Analyse de modèles d'écoulements multiphasiques

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Résumé

Modeling compressible multiphase flows is a major theoretical and numerical challenge, with significant applications in the energy sector. While significant progress has been made in the rigorous derivation of macroscopic models [4, 6], the description of complex interfacial dynamics and the construction of multi-velocity models remain challenging, particularly in highly heterogeneous regimes [5].
In this presentation, we study the viscous two-phase macroscopic model derived by homogenization [3] and its reformulation in Lagrangian coordinates. We propose implicit finite-difference schemes on a staggered grid : a semi-implicit scheme augmented with numerical viscosity, and a fully implicit scheme. For these schemes, we establish discrete energy inequalities demonstrating the dissipation of total energy (under a stability condition for the semi-implicit scheme, and without a restrictive CFL condition for the fully implicit scheme). The analysis is complemented by a continuous energy balance for the Lagrangian model, including the cases $\mu_+ = \mu_-$ and $\mu_+ \neq \mu_-$. These results provide a robust analytical and numerical basis for approximating multiphase models with dissipative structures.

Références

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