Title & Abstract

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Blowing-up the edge: connection formulae and stability chart of the Lamé equation

Connections between different areas of physics often provide new perspectives on difficult problems and suggest guiding principles for their solutions. In this work, we show how the correspondence between 4d $\mathcal{N} = 2$ supersymmetric gauge theories, 2d conformal field theories and quantum integrable systems can be used to study periodic spectral problems, with a particular focus on the Lamé equation.

After introducing the key ingredients, we use 2d CFT techniques to solve the connection problem of the Lamé equation in terms of semiclassical Virasoro blocks. We then analyze their analytic structure, showing how apparent poles turn into branch points through a partial resummation that combines the AGT correspondence—relating 2d conformal blocks to 4d Nekrasov partition functions—with a specific limit of the $\mathbb{C}^2$ blow-up equations satisfied by these functions. Finally, we apply these results to the study of the periodic spectral problems of the Lamé equation, highlighting the new insights gained from this perspective.