Modern aspects of Sobolev inequalities

Monday, July 6, 2026 - Friday, July 10, 2026

IHP

Scientific Program

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\mbox{Modern aspects of Sobolev inequalities}
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& \mbox{Mon 06.07} & \mbox{Tue 07.07} & \mbox{Wed 08.07} & \mbox{Thu 09.07} & \mbox{Fri 10.07} \\
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\mbox{10h-12h} &
A. Cianchi & A. Cianchi & A. Cianchi & A. Cianchi & A. Cianchi \\
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\mbox{13h45-15h45} & J. Dolbeault & J. Dolbeault & J. Dolbeault & J. Dolbeault & J. Dolbeault \\
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The Sobolev inequality,
$$ \lvert\lvert \nabla f \rvert\rvert_{L^2(\mathbb{R}^d)} \geq S_d \lvert\lvert f \rvert\rvert_{L^{2d/(d-2)}(\mathbb{R}^d)}, \quad f \in H^1(\mathbb{R}^d) \text{ and } d \geq 3, $$
is a valuable tool in many mathematical fields and appears in a variety of problems, perhaps most notably in the study of partial differential equations where it plays a vital role in understanding the existence, uniqueness, and regularity of solutions, but also differential geometry, geometric analysis, harmonic analysis, and probability theory. There are now several proofs of the Sobolev inequality, which highlight its ubiquity in mathematical analysis, as well as many generalizations, for instance to general L^p-spaces, to weighted settings, to the fractional setting, and to manifolds.

The first course by Andrea Cianchi will discuss reduction principles for Sobolev embeddings. The idea that certain Sobolev-type inequalities can be reduced to simpler one-dimensional inequalities via symmetrization methods is nowadays classical. It can be traced back at least to the works of T. Aubin, J. Moser, and G. Talenti in the seventies of the last century. Recent developments along diverse directions of this point of view will be presented. They include Sobolev embeddings for quite general families of norms, higher-order (possibly fractional) derivatives, and measures other than Lebesgue. Their treatment requires the use of various techniques, such as isoperimetric inequalities, interpolation arguments, and rearrangement methods. Special attention will be devoted to Orlicz-Sobolev spaces. The sharp form of Sobolev embeddings in the Gauss space will also be focused.

The second course by Jean Dolbeault will discuss entropy methods and stability in functional inequalities. Some functional inequalities of Sobolev-type play a crucial role in the study of nonlinear diffusion equations like the fast diffusion equation. As a typical example, optimal time dependent estimates for entropies are equivalent to optimal constants in some Gagliardo-Nirenberg inequalities. Reciprocally the fast diffusion equations can be used as a tool to investigate the set of optimal functions and the stability properties of these functions in strong norms using various extensions of the carré du champ method. Although limited to simple inequalities with good homogeneity and invariance properties, entropy methods do not rely on symmetrization and offer an alternative approach that can be used, for instance, to establish symmetry results and rigidity properties of the optimal functions in Caffarelli-Kohn-Nirenberg inequalities. The key points of these methods are the monotonicity properties of entropies, deficit and related functionals, which allow us to reduce a global problem to a local minimization issue that can be handled with spectral methods and (sometimes delicate) Taylor expansions. The goal of the lectures will be to give an overview of the strategies based on various examples and reduce the proofs to elementary questions that are easily accessible.