMBARO, J. RAUCH. *Diffraction behavior of the wave equation in periodic media: weak convergence analysis*. Annali di Matematica 188 (2009), 561-589.
- [4] S. BRAHIM-OTSMANE, G.A. FRANCFORT, F. MURAT. *Correctors for the homogenization of the wave and heat equations*. J. Math. Pures Appl. 71 (1992), 197-231.
- [5] M. BRASSART, M. LENZNER. *A two scale model for the periodic homogenization of the wave equation*. J. Math. Pur. Appl. 93 (2010), 474-517.
- [6] J. CASADO-DÍAZ, J. COUCE-CALVO, F. MAESTRE, J.D. MARTÍN-GÓMEZ. *Homogenization and correctors for the wave equation with periodic coefficients*. Math. Mod. Meth. Appl. Sci. 24 (2014), 1343-1388.
- [7] J. CASADO-DÍAZ, J. COUCE-CALVO, F. MAESTRE, J.D. MARTÍN-GÓMEZ. *A corrector for a wave problem with periodic coefficients in a 1D bounded domain*. ESAIM: Cont. Optim. Calc. Var. 21 (2015), 465-486.

[†]Faculty of Sciences of Tunis, University of Tunis El-Manar

[‡]Facultad de Matemáticas, University of Seville

[§]Faculty of Sciences of Tunis, University of Tunis El-Manar