Self-similarity and diffusive limits for linear kinetic equations: a Wild sum approach

José A. Cañizo Stéphane Mischler Niccolò Tassi*

Abstract

In this talk, I will present some recent results obtained with José Cañizo and Stéphane Mischler [4] regarding the asymptotic behaviour of some linear kinetic equations without confinement of the type

$$\partial_t f + v \cdot \nabla_x f = \mathcal{L}[f]$$

where $f = f(t, x, v)$; $t \in \mathbb{R}^+$ and $(x, v) \in \mathbb{R}^d \times \mathbb{R}^d$. $\mathcal{L}$ is a linear collisional operator acting only on the velocity variable. We present the results for three types of operators and their fractional variants. The unconfined setting prevents the use of standard hypocoercivity arguments. To the best of our knowledge, only recently have there been results on the decay of the norm of the solution in a unconfined setting in [2, 1]. We improve these results by providing a more detailed description of the asymptotics, not only establishing norm decay but also identifying the exact asymptotic profile of the solution. This, moreover, allows us to show, after rescaling, a mesoscopic-to-macroscopic limit uniformly in time, improving some of the results in [3].

References

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